CLASS XII MATHS

3 – D Geometry (Lines)

1. A vector $\vec{r}$ is inclined to x – axis at 45° and to z – axis at 60°. If $|\vec{r}|$ = 6 units find $\vec{r}$.

2. Write the vector equation of the following lines:

   i) $\frac{x-1}{5}=\frac{y+2}{3}=\frac{z+1}{-1}$ ii) $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$ iii) $\frac{x+3}{-2}=\frac{y-2}{1}=\frac{2-z}{3}$

   iv) $\frac{x}{1}=\frac{y+3}{-2}, z=3$ v) $\frac{1-x}{3}=\frac{7y-14}{21}=\frac{2z+3}{3}$ vi) $\frac{x+3}{2}=\frac{2y-5}{4}=\frac{6-z}{2}$

3. Write the Cartesian equation of the following lines:

   i) $\vec{r}=3\hat{i}-5\hat{j}+\hat{k}+\lambda(\hat{i}+2\hat{j}-\hat{k})$ ii) $\vec{r}=-2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

   iii) $\vec{r}=4\hat{i}-2\hat{j}-\hat{k}+\lambda(-\hat{i}+\hat{j}-2\hat{k})$ iv) $\vec{r}=-\hat{i}+3\hat{j}-4\hat{k}+\mu(\hat{i}-\hat{j}+4\hat{k})$

   v) $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ vi) $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

4. Find the equation of the line passing through the given point and parallel to the given line.

   i) (5, -2, 4); $2\hat{i}-\hat{j}+3\hat{k}$. ii)(2, 1, -3); $\hat{i}+2\hat{j}+\hat{k}$. iii)(-1, 4, 2); $\frac{x-1}{5}=\frac{y+2}{3}=\frac{z+1}{-1}$

   iv) (1, 2, 3); $\frac{x-6}{12}=\frac{y-2}{6}=\frac{z+7}{5}$ v) (1, 3, -5); $\frac{x-3}{1}=\frac{y+3}{5}=\frac{2z-5}{3}$

5. Find the equation of the line passing through the two points:

   i) (3, 4, - 7) and (5, 1, 6) ii) (3, 4, 1) and (5, 1, 6) iii) (2, -1, 1) and (1, 2, -3)

   iv) (- 2, 1, 3) and (3, 1, - 2) v) (1, - 2, 1) and (0, - 2, 3) vi) (0, 1, - 2) and (- 1, - 1, - 3)

6. Find the equation of the line joining the points whose position vectors are $3\hat{i}+2\hat{j}-4\hat{k}$. and $\hat{i}-2\hat{j}+3\hat{k}$.

7. Find the equation of the line passing through the point whose position vector is $3\hat{i}-\hat{j}+2\hat{k}$ and is parallel to the line which passes through the points whose position vectors are $2\hat{i}+\hat{j}-3\hat{k}$ and $4\hat{i}-2\hat{j}+\hat{k}$.

8. i) Find the coordinate of the point where the line through (5, 1, 6) & (3, 4, 1) crosses the yz – plane.

   ii) Find the coordinate of the point where the line through (5, 1, 6) & (3, 4, 1) crosses the xy – plane.

   iii) Find the coordinate of the point where the line through (1, - 1, 2) & (3, 2, - 3) meets the yz – plane.

   iv) Find the coordinate of the point where the line through (2, - 3, 1) & (3, 4, 5) cuts the plane 2x – 3y + 4z = 2.

9. Find the values of λ and μ if the following set of points are collinear:

   i) (- 1, 4, - 2), (0, 2, - 1) & (λ , μ , 1) ii) (λ , μ , - 6), (3, 2, - 4), (9, 8, - 10)

10. Find the angle between the following pairs of lines:

    i) $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda(3\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}-2\hat{k})$

    ii) $\vec{r}=4\hat{i}-\hat{j}+\lambda(\hat{i}+2\hat{j}-2\hat{k})$ and $\vec{r}=\hat{i}-\hat{j}+2\hat{k}-\mu(2\hat{i}+4\hat{j}-4\hat{k})$

    iii) $\vec{r}=2\hat{i}-3\hat{j}+t(-\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=5\hat{j}-2\hat{k}+s(3\hat{i}-\hat{j})$

iv) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{3-z}{3}$ & $\frac{x+3}{-1}=\frac{y+5}{8}=\frac{z-1}{4}$

v) $\frac{x-5}{2}=\frac{y+3}{1}=\frac{1-z}{3}$ & $\frac{x}{3}=\frac{y-1}{2}=\frac{5+z}{-1}$

vi) $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ & $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{5+z}{2}$

vii) $x=\frac{y}{0}=\frac{z}{-1}$ & $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

11. Show that the following lines are perpendicular:

i) $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$ & $\frac{x+2}{2}=\frac{y-4}{4}=\frac{5+z}{2}$

ii) 2x = 3y = - z & 6x = - y = - 4z

12. Find the equation of the line satisfying the given conditions:

i) through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ & $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

ii) through the point (-1, 2, 3) and perpendicular to the lines $\frac{x}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ & $\frac{x+3}{-1}=\frac{y+2}{2}=\frac{z-1}{3}$

iii) through the point (-1, 3, - 2) and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ & $\frac{x+2}{-3}=\frac{2y-1}{2}=\frac{z+1}{5}$

13. Find the shortest distance between the lines whose vector equations are:

i) $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

ii) $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

iii) $\vec{r}=(3-t)\hat{i}+(4+2t)\hat{j}+(t-2)\hat{k}$ and $\vec{r}=(1+s)\hat{i}+(3s-7)\hat{j}+(2s-2)\hat{k}$

iv) $\vec{r}=(3\hat{i}-15\hat{j}+9\hat{k})+\lambda(2\hat{i}-7\hat{j}+5\hat{k})$ and $\vec{r}=(-\hat{i}+\hat{j}+9\hat{k})+\mu(2\hat{i}+\hat{j}-3\hat{k})$

v) $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}+\hat{j}-\hat{k})$

vi) $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

vii) $\vec{r}=(\lambda-1)\hat{i}+(\lambda+1)\hat{j}-(\lambda+1)\hat{k}$ and $\vec{r}=(1-\mu)\hat{i}+(2\mu-1)\hat{j}+(\mu+2)\hat{k}$

viii) $\vec{r}=(2\lambda+1)\hat{i}-(\lambda+1)\hat{j}+(\lambda+1)\hat{k}$ and $\vec{r}=(3\mu+2)\hat{i}-(5\mu+5)\hat{j}+(2\mu-1)\hat{k}$

ix) $\vec{r}=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

x) $\vec{r}=(\hat{i}-2\hat{j}+3\hat{k})+\lambda(-\hat{i}+\hat{j}-2\hat{k})$ and $\vec{r}=(\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}-2\hat{k})$

14. Find the shortest distance between lines whose Cartesian equations are:

i) $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ and $x=\frac{y-2}{2}=\frac{z-3}{3}$

ii) $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{x-1}{1}$ and $\frac{x-2}{2}=\frac{y+1}{1}=\frac{z+1}{2}$

iii) $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{5-y}{2}=\frac{z-7}{1}$

iv) $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$

v) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ & $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

vi) $\frac{x-2}{-3}=\frac{y+2}{-1}=\frac{3z+1}{3}$ & $\frac{2x-2}{6}=\frac{y+4}{4}=\frac{2-z}{2}$

15. Determine whether the following lines intersect. If so, find the point of intersection.

i) $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}-\hat{k})$

ii) $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k})$

iii) $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$

iv) $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ & $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

v) $\frac{6-x}{6}=\frac{y+4}{4}=\frac{4-z}{8}$ & $\frac{x+1}{2}=\frac{y+2}{4}=\frac{z+3}{-2}$

vi) $x=\frac{y-2}{2}=\frac{z+3}{2}$ & $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$

vii) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y+1}{5}=z$

16. Show that the following lines are coplanar:

i) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ & $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}$

ii) $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ & $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

iii) $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ & $\frac{x}{1}=\frac{7-y}{3}=\frac{z+7}{2}$

iv) $\frac{x-1}{2}=\frac{y-3}{4}=-z$ and $\frac{x-4}{3}=\frac{1-y}{2}=z-1$

17. Find the length and foot of the perpendicular of the given point from the given line:

i) (3, 4, 5) ; $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-1}{3}$

ii) (- 1, 3, 9); $\frac{13-x}{5}=\frac{y+8}{8}=\frac{31-z}{1}$

iii) (2, 4, -1) ; $\vec{r}=(-5\hat{i}-3\hat{j}+6\hat{k})+\lambda(\hat{i}+4\hat{j}-9\hat{k})$

iv) (0, 2, 3); $\vec{r}=(-3\hat{i}+\hat{j}-4\hat{k})+\lambda(5\hat{i}+2\hat{j}+3\hat{k})$

v) (2, -1, 5) ; $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$

vi) (2, 3, 4) ; $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$

vii) (1, 0, 0) ; $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$

18. Find the image of the given point in the given mirror line:

i) (1, 6, 3) ; $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

ii) (1, -2, 1); $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z+3}{2}$

iii) (0, 2, 3); $\vec{r}=(-3\hat{i}+\hat{j}-4\hat{k})+\lambda(5\hat{i}+2\hat{j}+3\hat{k})$

iv) (1, 2, 1); $\vec{r}=(\hat{i}+4\hat{j}+6\hat{k})+\lambda(2\hat{i}-\hat{k})$

19. Find value of k so that the following pair of lines are perpendicular:

i) $\frac{1-x}{3}=\frac{7y-14}{2k}=\frac{z-3}{2}$ and $\frac{7-7x}{3k}=\frac{y-5}{1}=\frac{6-z}{5}$

ii) $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{6-z}{5}$

20. Find the distance between the following lines:

$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

Answers:

1. $3\sqrt{2}\hat{i}\pm 3\hat{j}+3\hat{k}$

2. i) $\vec{r}=(\hat{i}-2\hat{j}-\hat{k})+\lambda(5\hat{i}+3\hat{j}-\hat{k})$ ii) $\vec{r}=(-3\hat{i}+5\hat{j}-6\hat{k})+\lambda(2\hat{i}+4\hat{j}+2\hat{k})$

iii) $\vec{r}=(-3\hat{i}+2\hat{j}+2\hat{k})+\lambda(-2\hat{i}+\hat{j}-3\hat{k})$ iv) $\vec{r}=(-3\hat{j}+3\hat{k})+\lambda(\hat{i}-2\hat{j})$ v) $\vec{r}=(\hat{i}+2\hat{j}-\frac{3}{2}\hat{k})+\lambda(-3\hat{i}+3\hat{j}+\frac{3}{2}\hat{k})$

vi) $\vec{r} = (-3\hat{i} + \frac{5}{2}\hat{j} + 6\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ **3**. i) $\frac{x-3}{1} = \frac{y+5}{2} = \frac{z-1}{-1}$ ii) $\frac{x+2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$

iii) $\frac{x-4}{-1} = \frac{y+2}{1} = \frac{z+1}{-2}$ iv) $\frac{x+1}{1} = \frac{y-3}{-1} = \frac{z+4}{4}$ v) $\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$

vi) $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$ **4.** i) $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$ ii) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{1}$

iii) $\frac{x+1}{5} = \frac{y-4}{3} = \frac{z-2}{-1}$ iv) $\frac{x-1}{12} = \frac{y-2}{6} = \frac{z-3}{5}$ v) $\frac{x-1}{1} = \frac{y-3}{5} = \frac{z+5}{3/2}$

5. i) $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z+7}{13}$ ii) $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ iii) $\frac{x-2}{-1} = \frac{y+1}{3} = \frac{z-1}{-4}$

iv) $\frac{x+2}{5} = \frac{y-1}{0} = \frac{z-3}{-5}$ v) $\frac{x-1}{-1} = \frac{y+2}{0} = \frac{z-1}{2}$ vi) $\frac{x}{-1} = \frac{y-1}{-2} = \frac{z+2}{-1}$

6. $\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(-2\hat{i} - 4\hat{j} + 7\hat{k})$ **7.** $\vec{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k})$

8. i) (0, 17/2, -13/2) ii) (13/5, 23/5, 0) iii) (0, -5/2, 9/2) iv) (7, 32, 21)

9. i) 2 & -2 ii) 5 & 4 **10.** i) 90º ii) 180º iii) $\cos^{-1}\left(\frac{-\sqrt{15}}{6}\right)$ iv) $\cos^{-1}\left(\frac{5\sqrt{22}}{99}\right)$

v) $\cos^{-1}\left(\frac{11}{14}\right)$ vi) 90º vii) $\cos^{-1}\left(\frac{-1}{5}\right)$ **12.** i) $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda\ (2\hat{i} - 7\hat{j} + 4\hat{k})$

ii) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\ (-5\hat{i} - 4\hat{j} + \hat{k})$ iii) $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda\ (\hat{i} - 2\hat{j} + \hat{k})$

13. i) $\frac{3}{\sqrt{2}}$ ii) $\frac{10}{\sqrt{59}}$ iii) $\sqrt{35}$ iv) $4\sqrt{3}$ v) $\frac{1}{\sqrt{14}}$ vi) $\frac{20}{\sqrt{219}}$ vii) $\frac{5}{\sqrt{2}}$

viii) $\frac{21}{\sqrt{59}}$ ix) 14 x) $\frac{14}{\sqrt{29}}$ **14.** i) $\sqrt{6}$ ii) $\frac{3}{\sqrt{2}}$ iii) $2\sqrt{29}$ iv) $\frac{9}{\sqrt{195}}$

v) $\frac{3}{\sqrt{6}}$ vi) $\frac{13}{\sqrt{94}}$ **15.** i) no ii) Yes; (4, 0, -1) iii) no iv) yes; (1, 3, 2) v) yes; (0, 0, -4)

vi) yes; (2, 6, 3) vii) yes; (-1, -1, -1) **17.** i) $\frac{\sqrt{17}}{2}$ ii)21 iii) 7 iv) $\sqrt{21}$; (2, 3, -1) v) $\sqrt{14}$; (1, 2, 3)

vi) $\frac{3\sqrt{101}}{7};\left(\frac{170}{49}, \frac{78}{49}, \frac{10}{49}\right)$ vii) $\sqrt{24}$; (3, - 4, - 2) **18.** i) (1, 0, 7) ii) $\left(\frac{39}{7}, \frac{-6}{7}, \frac{-37}{7}\right)$ iii) (4, 4, -5)

iv) (5, 6, 9) **19.** i) k = 70/11 ii) k = - 10/7 **20.** $\frac{\sqrt{293}}{7}$