Vector Algebra Class XII

One Mark Questions

1. Find the direction ratios and direction cosines of $\vec{PQ}$ if P (1, 4, 7) and Q(2, 3, 5).
2. Using vectors show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.
3. Find a unit vector in the direction of the sum of the vectors a $\vec{a}=3\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+5\hat{k}$.
4. Find the value of p for which $(\hat{i}+\hat{j}+\hat{k})p$ is a unit vector.
5. Find a vector of magnitude 7 units in the direction of $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$.
6. If the vector a = 2î - 3ĵ and b = -6î + mĵ are collinear , find the value of m.
7. If a vector makes angles **α, β, γ** with x-axis, y-axis, z-axis respectively, then what is the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.
8. If the position vector "a" of the point (5,n) is such that | a | = 13, find the value of n.
9. Find the value of **'x'** for which **x(**î + ĵ + k **)** is a unit vector.
10. Find the position vector of the mid point of the vector joining the points P(2,3,4) and Q(4,1,-2).
11. Find position vectors of the points which divides the join of the points 2a - 3b and 3a - 2b externally in the ratio 2:3.
12. For what values of **'λ',** the vectors (2î - 3 ĵ) and (λî - 6ĵ) are parallel ?
13. Find a unit vector perpendicular to both $\vec{a}=3\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+4\hat{k}$.
14. Find a vector of magnitude 6 units perpendicular to both $\vec{a}=3\hat{i}-2\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+4\hat{k}$.
15. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-5\hat{k}$.
16. For what value of λ are the vectors $\vec{a}=\hat{i}-\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-5\hat{k}$ orthogonal?
17. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ show that $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$.
18. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}-\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$.
19. Find the projection of $\hat{i}+3\hat{j}+7\hat{k}$ on $7\hat{i}-\hat{j}+8\hat{k}$.
20. Find the value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$
21. Define vector product of two vectors . If $|\vec{a}|$ =2, $|\vec{b}|$ = 5 and $|\vec{a}\times\vec{b}|=8$, find $\vec{a}.\vec{b}$
22. i) If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ find the angle between $\vec{a}$ and $\vec{b}$.

    ii) If $|\vec{a}|=\sqrt{3}$ and $|\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is 60^0 find $\vec{a}\bullet\vec{b}$

    iii) If $|\vec{a}|$ =5, $|\vec{b}|$ = 13 and $|\vec{a}\times\vec{b}|=25$, find $\vec{a}.\vec{b}$

    iv)) If $|\vec{a}|$ = 2, $|\vec{b}|$ = 5 and $\vec{a}.\vec{b}$ =10 find $|\vec{a}-\vec{b}|$.

    v) If $|\vec{a}|$ = 3, $|\vec{b}|$ = 3 and $\vec{a}.\vec{b}$ =1, find the angle between $\vec{a}$ and $\vec{b}$.

    vi) If $|\vec{a}|$ = 2, $|\vec{b}|$ = 3 and $\vec{a}.\vec{b}$ =3, find the projection of $\vec{b}$ on $\vec{a}$.

    vii) If $|\vec{a}|$ = 1, $|\vec{b}|$ = 2 and $|\vec{a}\times\vec{b}|=\sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

viii) If $|\vec{a} \bullet \vec{b}| = |\vec{a} \times \vec{b}|$ find the angle between $\vec{a}$ and $\vec{b}$.

ix) If $|\vec{a}|$ = 3, $|\vec{b}|$ = 2 and $\vec{a}.\vec{b}$ = 6 find $|\vec{a}+\vec{b}|$.

x) If $|\vec{a}|$ = 2, $|\vec{b}|$ = $\sqrt{3}$ and $\vec{a}.\vec{b}$ = $\sqrt{3}$, find $|\vec{a} \times \vec{b}|$

xi) If $|\vec{a}|$ = 10, $|\vec{b}|$ = 2 and $\vec{a}.\vec{b}$ = 12, find $|\vec{a} \times \vec{b}|$.

xii) If $(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+3\hat{j}+p\hat{k}) = 0$, find the value of p.

23. If $\vec{a} = \hat{i}+2\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}+\hat{j}-\hat{k}$ find a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

24. i) Find $|\vec{a}|$ and $|\vec{b}|$ if $(\vec{a}+\vec{b}) \bullet (\vec{a}-\vec{b}) = 3$ and $|\vec{a}| = 2|\vec{b}|$

ii) Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x}+\vec{a}) \bullet (\vec{x}-\vec{a}) = 8$.

25. Find the value of λ for which the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}+\lambda\hat{j}+3\hat{k}$ are i) parallel ii) perpendicular.

4 Marks Questions

26. If $\vec{a} = 3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}-2\hat{j}+4\hat{k}$ find i) a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

ii) a vector of magnitude 10 units in the direction of $\vec{a}-\vec{b}$.

iii) the area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$.

iv) the area of a parallelogram whose diagonals are $\vec{a}$ and $\vec{b}$.

27. The vertices of a triangle are A(0, -1, -2), B(3, 1, 4) and C(5, 7, 1). Find the i) measure of angle ABC ii) area of ABC.

28. If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

29. If $\vec{a} = \hat{i}-\hat{j}, \vec{b} = 3\hat{j}-\hat{k}$ and $\vec{c} = 7\hat{i}-\hat{k}$. Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 1$

30. If $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$, $\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$. Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 18$.

31. The dot product of a vector with the vectors $\hat{i}-\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}-3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$ are 4, 0 and 2 respectively. Find the vector.

32. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. Find the value of λ.

33. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ and $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$ find the value of λ.

34. Find a unit vector perpendicular to the plane ABC where position vector of points A, B and C are $\hat{i}-2\hat{j}-\hat{k}$, $\hat{i}+2\hat{j}-4\hat{k}$ *and* $2\hat{i}+2\hat{j}-3\hat{k}$ respectively.
35. Show that the area of the parallelogram with diagonals $3\hat{i}+\hat{j}-2\hat{k}$ *and* $\hat{i}-3\hat{j}+4\hat{k}$ *is* $5\sqrt{3}$ square units.
36. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ *and* $3\hat{i}-4\hat{j}-4\hat{k}$ form the sides of a right angled triangle.
37. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}\,|=3$ *and* $|\vec{b}|=5$ *and* $|\vec{c}|=7$, show that the angle between $\vec{a}$ *and* $\vec{b}$ is 60^0.
38. If $\vec{a}, \vec{b}, \vec{c}$ are such that each is perpendicular to the sum of the other two and $|\vec{a}|$ =3 $|\vec{b}|$ = 4 and $|\vec{c}|$ = 5 ,find $|\vec{a}+\vec{b}+\vec{c}|$
39. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}\,|=3$ *and* $|\vec{b}|=4$ *and* $|\vec{c}|=5$,find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.
40. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then prove that $\vec{a}+\vec{b}+\vec{c}$ makes equal angles with $\vec{a}, \vec{b}$ *and* $\vec{c}$.
41. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that each one of them is perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.
42. Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5\hat{k}$ as a sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+k$ and the other is perpendicular to $\vec{b}$.
43. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.
44. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are the position vectors of the vertices A,B,C of a triangle ABC, prove that its area is $\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$
45. Prove that for any vector $\vec{a}$, $\vec{a}$ = $(\vec{a}.\hat{i})i+(\vec{a}.\hat{j})\hat{j}+(\vec{a}.\hat{k})\hat{k}$
46. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}; \vec{b}\times\vec{c}=\vec{a}$,Prove that $\vec{a}, \vec{b}$ *and* $\vec{c}$ are mutually perpendicular and $|\vec{b}|=1, \quad |\vec{c}|=|\vec{a}|$
47. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ *and* $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}, \vec{a}\neq 0,$ *then show that* $\vec{b}=\vec{c}$.
48. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ *and* $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then prove that $\vec{a}-\vec{d}$ *is parallel* to $\vec{b}-\vec{c}$.
49. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}\times\vec{b}|^2=\vec{a}^2\vec{b}^2-(\vec{a}.\vec{b})^2$
50. The sum of two unit vectors is a unit vector. Prove that magnitude of their difference is $\sqrt{3}$.