- **Sets and their Representations:**
- A set is a well-defined collection of objects.
- Objects, elements and members of a set are synonymous terms.
- Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.
- The elements of a set are represented by small letters a, b, c, x, y, z, etc.
- If 'a' is an element of a set A, we say that " a belongs to A" the Greek symbol ∈ (epsilon) is used to denote the phrase 'belongs to'. Thus, we write $a \in A$. If 'b' is not an element of a set A, we write $b \notin A$ and read "b does not belong to A". Thus, in the set V of vowels in the English alphabet, $a \in V$ but $b \notin V$. In the set P of prime factors of 30, $3 \in P$ but $15 \notin P$.
- **Methods Of Representing A Set :**
- **Roster (or tabular form or list form)**
- **Set-builder form (or rule form).**
  - In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }. For example, the set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.
  - In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set. For example, in the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property. Denoting this set by V, we write
    V = {x : x is a vowel in English alphabet}
- **TYPES OF SETS**
- **The Empty set :** A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol ϕ or { }. A few examples of empty sets are given below.
  - Let A = {x : 1 < x < 2, x is a natural number}. Then A is the empty set, because there is no natural number between 1 and2.
  - B = {x : $x^2 - 2 = 0$ and x is rational number}. Then B is the empty set because the equation $x^2 - 2 = 0$ is not satisfied by any rational value of x.
  - C = {x : x is an even prime number greater than 2}. Then C is the empty set, because 2 is the only even prime number.
  - D = { x : $x^2 = 4$, x is odd }. Then D is the empty set, because the equation $x^2 = 4$ is not satisfied by any odd value of x.
- **Finite and Infinite Sets :** A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.
  Consider some examples :
  - Let W be the set of the days of the week. Then W is finite.
  - Let S be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.
  - Let G be the set of points on a line. Then G is infinite.

When we represent a set in the roster form, we write all the elements of the set within braces { }. It is not possible to write all the elements of an infinite set within braces { } because the numbers of elements of such a set is not finite. So, we represent some infinite set in the roster form by writing a few elements which clearly indicate the structure of the set followed (or preceded) by three dots. For example, {1, 2, 3 . . .} is the set of natural numbers, {1, 3, 5, 7, . . .} is the set of odd natural numbers,{. . .,–3, –2, –1, 0,1, 2 ,3, . . .} is the set of integers. All these sets are infinite.

- **Equal sets :** Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write $A \neq B$.
  - Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.
  - If A = {1, 2, 3, 4} and B = {1, 1, 2, 2, 2, 3, 4, 4, 4}. Then also A = B.
  - Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

- **Singleton set :** A set, consisting of a single element is called a singleton set. The sets {0}, {5}, {–7} are singleton sets. $\{x : x + 6 = 0, x \in Z\}$ is a singleton set, because this set contains only integer namely, – 6.

- **Disjoint Sets:** Two sets are said to be disjoint iff' they have no common element.

➢ **SUBSETS:** A set A is said to be a subset of a set B if every element of A is also an element of B. In other words, $A \subset B$ if whenever $x \in A$, then $x \in B$. It is often convenient to use the symbol "⇒" which means implies. Using this symbol, we can write the definition of subset as follows:
$A \subset B$ if $x \in A \Rightarrow x \in B$. We read the above statement as "A is a subset of B if x is an element of A implies that x is also an element of B". If A is not a subset of B, we write $A \not\subset B$. For example :
  - The set Q of rational numbers is a subset of the set R of real numbers, and we write $Q \subset R$.
  - If A is the set of all divisors of 56 and B the set of all prime divisors of 56, then B is a subset of A and we write $B \subset A$.
  - Let A = {1, 3, 5} and B = {x : x is an odd natural number less than 6}. Then $A \subset B$ and $B \subset A$ and hence A = B.
  - Let A = { a, e, i, o, u} and B = { a, b, c, d}. Then A is not a subset of B, also B is not a subset of A.

➢ **Proper subset :** If $A \subset B$ and $A \neq B$, then A is called a proper subset of B, written as $A \subset B$.
  - For example : Let A = {x : x is an even natural number} and B = {x : x is a natural number}. Then, A = {2, 4, 6, 8, ...} and B = {1, 2, 3, 4, 5, ...} $\Rightarrow A \subset B$.

➢ **Theorems on subsets :**
  - **Theorem 1 :** Every set is a subset of itself.
    Proof : Let A be any set. Then, each element of A is clearly in A. Hence $A \subseteq A$.
  - **Theorem 2 :** The empty set is a subset of every set.
    Proof : Let A be any set and φ be the empty set. In order to show that $\varphi \subseteq A$, we must show that every element of φ is an element of A also. But, φ contains no element, So, every element of φ is in A. Hence $\varphi \subset A$.
  - **Theorem 3 :** The total number of subsets of a finite set containing n elements is 2^n.
    Proof : Let A be a set of n elements.
    The null set is a subset of A containing no element.
    ∴ Number of subsets of A containing no element = $1 = {}^nC_0$.
    Number of subsets of A containing 1 element = Number of groups of n elements taking 1 at a time = ${}^nC_1$.
    Number of subsets of A containing 2 elements = Number of groups of n elements taking 2 at a time = ${}^nC_2$.
    ...
    ...

    Number of subsets of A containing n elements i.e. A = $1 = {}^nCn$
    Total number of subsets of A = ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots\dots\dots + {}^nCn = 2^n$ [Using binomial theorem]

➢ **Some properties of subsets :**
  - If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
    Let $x \in A \Rightarrow x \in B$ ($A \subseteq B$) and $x \in B \Rightarrow x \in C$ ($B \subseteq C$) $\Rightarrow A \subseteq C$
  - If A = B if and only if $A \subseteq B$ and $B \subseteq A$.
    Let $A \subseteq B$ and $B \subseteq A$ ∴ $x \in A \Rightarrow x \in B$ ($A \subseteq B$) and $x \in B \Rightarrow x \in A$ ($B \subseteq A$)
    ∴ A = B
    Conversely, Let $A \subseteq B$
    ∴ $x \in A \Rightarrow x \in B$ (A = B) ∴ $A \subseteq B$
    Similarly, $x \in B \Rightarrow x \in A$ (A = B)
    ∴ $B \subseteq A$

➢ **Intervals as subsets of R :** Let a, b ∈ R and a < b. Then the set of real numbers { y : a < y < b} is called an **open interval** and is denoted by (a, b). All the points between a and b belong to the open interval (a, b) but a, b themselves do not belong to this interval. The interval which contains the end points also is called **closed interval** and is denoted by [a, b]. Thus
  - $[a, b] = \{x : a \le x \le b\}$ We can also have intervals closed at one end and open at the other, i.e.,
  - $[a, b) = \{x : a \le x < b\}$ is an open interval from a to b, including a but excluding b.
  - $(a, b] = \{ x : a < x \le b \}$ is an open interval from a to b including b but excluding a.

- $(a, b) = \{ x : a < x < b \}$ is an open interval from a to b excluding both a and b.

- **POWER SET :** The collection of all subsets of a set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set. If a set A has n elements then its power set has 2^n elements.
  If $n(A) = x$ then $n(P(A)) = 2^x$ and $n[P(P(A))] = 2^{2^x}$

- **UNIVERSAL SET :** If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set, denoted by U.

- **OPERATIONS ON SETS**

- **Union Of Sets** : The union of two sets A and B is the set of all those elements which are either in A or in B or in both. It is represented as $A \cup B$. $A \cup B = \{x: x \in A \text{ or } x \in B\}$
  - **Eg**. If $A = \{2, 3, 6, 7\}$ and $B = \{3, 4\}$ then $A \cup B = \{2, 3, 4, 6, 7\}$
- **Intersection Of Sets** : The intersection of two sets A and B is the set of all elements which are common. It is represented as $A \cap B$. $A \cap B = \{x: x \in A \text{ and } x \in B\}$
  - **Eg**. If $A = \{2, 3, 6, 7\}$ and $B = \{3, 4\}$ then $A \cap B = \{3\}$
  - **Note** : if two sets are disjoint then $A \cap B = \{ \}$ or φ
- **Difference Of Sets** : The difference of two sets A and B in this order is the set of elements which belong to A but not to B. $A - B = \{x: x \in A \text{ but } x \notin B\}$
  - Eg. If $A = \{2, 3, 6, 7\}$ and $B = \{3, 4, 6\}$ then $A - B = \{2, 7\}$ and $B - A = \{4\}$
- **Complement Of a Set :** The complement of a subset A of universal set U is the set of all elements of U which are not the elements of A. It is represented as A^C or A'. $A' = \{x: x \in U \text{ but } x \notin A\}$
  - Eg. If $U = \{1, 2, 3, \ldots, 9\}$ and $A = \{2, 4, 6\}$ then A^C (or A') $= \{1, 3, 5, 7, 8, 9\}$
- **Properties of Union Of Sets**
  - $A \cup B = B \cup A$ (Commutative law)
  - $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
  - $A \cup \varphi = A$ (Law of identity element)
  - $A \cup A = A$ (Idempotent law)
  - $U \cup A = U$ (Law of U)
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive law)
- **Properties of Intersection Of Sets**
  - $A \cap B = B \cap A$ (Commutative law)
  - $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
  - $A \cap \varphi = \varphi$ (Law of φ)
  - $U \cap A = A$ (Law of U)
  - $A \cap A = A$ (Idempotent law)
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)
- **Properties of Complement Sets**
  - $A \cup A' = U$ (Complement law)
  - $A \cap A' = \varphi$ (Complement law)
  - $(A \cup B)' = A' \cap B'$ (De Morgan's law)
  - $(A \cap B)' = A' \cup B'$ (De Morgan's law)
  - $(A')' = A$ (Law of double complementation)
  - $U' = \varphi$
  - $\phi' = U$

- **Practical Applications**
  - If A and B are finite sets such that $A \cap B = \varphi$, then $n(A \cup B) = n(A) + n(B)$.
  - If $A \cap B \neq \varphi$, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  - $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- **Venn Diagrams: (will discuss in the class)**