Class : XII Mathematics Assignment: 3 – D GEOMETRY

1. Find the Cartesian equation of the following planes:
   i) $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=2$ ii) $\vec{r}\cdot(3\hat{i}+2\hat{j}-4\hat{k})=1$ iii) $\vec{r}\cdot(3\hat{i}-4\hat{j}+\hat{k})=3$ iv) $\vec{r}\cdot(-\hat{i}+2\hat{j}+3\hat{k})=\left(2\hat{i}-\hat{k}\right).\left(-\hat{i}+2\hat{j}+3\hat{k}\right)$
2. Write the vector equation of the following planes:
   i) 3x – 5y + 4z = 1 ii) 2x – z + 4 = 0 iii) x – 2y + 2z – 9 = 0
3. Find a unit normal vector to the following planes:
   i) x + 2y + 3z – 6 = 0 ii) 2x – 3y + 6z + 14 = 0 iii) $\vec{r}\cdot(5\hat{i}-4\hat{j}+20\hat{k})+3=0$
4. Reduce the following equations to normal form and hence find the length of perpendicular from the origin to the plane:
   i) x – 2y + 2z – 9 = 0 ii) $\vec{r}\cdot(2\hat{i}+3\hat{j}-6\hat{k})+6=0$ iii) 2x – 3y + 6z + 14 = 0
5. Find the vector equation of the plane in the dot product form, through the point (2, -1, 5) and perpendicular to the vector $\hat{i}+2\hat{j}+\hat{k}$.
6. Find the cartesian equation of the plane in the normal form, through the point (- 2, - 6, 5) and perpendicular to the vector $3\hat{i}+2\hat{j}+6\hat{k}$.
7. Find the vector equation of a plane which is at the given distance from the origin and normal to the given vector.
   i) 5 units, $2\hat{i}-\hat{j}+3\hat{k}$ ii) 3 units, $\hat{i}+\hat{k}$ iii) 5 units, $3\hat{i}-2\hat{j}+4\hat{k}$
8. Find the equation of the plane passing through the given point and perpendicular to the given vector:
   i) (2, 1, -1), $\hat{i}-2\hat{j}+3\hat{k}$ ii) origin, $5\hat{i}-3\hat{k}$
   iii) (2, -3, 1); normal passes through (3, 4, - 1) and (2, -1, 5).
9. Find the equation of the plane passing through the point (- 1, 2, 1) and perpendicular to the line through the points (- 3, 1, 2) and (2, 3, 4).
10. Find the coordinates of the foot of the perpendicular drawn from the origin:
    i) 2x + 3y + 4z - 58 = 0 ii) 3y + 4z + 50 = 0 iii) x + y + z + 3 = 0
11. Find the equation of the plane when the foot of the perpendicular drawn from origin to the plane is given:
    i) foot of perpendicular = (3, - 4, 2) ii) (4, - 2, - 5)
12. Find the equation of the plane passing through the following points:
    i) (1, 1, 1), (1, -1, 1) and (- 7, - 3, - 5) ii) (1, - 2, 5), (0, - 5, - 1) and (- 3, 5, 0)
    iii) (0, 0, 0), (2, 1, 0) and (1, 1, 2) iv) (0, 3, 0), (2, 1, 0) and (1, 1, 2)
    v) (4, 5, 1), (3, 9, 4) and (- 4, 4, 4) vi) (3, - 1, 2), (4, -1, - 1) and (2, 0, 2)
    vii) (0, 6, 4), (1, 5, 4) and (4, - 2, 0) viii) (2, 5, - 3) , (- 2, - 3, 5) and (5, 3, -3).
13. Find the vector equation of the plane in the dot product form, through the points (2, - 1, 5), (1, - 3, 4) and (4, - 2, 0).
14. Prove that the points (0, 6, 4), (1, 5, 4) and (2, 4, 4) and (2, 6, 2) are coplanar. Also find the equation of the plane containing these points.
15. Show that the following points are coplanar: Also find the equation of the plane containing these points.
    i) (0, - 1, 0), (2, 1, - 1), (1, 1, 1) and (3, 3, 0) ii) (- 1, 4, - 3), (3, 2, - 5), (- 3, 8, - 5) and (- 3, 2, 1)
16. Prove that the points with position vectors $6\hat{i}+2\hat{j}-\hat{k}$, $2\hat{i}-\hat{j}+3\hat{k}$, $-\hat{i}+2\hat{j}-4\hat{k}$ and $-12\hat{i}-\hat{j}-3\hat{k}$ are coplanar.

17. Find the value of λ so that the points with the position vectors $-\hat{j}+\hat{k},\ 2\hat{i}-\hat{j}-\hat{k},\ \hat{i}+\lambda\hat{j}+\hat{k}$ and $3\hat{j}+3\hat{k}$ are coplanar.
18. Show that the plane through (1, 1, 1), (1, -1, 1) and (- 7, - 3, - 5) is perpendicular to xz – plane.
19. Find the intercepts cut off by the following planes with the axes:
    i) x + 2y – 3z = 9 ii) 2x – 3y + z = - 6 iii) 2x + 6y – 3z + 3 = 0
20. A plane meets the coordinate axes at A, B and C such that centroid of Δ ABC is (3, 4, - 6). Find the equation of plane.
21. A plane meets the coordinate axes at A, B and C such that the centroid of Δ ABC is (- 3, 2, 4). Find equation of the plane.
22. Find the equation of the plane which makes equal intercepts on the axes and passes through the point (1, - 3, 6).
23. A plane meets the x, y and z axes at A, B and C respectively such that the centroid of the triangle ABC is (p, q, r).

    Prove that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$
24. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B and C. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$
25. Find the equation of the plane containing the following lines:

    i) $\vec{r}=(3\hat{i}-\hat{j}+4\hat{k})+\lambda(12\hat{i}-\hat{k})$
    $\vec{r}=(\hat{i}+\hat{j}-2\hat{k})+\mu(\hat{j}+2\hat{k})$

    ii) $\vec{r}=(3\hat{i}-3\hat{j})+\lambda(\hat{j}+\hat{k})$
    $\vec{r}=(2\hat{i}+\hat{j}-3\hat{k})+\mu(\hat{i}+\hat{j}+\hat{k})$

    iii) $\vec{r}=3\hat{i}+t(\hat{i}-\hat{j}+\hat{k})$
    $\vec{r}=(\hat{j}+2\hat{k})+s(2\hat{i}+3\hat{k})$

    iv) $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z}{2}$ and $\frac{x+2}{0}=y=\frac{z+1}{3}$

    v) $\frac{x}{1}=\frac{y}{2}=\frac{z-1}{0}$ and $\vec{r}=3\hat{i}-2\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$

    vi) $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z}{-1}$ and $\frac{x+2}{3}=\frac{y-2}{-2}=\frac{z+2}{3}$

    vii) $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

    viii) $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$

    ix) $\vec{r}=\hat{i}+\hat{j}+t(-\hat{i}+\hat{j}-2\hat{k})$
    $\vec{r}=\hat{i}+\hat{j}+s(\hat{i}+2\hat{j}-\hat{k})$

    x) $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$

    xi) $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(4\hat{i}-2\hat{j}+2\hat{k})$ & $\frac{x-2}{2}=\frac{y-4}{-1}=\frac{z-6}{1}$
26. Find the equation of the plane through the point of intersection of the planes x + 2y – z = 1 and x + y + 2z + 1 = 0 perpendicular to the plane 2x + 2y – z = 0.
27. i) Find the equation of the plane passing through the origin and the line of intersection of the planes x – 2y + 3z + 4 = 0 and x – y + z + 3 = 0.

    ii) Find the equation of the plane passing through the origin and the line of intersection of the planes x + 2y – 5z + 1 = 0 and 2x – y + 3z – 11 = 0.

    iii) Find the equation of the plane passing through (2, 1, - 1) and the intersection of the planes

$\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=0$ and $\vec{r}\cdot(\hat{j}+2\hat{k})=0$

iv) Find the equation of the plane passing through (1, 2, - 3) and the intersection of the planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=5$ and $\vec{r}\cdot(\hat{i}-3\hat{k})=2$.

v) Find the equation of the plane passing through (1, 1, 1) and the intersection of the planes $\vec{r}\cdot(\hat{i}-\hat{j}+3\hat{k})+1=0$ and. $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})-5=0$.

vi) Find the equation of the plane passing through (- 2, 1, 3) and the intersection of the planes $\vec{r}\cdot(-\hat{i}+3\hat{j})=2$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+5\hat{k})-1=0$.

vii) Find the equation of the plane passing through (1, 1, 1) and the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$.

viii) Find the equation of the plane passing through (0, 0, 1) and the intersection of the planes $2x + 3y - z = - 4$ and $3x - 2y + 4z - 1 = 0$.

ix) Find the equation of the plane through the point (1, - 2, 0) and through the intersection of the planes $3x + 2y – z = 4$ and $x + y – 2z – 2 = 0$.

x) Find the equation of the plane through the intersection of the planes $x + 2y + z = 4$ and $3x + y – z – 1 = 0$ and through the origin.

28. i) Find the equation of the plane passing through the line of intersection of the planes $x + y + z + 1 = 0$ and $2x - y + 3z + 4 = 0$ and parallel to the line $\vec{r}=(3\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}+\hat{j}-3\hat{k})$.

ii) Find the equation of the plane passing through the line of intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to the line $\vec{r}=\hat{i}+3\hat{j}+5\hat{k}+\lambda(2\hat{i}+4\hat{j}+5\hat{k})$.

iii) Find the equation of the plane passing through the line of intersection of the planes $2x – 3y + 4z = 0$ and $3x + y – z = 5$ and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-2}=\frac{z-1}{3}$.

iv) Find the equation of the plane passing through the line of intersection of the planes $4x – y + z = 0$ and $x + y – z = 4$ and parallel to the line with direction ratios 2, 1, 1.

29. i) Find the equation of the plane passing through the line of intersection of the planes $2x – 3y + z – 4 = 0$ and $x – y + z + 1 = 0$ and perpendicular to the plane $x + 2y – 3z + 6 = 0$.

ii) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z – 4 = 0$ and $2x + y – z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 1 = 0$.

iii) Find the equation of the plane passing through the line of intersection of the planes $x + y + z = 0$ and $3x – y +2z – 1 = 0$ and perpendicular to the plane $2x + 3y – 5z + 10 = 0$.

iv) Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+\hat{k})+8=0$.

v) Find the equation of the plane through the intersection of the planes $x + 2y – z = 1$ and $x + y + 2z + 1 = 0$ and perpendicular to the plane $2x + 2y – z = 0$.

30. i) Find the equation of the plane passing through (3, 4, -1) and parallel to the plane $\vec{r}\cdot(2\hat{i}-3\hat{j}+5\hat{k})+7=0$

ii) Find the equation of the plane passing through (1, 4, - 2) and parallel to the plane – 2x + y – 3z = 0.

iii) Find the equation of the plane passing through (1, 4, - 2) and parallel to the plane – 2x + y – 3z = 7.

31. i) Find the equation of the plane passing through (2, - 3, 0) and parallel to the lines

$\vec{r}=(3\hat{i}-2\hat{j}+3\hat{k})+\lambda(-\hat{i}+\hat{j}-2\hat{k})$ and $\vec{r}=2\hat{i}+\hat{k}+\mu(3\hat{j}+4\hat{k})$

ii) Find the equation of the plane passing through (1, 0, 1) and parallel to the lines

$\vec{r}=\hat{i}-2\hat{j}+\hat{k}+\lambda(2\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=\hat{j}+2\hat{k}+\mu(\hat{i}-\hat{j}+2\hat{k})$

iii) Find the equation of the plane passing through (1, 1, 1) and parallel to the lines

$\frac{x-3}{1}=\frac{y}{-2}=\frac{z+1}{3}$ and $\frac{x}{2}=\frac{y+2}{3}=\frac{z}{-4}$

iv) Find the equation of the plane passing through (1, 0, 0) and parallel to the lines

$\vec{r}=-3\hat{i}+2\hat{j}+\lambda(\hat{i}-\hat{j})$ and $\frac{x+1}{-2}=\frac{1-y}{1}=\frac{z}{4}$

v) Find the equation of the plane passing through (2, - 3, 2) and parallel to the lines

$\vec{r}=2\hat{i}-\hat{k}+\lambda(3\hat{i}+2\hat{k})$ and $\vec{r}=\hat{j}+3\hat{k}+\mu(\hat{i}-\hat{j}-\hat{k})$

vi) Find the equation of the plane passing through (2, 0, - 1) and parallel to the lines

$\frac{x}{-3}=\frac{y-2}{4}=\frac{z+1}{1}$ and $\frac{x-4}{1}=\frac{1-y}{2}=2z$

32. i) Find the equation of the plane passing through (1, - 1, 2) and perpendicular to each of the planes 3x + 2y – 3z – 1 = 0 and 5x – 4y + z = 5.

ii) Find the equation of the plane passing through (- 1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

iii) Find the equation of the plane passing through (2, - 1, 5) and perpendicular to each of the planes

$\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=1$ and $\vec{r}\cdot(3\hat{i}-4\hat{j}+\hat{k})=5$

iv) Find the equation of the plane passing through (1, 1, - 1) and perpendicular to each of the planes

$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=7$ and $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=0$

v) Find the equation of the plane passing through a point with position vector $\vec{a}=2\hat{i}+5\hat{j}-8\hat{k}$ and perpendicular to each of the planes $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})+1=0$ and $\vec{r}\cdot(4\hat{i}+\hat{j}-2\hat{k})+6=0$

vi) Find the equation of the plane passing through origin and perpendicular to each of the planes

$\vec{r}\cdot(\hat{i}+2\hat{j}+2\hat{k})-5=0$ and $\vec{r}\cdot(3\hat{i}+3\hat{j}-2\hat{k})-8=0$

vii) Find the equation of the plane through the points (1, - 3, 4) and perpendicular to the planes

$\vec{r}\cdot(3\hat{i}+2\hat{j}+6\hat{k})=3$ and 3x – 2y + 2z = 1

33. i) Find the equation of the plane through the point (- 1, - 2, 0) and containing the line $\frac{x+1}{-2}=\frac{y-2}{3}=\frac{z+3}{2}$

ii) Find the equation of the plane through the point (2, - 2, 1) and containing the line

$\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(\hat{i}+2\hat{j})$

iii) Find the equation of the plane passing through (0, 7, - 7) and containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$

34. Find the equation of the plane passing through the point of intersection of lines $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z+1}{2}$ and $\frac{x+3}{2}=\frac{y-2}{3}=\frac{z+7}{-2}$ and is parallel to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-1}=\frac{y}{4}=\frac{z+2}{-3}$

35. i) Find the equation of the plane passing through the points (3, 2, 2) and (1, 0, - 1) and parallel to the line $\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{3}$

ii) Find the equation of the plane through the points (0, 6, 4), (1, 5, 4) and parallel to the line $\vec{r}=(1-2t)\hat{i}+(3-2t)\hat{j}+(3t-2)\hat{k}$

iii) Find the equation of the plane passing through the points (2, 2, - 1) and (3, 4, 2) and parallel to the line with direction ratios 7, 0 and 6.

iv) Find the equation of the plane passing through the points (- 1, 1, 1) and (1, - 1, 1) and perpendicular to the plane x + 2y + 2z = 5.

v) Find the equation of the plane through the points (0, 6, 4) and (1, 5, 4) and perpendicular to the plane $\vec{r}\cdot(3\hat{i}+2\hat{j}+6\hat{k})=3$

vi) Find the equation of the plane through the points (0, 6, 4), (1, 5, 4) and perpendicular to the plane 3x + 2y – 3z = 4.

36. Find the angle between the following planes:

i) 2x – y + z = 6 & x + y + 2z = 7 ii) 2x – 3y + 4z – 1 = 0 & - x + y = 4

iii) $\vec{r}\cdot(\hat{i}+\hat{j})=1$ & $\vec{r}\cdot(\hat{i}+\hat{k})=3$ iv) $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})-1=0$ and $\vec{r}\cdot(4\hat{i}-2\hat{j}-\hat{k})+2=0$

37. Find the angle between the following lines & planes:

i) $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ & 2x + y – 3z + 4 = 0 ii) $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ & 10x + 2y – 11z = 3

iii) $\frac{x+3}{3}=\frac{y-2}{2}=\frac{z+3}{6}$ $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})=6$

iv) $\vec{r}=(3-t)\hat{i}+(4+2t)\hat{j}+(t-2)\hat{k}$ & $\vec{r}\cdot(3\hat{i}+2\hat{j}-\hat{k})=6$

v) $\vec{r}=5\hat{i}-\hat{j}-4\hat{k}+\lambda(2\hat{i}-\hat{j}+3\hat{k})$ & $\vec{r}\cdot(3\hat{i}+4\hat{j}+\hat{k})+5=0$

vi) $\vec{r}=\hat{i}-\hat{j}+\hat{k}+\lambda(2\hat{i}-\hat{j}+3\hat{k})$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=4$

38. Find the distance of the following points from the given planes:

i) (2, 1, - 1) ; x – 2y + 4z = 9 ii) $(3,-2,4)$; $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})+2=0$

iii) (3, 4, 5) ; $\vec{r}\cdot(2\hat{i}-5\hat{j}+3\hat{k})=29$

39. Find the cartesian equation of the plane in the normal form, through the point (- 2, - 6, 5) which is also the foot of the perpendicular from the point (2, 3, 2).

40. Find the length and foot of the perpendicular from the point:

i) (7, 14, 5) to the plane 2x + 4y – z = 2 ii) origin to the plane 2x – 3y + 4z – 6 = 0

iii) (1, 2, 3) to 3x + y – 2z + 15 = 0 iv) (0, 1, 3) to $\vec{r}\cdot(3\hat{i}-4\hat{j}+5\hat{k})-1=0$

v) $\hat{i}+\hat{j}+2\hat{k}$ to $\vec{r}\cdot(2\hat{i}-2\hat{j}+4\hat{k})+5=0$

41. Find the image of the given point in the given plane:

i) (5, - 4, 2) in 8x – 5y – z – 13 = 0 ii) (1, 3, - 4) in 3x + y – 2z = 0

iii) (1, 3, 4) in 2x – y + z + 3 = 0 iv) (- 1, - 1, 3) in 2x + 3y – 4z – 10 = 0

v) (- 1, 2, 0) in the plane x – 2y + 2z = 1 vi) (2, - 1, 5) from the plane x + 2y – 2z = 4

42. Find the distance between the planes x + 2y – 2z = 4 and $\vec{r}\cdot(2\hat{i}+4\hat{j}-4\hat{k})=6$

43. i) Find the distance of the point (0, - 3, 2) from the plane 2x + y – z = 5 measured parallel to the line $\frac{x}{-2}=\frac{y}{3}=\frac{z-1}{6}$

ii) Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z + 5 = 0 measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$

iii) Find the distance of the point (2, - 1, 1) from the plane x + 2y – 2z = 4 measuring parallel to the line $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{2}$

iv) Find the distance of the point ($\hat{i}+2\hat{j}+3\hat{k}$) from the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=5$ measured parallel to the vector $2\hat{i}+3\hat{j}-6\hat{k}$

v) Find the distance of the point (2, 0, 1) from the plane x + 2y – 2z = 4 measuring parallel to the line $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+1}{2}$

vi) Find the distance of the point (2, - 1, 5) from the plane x + 2y – 2z = 4 measured parallel to the line $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{2}$

44. Find the distance of the point (2, - 1, 5) from the line $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{2}$ measuring along the plane $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})-1=0$

45. Find the distance between the parallel planes:

i) $\vec{r}\cdot(2\hat{i}-\hat{j}+3\hat{k})+5=0$ and $\vec{r}\cdot(4\hat{i}-2\hat{j}+6\hat{k})+8=0$ ii) 2x – y + 3z – 4 = 0 and 6x – 3y + 9z + 13 = 0

46. i) Show that the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}\cdot(\hat{i}+5\hat{j}+\hat{k})=5$ and find the distance between them.

ii) Show that the line $\vec{r}=-\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=1$ and find the distance between them.

47. i) Find the point of intersection between the lines $\frac{x+1}{1}=\frac{y}{-2}=\frac{z-1}{2}$ and the plane 2x – y + 2z = 0.

ii) Find the point of intersection between the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k})=6$

48. Find the equation of the plane through the point (1, 5, 4) perpendicular to the plane 3x + 2y – 3z = 4 and parallel to the line $\vec{r}=(3+2s)\hat{i}+(s-7)\hat{j}+(2s-2)\hat{k}$

49. Find the vector equation of the following plane in the dot product form

$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

50. Find the vector equation of the following plane in the dot product form

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$$

3D GEOMETRY – TEST YOURSELF

1. Find vector and Cartesian equations of the median AD of the triangle ABC with the vertices at A(2, 3, 4), B(1, 6, 9) and C(0, 2, 3).
2. Find the angle between the line $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 2\hat{k})$ **and** $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} - 2\hat{k})$
3. Find the foot of the perpendicular from the point (2, - 1, 5) in the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$

   And also find the length of the perpendicular.
4. Find the image of the point (1, 0, 4) in the line $\dfrac{x-2}{1} = \dfrac{y-4}{-5} = \dfrac{z}{5}$
5. Find the point of intersection between the lines $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7}$ and $\dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5}$

   if they intersect.
6. Find the distance between the point (2, - 1, 5) and the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$
7. Find the shortest distance between the lines whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

   and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$
8. Find the shortest distance between lines whose equations are $\dfrac{x-2}{2} = \dfrac{y-6}{3} = \dfrac{z-3}{4}$ and

   $x = \dfrac{y-2}{2} = \dfrac{z+3}{3}$
9. Find the shortest distance between lines whose equations are $\dfrac{x-2}{2} = \dfrac{y-6}{3} = \dfrac{z-3}{4}$ and

   $\dfrac{x-2}{2} = \dfrac{y+3}{3}; z = 2$
10. Find the shortest distance and the equation of line of line of shortest distance between lines whose equations are $\vec{r} = (1+s)\hat{i} + (3s - 7)\hat{j} + (2s-2)\hat{k}$ and $\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$
11. Find λ so that the lines $\dfrac{2x-4}{2\lambda} = \dfrac{1-y}{3} = \dfrac{2z-3}{3}$ *and* $\dfrac{5x-10}{2} = \dfrac{7-7y}{2\lambda} = \dfrac{z-1}{2}$ are orthogonal

12. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$

$and\ \frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find the point of intersection.

Planes

13. Find the vector and Cartesian equation of the plane , through the point (2, -1, 5) and perpendicular to the vector $\hat{i}+2\hat{j}+\hat{k}$
14. Find the Cartesian equation of the plane, through the point (- 2, - 6, 5) which is also the foot of the perpendicular from the point (2, 3, 2)
15. Find the vector equation of the plane , through the points (2, -1, 5), (1, -3, 4) and (4, -2, 0).
16. Prove that the points (0, 6, 4), (1, 5, 4) and (2, 4, 4) and (2, 6, 2) are coplanar. Also find the equation of the plane containing these points.
17. Find the equation of the plane through the points (0, 6, 4), (1, 5, 4) and perpendicular to the plane 3x + 2y - 3z = 4
18. Find the equation of the plane through the points (1, - 3, 4) and perpendicular to the planes $\vec{r}\cdot(3\hat{i}+2\hat{j}+6\hat{k})=3$ and 3x - 2y + 2z = 1
19. Find the point of intersection between the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k})=6$
20. Find the angle between the line $\frac{x+3}{3}=\frac{y-2}{2}=\frac{z+3}{6}$ and the plane $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})=6$
21. Find the angle between the line $\vec{r}=(3-t)\hat{i}+(4+2t)\hat{j}+(t-2)\hat{k}$ and the plane $\vec{r}\cdot(3\hat{i}+2\hat{j}-\hat{k})=6$
22. Find the image of the point (-1, 2, 0) in the plane x - 2y + 2z = 1
23. Find the equation of the plane through the points (0, 6, 4), (1, 5, 4) and parallel to the line $\vec{r}=(1-2t)\hat{i}+(3-2t)\hat{j}+(3t-2)\hat{k}$
24. Find the equation of the plane through the point (-1, -2, 0) and containing the line $\frac{x+1}{-2}=\frac{y-2}{3}=\frac{z+3}{2}$
25. Find the equation of the plane containing the lines $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(4\hat{i}-2\hat{j}+2\hat{k})$ and $\frac{x-2}{2}=\frac{y-4}{-1}=\frac{z-6}{1}$
26. Find the equation of the plane through the point (1, 5, 4) perpendicular to the plane 3x + 2y - 3z = 4 and parallel to the line $\vec{r}=(3+2s)\hat{i}+(s-7)\hat{j}+(2s-2)\hat{k}$
27. Find the distance between the point (2, - 1, 5) and the plane x + 2y - 2z = 4
28. Find the distance of the point (2, 0, 1) from the plane x + 2y - 2z = 4 measured parallel to the line $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+1}{2}$

29. Find the distance of the point (2, - 1, 5) from the line $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{2}$ measured along the plane $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})-1=0$

30. Find the equation of the plane through the point (1, - 2, 0) and through the point of intersection of the planes to the planes 3x + 2y – z = 4 and x + y - 2z – 2 = 0.

31. Find the equation of the plane through the point of intersection of the planes to the planes x+2y-z=1 and x+y+2z+1=0 perpendicular to the plane 2x+2y-z=0.

32. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B and C. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

33. Find the coordinates of the point where the line through (3, - 4, - 5) and (2, - 3, 1) crosses the plane determined by the points A(1, 2, 3), B(2, 2, 1) and C(- 1, 3, 6)

34. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5};\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing them.

35. Find the distance between the point P(6, 5, 9) and the plane determined by the points (3, -1, 2), (5, 2, 4) and (-1, -1, 6).

36. Find the distance of the point (- 2, 3, - 4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y - 3z + 1 =0

37. Find the point through which the line joining the points (-1, 2 , 3) and (4, 5, 1) cross the XZ plane.

38. Find the equation of a line passing through the point (1, 2, 3) and parallel to the planes 2x + 3y - 4z + 5 = 0 and x - 2y + 5z – 1 = 0

39. Find the equation of a line through the point (-1, 2, 3) which is perpendicular to the lines $\frac{x-1}{2}=\frac{2y-1}{-4}=\frac{z+2}{-1}$ and $\frac{x+3}{-1}=\frac{y+2}{2}=\frac{z-1}{3}$.