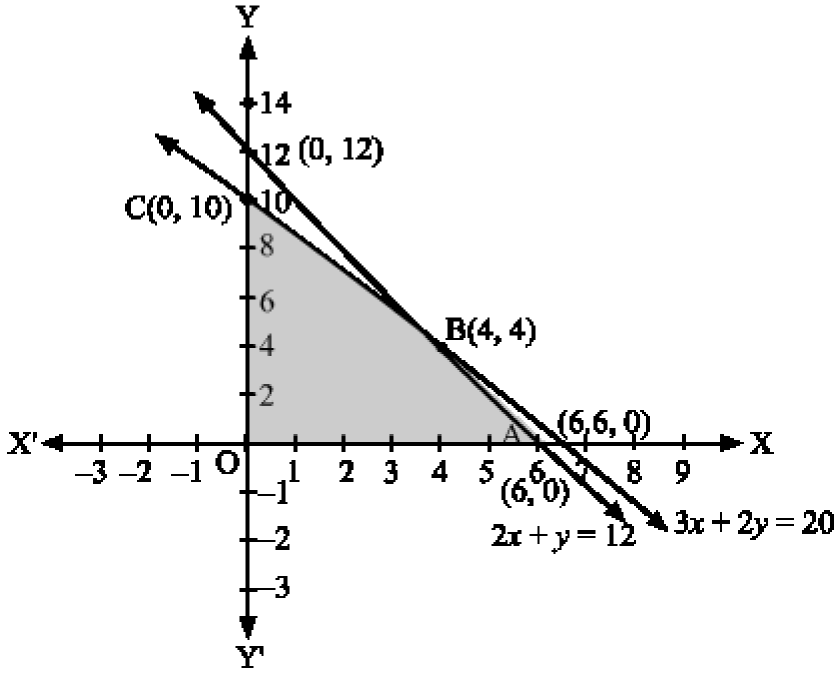
Solutions of LPP Board Exam questions

1. total profit = 25x + 15y i.e. $Z = 25x + 15y$ So, we have to maximise $Z = 25x+15y$, subject to the following constraints: $2x+y \leqslant 12$ $3x+2y \leqslant 20$ $x \geqslant 0, y \geqslant 0$ (*non−negative constraints*)



Corner Point	$Z=25x+15y$
$O(0,0)$	0
$A(6,0)$	150
$B(4,4)$	160
$C(0,10)$	150

So, maximum value of Z is 160 at $B(4,4)$. Hence, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximise his profit.

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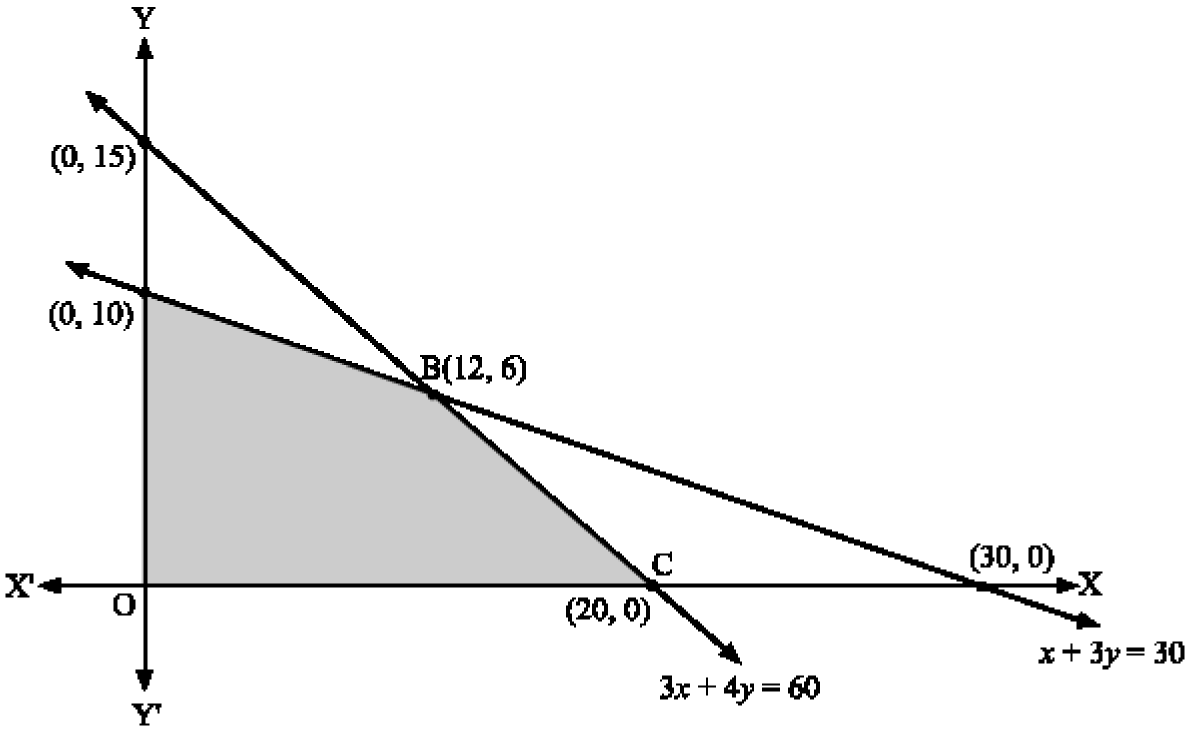
2. Maximise Z = $80x + 120y$, subject to the constraints $3x + 4y \leqslant 60$ $x + 3y \leqslant 30$ and $x \geqslant 0, y \geqslant 0$



The values of the objective function at these points are given in the following table:

Point (x, y)	Value of objective function $Z = 80x + 120y$
A (0, 10)	$Z = 80 \times 0 + 120 \times 10 = 1200$
B (12, 6)	$Z = 80 \times 12 + 120 \times 6 = 1680$
C (20, 0)	$Z = 80 \times 20 + 120 \times 0 = 1600$

Clearly, Z is maximum at (12, 6). The maximum value of Z is 1680.

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3. Let Z denote the total profit. Then $Z = 22x + 18y$

Maximize $Z = 22x + 18y$ Subject to the constraints:

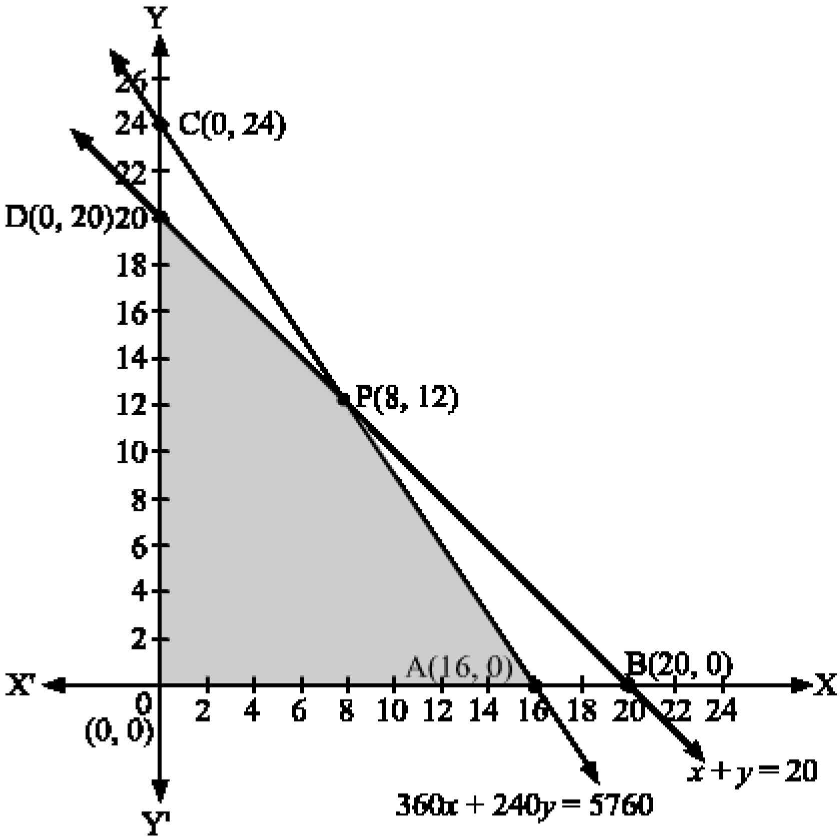
$x + y \leq 20$ ...(i)

$360x + 240y \leq 5760$ ...(ii)

and $x, y \geq 0$ ...(iii)

Let us draw the graph of constraints (i), (ii) and (iii).

APDO, as shown in the figure below, is the feasible region (shaded) determined by the constraints (i), (ii) and (iii).



The corner points of the feasible region are A(16, 0), P(8, 12), D(0, 20) and O(0, 0).

Let us evaluate $Z = 22x + 18y$ at these corner points.

Corner point	$\mathbf{Z = 22x + 18y}$
(16, 0)	352
(8, 12)	392
(0, 20)	360
(0, 0)	0

We see that the point (8, 12) is giving the maximum value of Z.

Hence, the dealer should purchase 8 electronic sewing machines and 12 manually operated sewing machines to obtain the maximum profit under the given conditions.

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4. total revenue $R = 100x + 120y$.

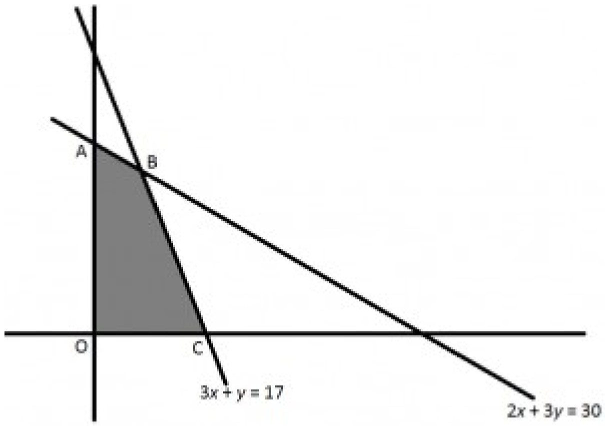
Total number of workers used in the production of given units of A and B = $2x + 3y$.

Total capital used in the production of given units of A and B = $3x + y$.

As per the information given in the question, the following must hold true:

$2x + 3y \leq 30$ *and* $3x + y \leq 17$

The problem is to maximize the value of R such that $2x + 3y \leq 30, 3x + y \leq 17, x \geq 0$ and $y \geq 0$.



The coordinates of the corner points A, B, C, and O are (0, 10); (3, 8); (17/3, 0) and (0, 0) respectively.
The value of R at the points A, B, C, and O are Rs 1200, Rs 1260, Rs 566.67, and Rs 0 respectively.
Therefore, maximum revenue would be obtained when 3 units of A and 8 units of B are produced. In doing so, 30 workers and 17 units of capital must be used.

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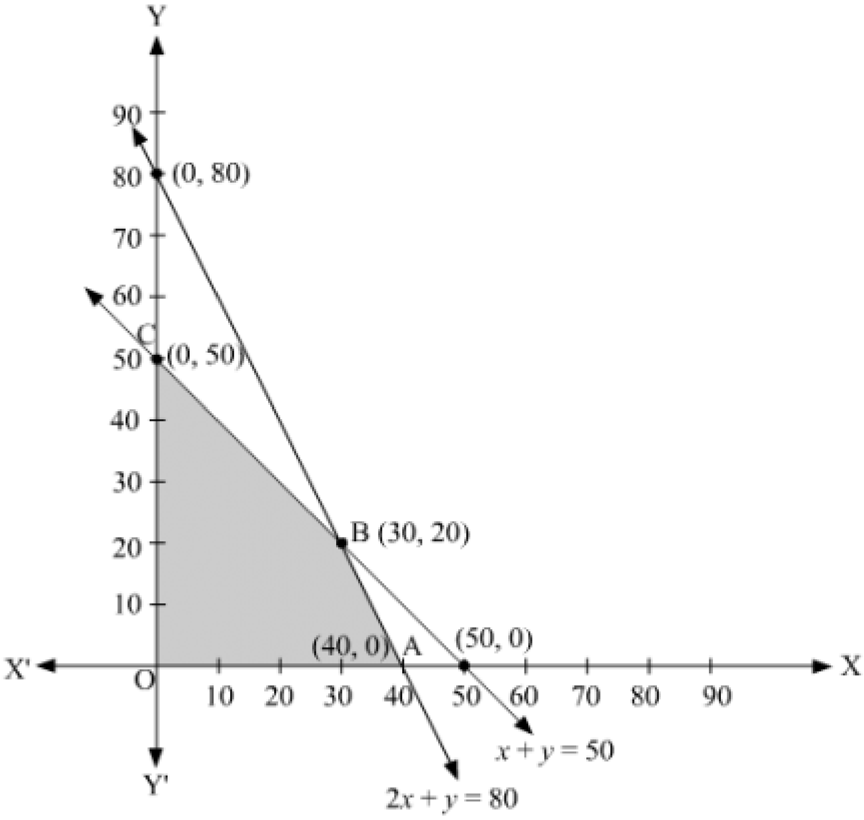
5. total profit = Rs $(10{,}500x + 9{,}000y)$ = Rs $1500\,(7x + 6y)$
Therefore, the mathematical formulation of the given problem is
Maximize $Z = 1500\,(7x + 6y)$ subject to the constraints
$x + y \le 50$ … (1)
$2x + y \le 80$ … (2)
$x \ge 0$ … (3)
$y \ge 0$ … (4)
The feasible region determined by constraints (1), (2), (3) and (4) is represented by the shaded region in the following graph:



The corner points of the feasible region are O (0, 0), A (40, 0), B (30, 20) and C (0, 50).
The values of Z at these corner points are calculated as:

Corner point	**Z** = 1500 $(7x + 6y)$	
O (0, 0)	0	
A (40, 0)	420000	
B (30, 20)	495000	← Maximum
C (0, 50)	420000	

The maximum profit is at point B (30, 20).
Therefore, 30 hectares of land should be allocated for crop **A** and 20 hectares of land should be allocated for crop **B**.
The maximum profit is Rs 495000.
Yes, we agree that the protection of wildlife is utmost necessary to preserve the balance in environment.

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6. Maximize $z = 1000x + 600y$ … (1) subject to the constraints,

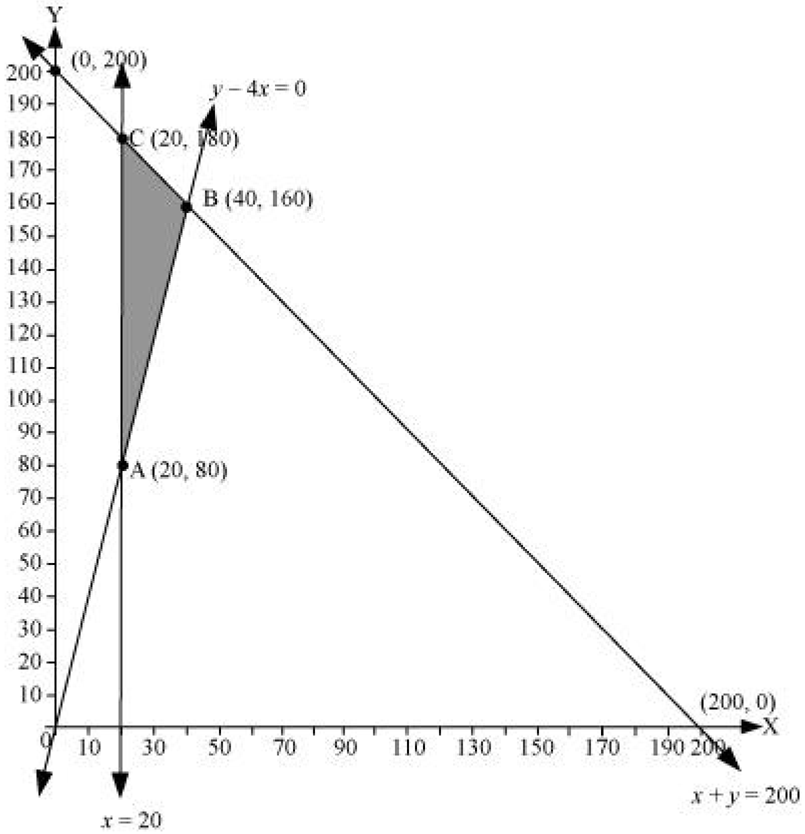
$x + y \le 200$ ...(2)

$x \ge 20$ ...(3)

$y - 4x \ge 0$ ...(4)

$x, y \ge 0$ ...(5)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

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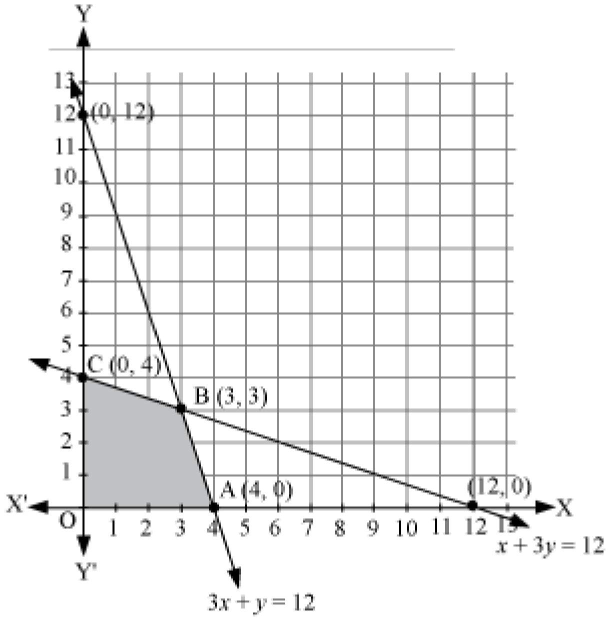
7. Maximise $Z = 17.5x + 7y$ … (1) subject to the constraints:

$x + 3y \le 12$ … (2)

$3x + y \le 12$ … (3)

$x, y \ge 0$ … (4)

The feasible region determined by the system of constraints is:



The corner points are A (4, 0), B (3, 3) and C (0, 4).
The value of Z at these corner points are as follows:

Corner points	$\mathbf{Z = 17.5x + 7y}$	
O(0, 0)	0	
A(4, 0)	70	
B(3, 3)	73.5	→ Maximum
C(0, 4)	28	

The maximum value of Z is Rs 73.50 at (3, 3).
Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit as Rs 73.50

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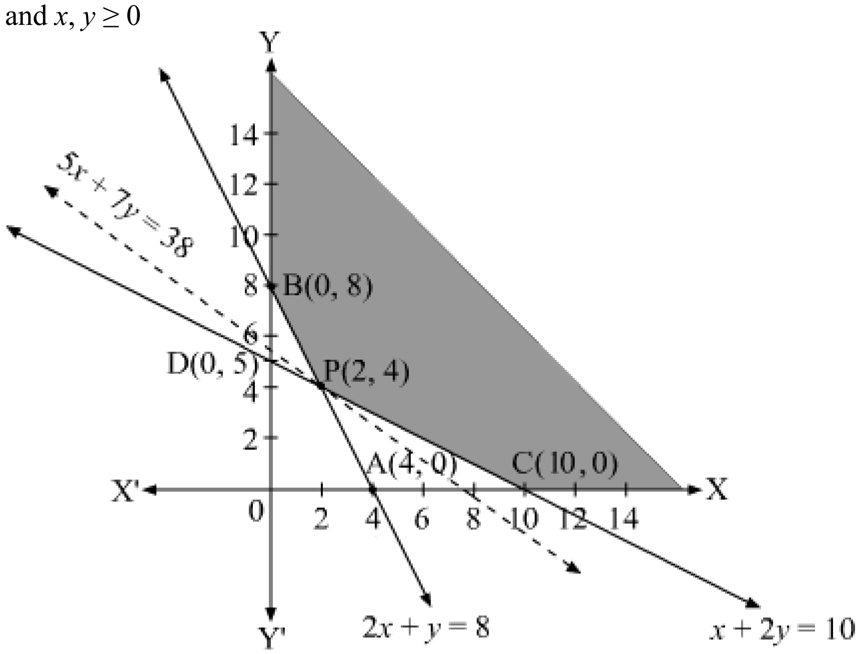
8. Minimize $Z = 5x + 7y$ Subject to the constraints
$2x + y \geq 8$
$x + 2y \geq 10$
and $x, y \geq 0$



The coordinates of the corner points of the feasible region are B (0, 8), P (2, 4) C (10, 0).
Value of Z at B (0, 8) = $5\times 0 + 7\times 8 = 56$
Value of Z at P (2, 4) = $5\times 2 + 7\times 4 = 38$
Value of Z at C (10, 0) = $5\times 10 + 7\times 0 = 50$
Thus, the minimum value of Z is 38.
Since the feasible region is unbounded, we need to verify whether Z = 38 is minimum value of given objective function or not.
For this, draw a graph of $5x + 7y < 38$.
We observe that the open half plane determined by $5x + 7y < 38$ has no points in common with the feasible region. Z is minimum for $x = 2$ and $y = 4$ and the minimum value of Z is 38.
Thus, the minimum cost of the mixture is Rs 38.

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9. Minimize Z = 10x + 4y subject to the constraints $4x + y \geq 80$, $2x + y \geq 60$ and $x \geq 0$, $y \geq 0$

Corner points are A(0, 80), B(10, 40) and C(30, 0) Minimum is Rs 260 when x = 10 and y = 40

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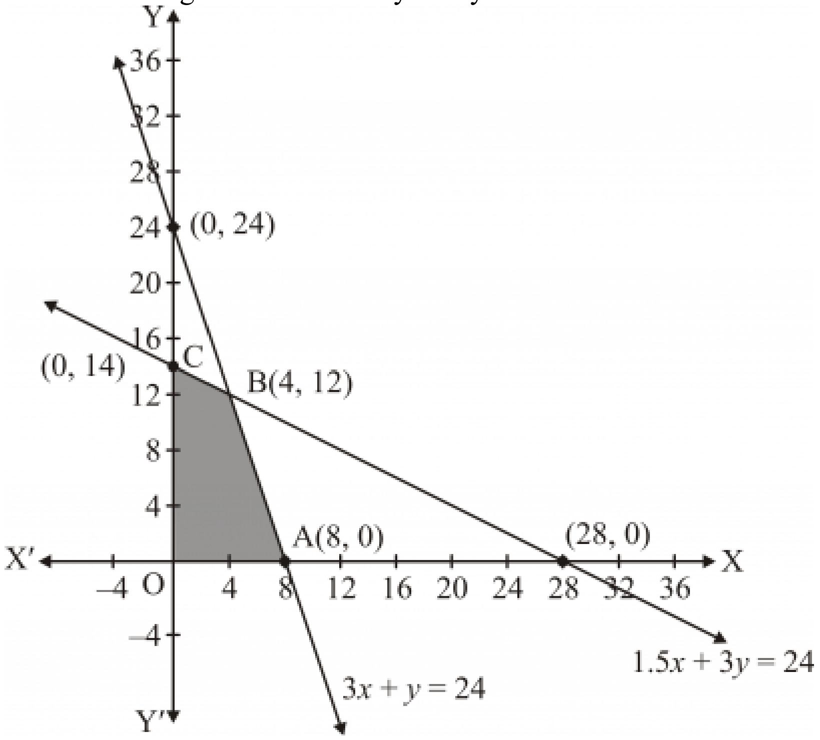
10. Maximize $Z = 20x + 10y$ … (1) Subject to the constraints,

$1.5x + 3y \leq 42$ ... (2)

$3x + y \leq 24$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows:



The corner points are A (8, 0), B (4, 12), C (0, 14), and O (0, 0).

The values of Z at these corner points are as follows:

Corner point	$Z = 20x + 10y$	
A(8, 0)	160	
B(4, 12)	200	→ Maximum
C(0, 14)	140	
O(0, 0)	0	

The maximum value of Z is 200, which occurs at $x = 4$ and $y = 12$.

Thus, the factory must produce 4 tennis rackets and 12 cricket bats to earn the maximum profit.

The maximum obtained profit earned by the factory by producing these items is Rs 200.

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11. Let the merchant stock x desktop models and y portable models. Therefore,

$x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs 70 lakhs.

$\therefore 25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

$\therefore x + y \leq 250$

The profit on a desktop model is Rs 4500 and the profit on a portable model is Rs 5000.

Total profit, $Z = 4500x + 5000y$

Thus, the mathematical formulation of the given problem is

Maximum $Z = 4500x + 5000y$ ...(1)

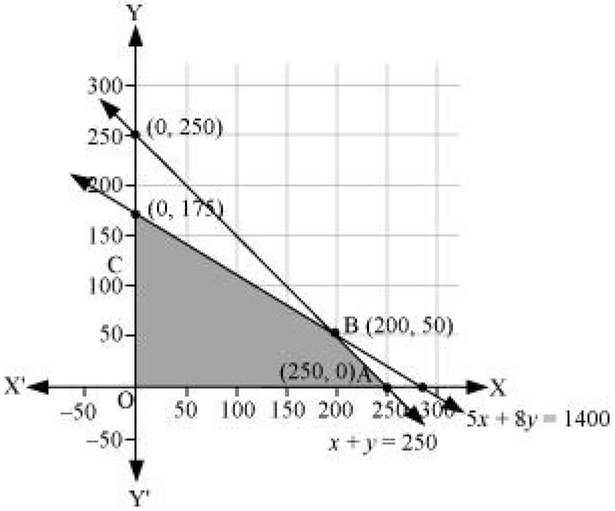
subject to the constraints,

$5x + 8y \leq 1400$ ...(2)

$x + y \leq 250$ ...(3)

$x, y \geq 0$ ...(4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	Z = 4500x + 5000y	
A(250, 0)	1125000	
B(200, 50)	1150000	→ Maximum
C(0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

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12. Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, Z = $5x + 3y$

The mathematical formulation of the given problem is

Maximize Z = $5x + 3y$ … (1)

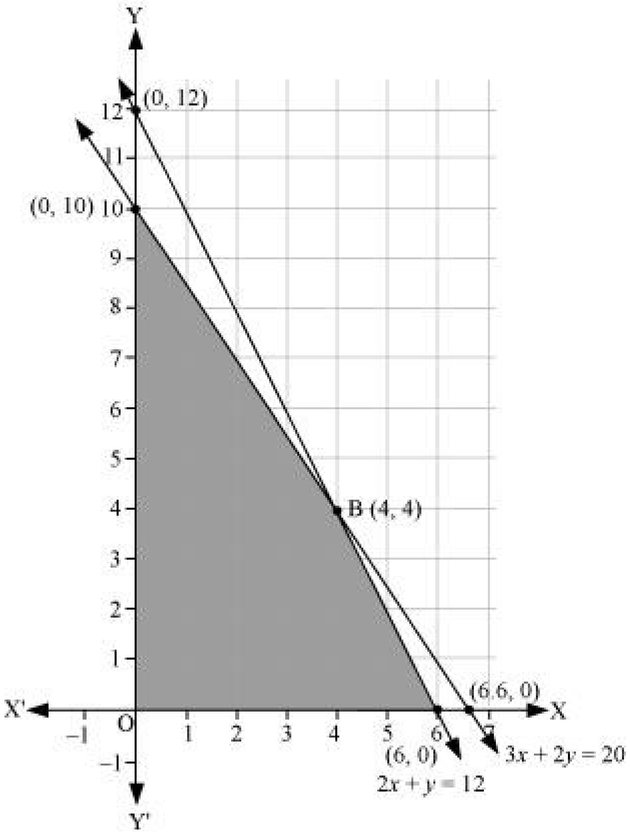
subject to the constraints,

$2x + y \leq 12$ … (2)

$3x + 2y \leq 20$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	Z = 5x + 3y	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

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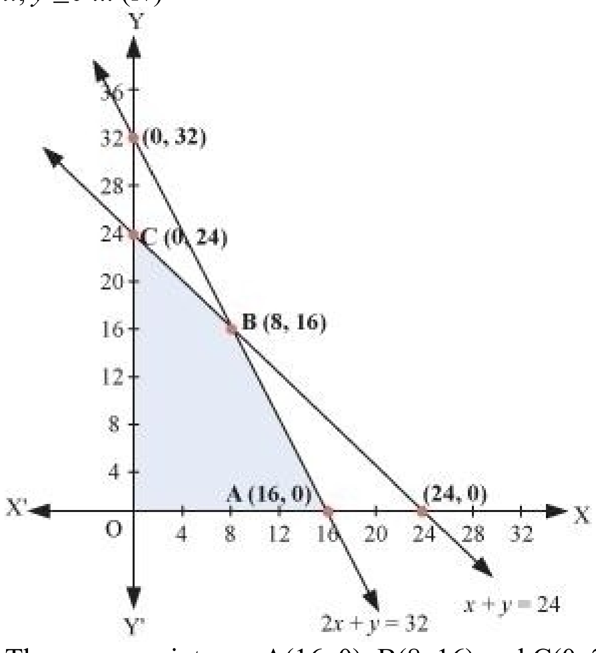
13. Maximise $z = 300x + 190y$ ... (i) Subject to the constraints:

$x + y \leq 24$ ... (ii)

$2x + y \leq 32$ ... (iii)

$x, y \geq 0$ ... (iv)



The corner points are A(16, 0), B(8, 16) and C(0, 24).

Corner point	$z = 300x + 190y$	
A(16, 0)	4800	
B(8, 16)	5440	→ Maximum
C(0, 24)	4560	

Thus, the maximum value of z is 5440 at (8, 16)
Thus, 8 gold rings and 16 chains should be manufactured per day to maximise the profits.

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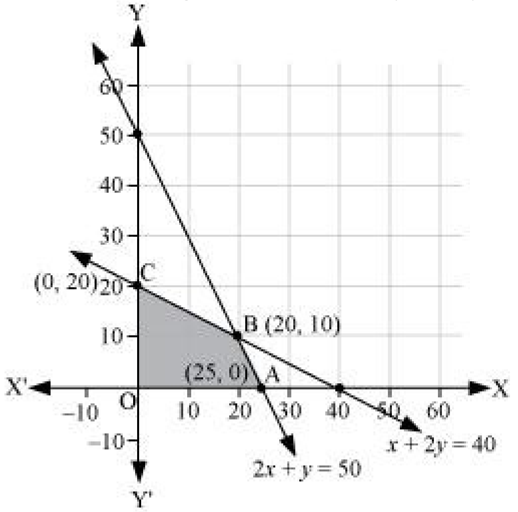
14. Maximize $Z = x + y$ ... (1) Subject to constraints,
$2x + y \le 50$ ... (2)
$x + 2y \le 40$ ... (3)
$x, y \ge 0$ ... (4)
The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0) and C (0, 20).

Corner points	$Z = x + y$	
A (25, 0)	25	
B (20, 10)	30	→ Maximum
C (0, 20)	20	
O (0, 0)	0	

Thus, maximum numbers of cakes that can be made are 30, 20 of first kind and 10 of the other kind.

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15. Let the company manufacture x souvenirs of type A and y souvenirs of type B. Therefore,
$x \ge 0$ and $y \ge 0$
The given information can be complied in a table as follows.

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are
$5x + 8y \le 200$
$10x + 8y \le 240$ i.e., $5x + 4y \le 120$
Total profit, $Z = 5x + 6y$
The mathematical formulation of the given problem is
Maximize $Z = 5x + 6y$ … (1)

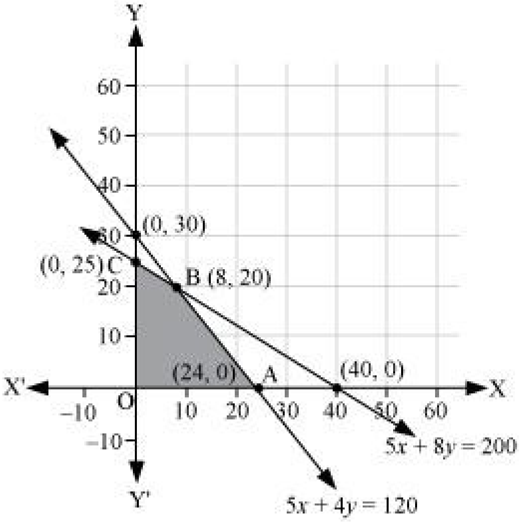
subject to the constraints,

$5x+8y \leq 200$ … (2)

$5x+4y \leq 120$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	**Z = 5*x* + 6*y***	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

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16. Maximize $Z = x + y$ subject to the constraints

$6x + 4y \leq 96$ … i.e. $3x + 2y \leq 48$….. (1)

$x + 1.5y \leq 21$ … i.e. $2x + 3y \leq 42$….. (2)

$x \geq 0$ … (3) $y \geq 0$ … (4)

corner points are O(0, 0) , A(0, 14), B(12, 6) and C(16, 0)

Maximum no. of books is 18 at B(12, 6)

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17. Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit and of Food F_2 is Rs 6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, Z = $4x + 6y$

The mathematical formulation of the given problem is

Minimise Z = $4x + 6y$ … (1)

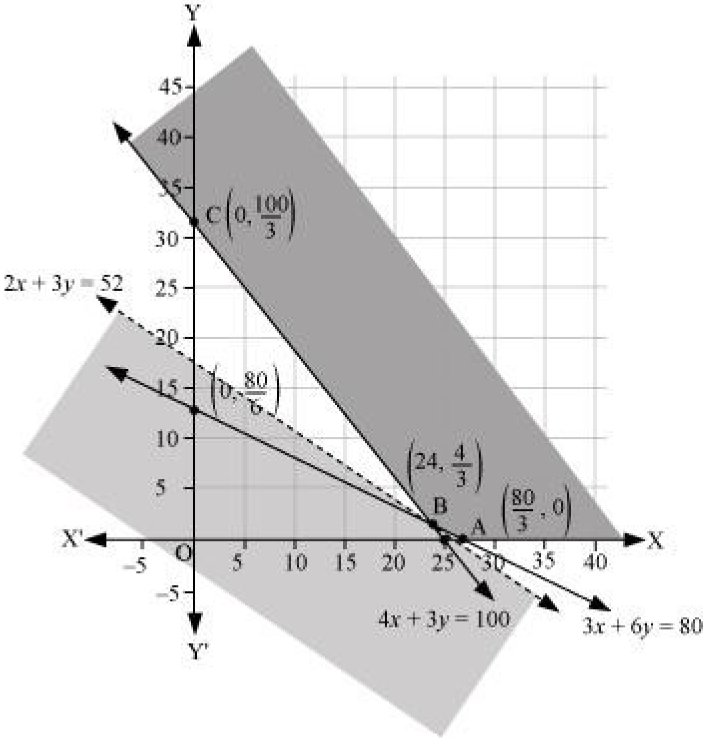
subject to the constraints,

$3x + 6y \geq 80$ … (2)

$4x + 3y \geq 100$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$.

The corner points are $A\left(\frac{80}{3}, 0\right)$, $B\left(24, \frac{4}{3}\right)$, and $C\left(0, \frac{100}{3}\right)$.

The values of Z at these corner points are as follows.

Corner point	**Z = 4*x* + 6*y***	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

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18. Let Z denote the total profit. Then $Z = 22x + 18y$

Maximize $Z = 22x + 18y$ Subject to the constraints:

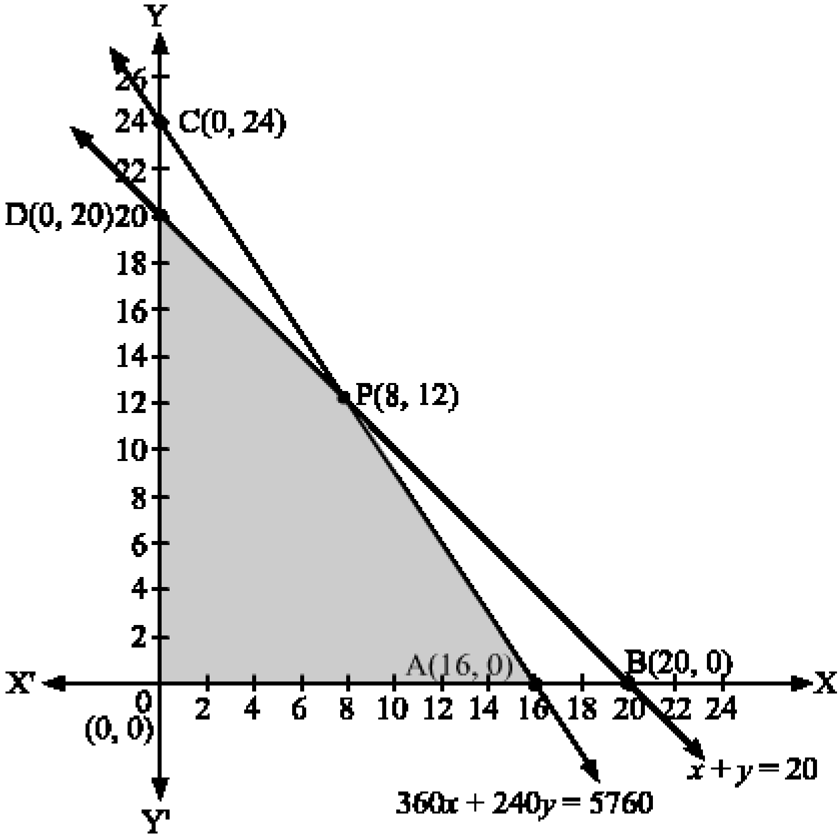
$x + y \le 20$ ...(i)

$360x + 240y \le 5760$ ...(ii)

and $x, y \ge 0$ ...(iii)

Let us draw the graph of constraints (i), (ii) and (iii).

APDO, as shown in the figure below, is the feasible region (shaded) determined by the constraints (i), (ii) and (iii).



The corner points of the feasible region are A(16, 0), P(8, 12), D(0, 20) and O(0, 0).

Let us evaluate $Z = 22x + 18y$ at these corner points.

Corner point	**$Z = 22x + 18y$**
(16, 0)	352
(8, 12)	392
(0, 20)	360
(0, 0)	0

We see that the point (8, 12) is giving the maximum value of Z.

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19. Let there be x cakes of first kind and y cakes of second kind. Therefore,

$x \ge 0$ and $y \ge 0$

The given information can be complied in a table as follows.

	Flour (g)	**Fat (g)**
Cakes of first kind, x	200	25
Cakes of second kind, y	100	50
Availability	5000	1000

$\therefore 200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

$25x + 50y \leq 1000$

$\Rightarrow x + 2y \leq 40$

Total numbers of cakes, Z, that can be made are, $Z = x + y$

The mathematical formulation of the given problem is

Maximize $Z = x + y$ … (1)

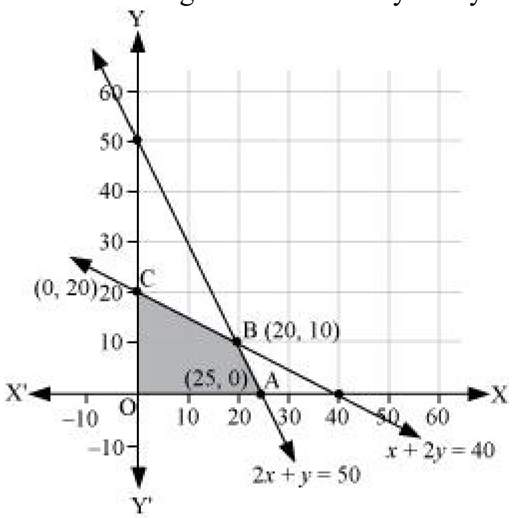
subject to the constraints,

$2x + y \leq 50$ ...(2)

$x + 2y \leq 40$ ...(3)

$x, y \geq 0$ ...(4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	$\mathbf{Z = x + y}$	
A(25, 0)	25	
B(20, 10)	30	→ Maximum
C(0, 20)	20	
O(0, 0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

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20. Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Nitrogen (%)	**Phosphoric Acid (%)**	**Cost (Rs/kg)**
$\mathbf{F_1}$ **(x)**	10	6	6
$\mathbf{F_2}$ **(y)**	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$\therefore$ 10% of x + 5% of $y \geq 14$

$$\frac{x}{10}+\frac{y}{20}\geq 14$$
$$2x+y\geq 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

∴ 6% of x + 10% of $y \geq 14$

$$\frac{6x}{100}+\frac{10y}{100}\geq 14$$
$$3x+56y\geq 700$$

Total cost of fertilizers, Z = $6x + 5y$

The mathematical formulation of the given problem is

Minimize Z = $6x + 5y$ … (1)

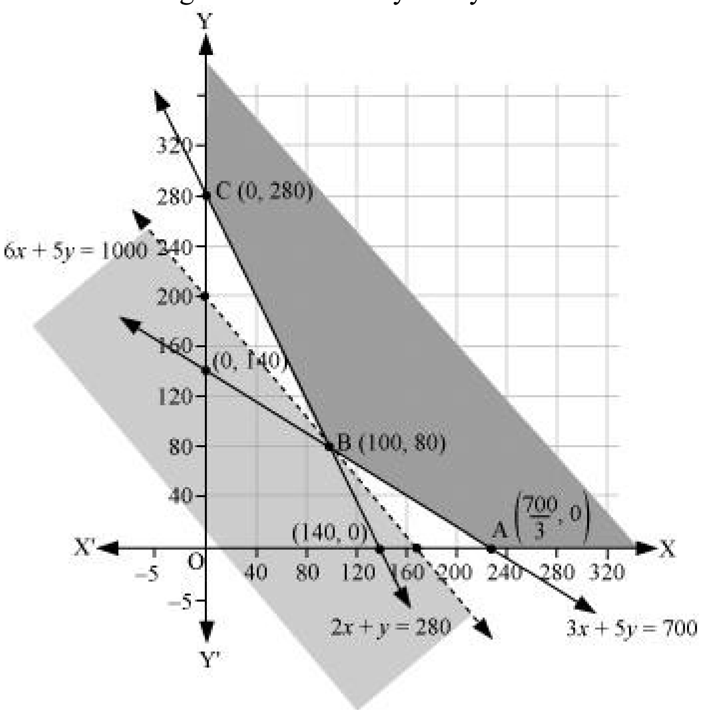
subject to the constraints,

$2x + y \geq 280$ … (2)

$3x + 5y \geq 700$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right), B(100,80)$, and $C(0,280)$.

The values of Z at these points are as follows.

Corner point	Z = 6x + 5y	
$A\left(\frac{700}{3},0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with

$6x + 5y < 1000$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

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21. Maximise $Z = 60x + 40y$ Subject to the constraints

$$1000x + 1200y \le 9000$$

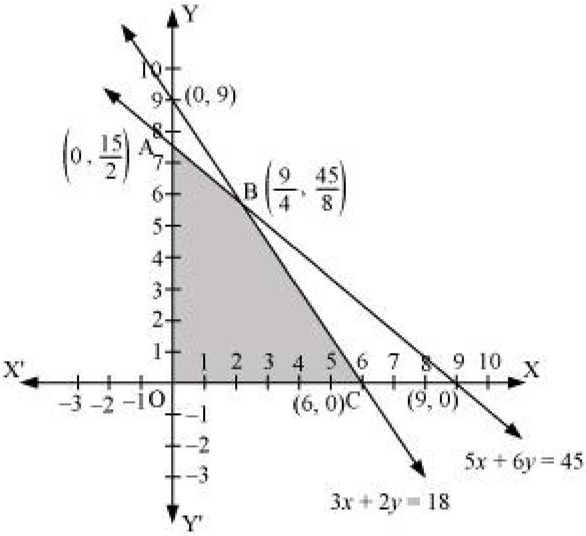
$$\Rightarrow 5x + 6y \le 45 \quad ...(1)$$

$$12x + 8y \le 72$$

$$\Rightarrow 3x + 2y \le 18 \quad ...(2)$$

$$x \ge 0, y \ge 0 \quad ...(3)$$

The inequalities (1) to (3) can be graphed as:



It is seen that the shaded portion OABC is the feasible region and the values of Z at the corner points are given by the following table.

Corner point	$Z = 60x + 40y$	
O (0, 0)	0	
$A\left(0, \frac{15}{2}\right)$	300	
$B\left(\frac{9}{4}, \frac{45}{8}\right)$	360 $\longrightarrow$	Maximum
$C(6,0)$	360 $\longrightarrow$	Maximum

The maximum value of Z is 360 units, which is attained at $B\left(\frac{9}{4}, \frac{45}{8}\right)$ and $C(6,0)$.

It is clear that the number of machines cannot be in fraction.