Class XII Mathematics Assignment
Relations, Functions, Binary operations and Inverse Trigonometric Functions

1. Prove that the relation R on the set Z defined by $(x,y) \in R \Leftrightarrow x - y$ is divisible by 3 is an equivalence relation.
2. Prove that the relation R on the set N X N defined by $(a, b) R (c,d) \Leftrightarrow a + d = b + c$ is an equivalence relation.
3. Prove that the relation R on the set N X N defined by $(a, b)R(c,d) \Leftrightarrow ad = bc$ is an equivalence relation.
4. Prove that the relation R on the set N X N defined by $(a, b)R(c,d) \Leftrightarrow ad(b + c) = bc(a + d)$ is an equivalence relation.
5. Check whether the relation R on the set of real numbers defined by $(a,b) \in R \Leftrightarrow 1+ ab > 0$ is equivalence relation.
6. Check whether the following functions are bijective if so, find their inverse also
   a). f: $R \to R$ defined by $f(x) = 4x^3 + 8$
   b). f: $R \to R$ defined by $f(x) = (x+1)^2 - 1$ $x,y \geq -1$
   c). f: $[0,2] \to :[0,2]$ defined by $f(x) = \sqrt{4-x^2}$
   d) f: $N \to N$ defined by $f(x) = \begin{cases} x+1 & if\ x\ is\ odd \\ x-1 & if\ x\ is\ even \end{cases}$
   e) f : $R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y-6}-1}{3}$
   f) f : $R_+ \to [4, \infty]$ given by $f(x) = x^2 + 4$ Show that f is invertible with $f^{-1}(y) = \sqrt{y-4}$
   g) f:$R \to R$ defined by $f(x) = |x| + 5$
7. If $f(x) = \dfrac{x-1}{x+1}$ prove that $f(2x) = \dfrac{3f(x)+1}{f(x)+3}$
8. Let f,g : $R \to R$ defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find fog and gof
9. Given $f(x) = \dfrac{2x+5}{4x+3}$ if fog (x) = x ,then find g
10. If $f(x) = \log\dfrac{1+x}{1-x}$ and $g(x) = \dfrac{3x+x^3}{1+3x^2}$ then prove that fog(x) = 3f(x)
11. Let f,g : $R \to R$ defined by f(x) = sinx and $g(x) = x^2$ find fog and gof
12. On the set Q^+ of all positive rational numbers define a binary operation * by a * b = ab/2.Show that * is commutative and associative. Also find the identity element, if any for* on Q^+, Also find the inverse of $a \in Q^+$ if it exists
13. Let * be a binary operation on Z defined by a*b = a + b + 1, Check whether * is commutative and associative. Find the identity element if any . find also the inverse element of $a \in Z$
14. Let A = {1,-1,i.-i} where $i = \sqrt{-1}$ .Draw an operation table for the operation 'multiplication'. Is it a binary operation? If so find the identity element and inverse of each element of A if they exist.
15. Let *: Q X Q: $\to$ Q defined by a*b =a + b – ab, Check whether * is commutative and associative. Also find the identity element and inverse of each a in Q if they exist.

16. Show that the operations "Union"and "Intersection" of sets defined on the powerset of a nonempty set A is a binary operation. Also prove that these operations are commutative and associative. Also find the identity elements for the operations.

17. Let A ={ a,b,c,d} Give an example for a relation in A which is

a. reflexive but neither symmetric nor transitive

b. reflexive and symmetric but not transitive

18. $f : R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$.Show that $g \circ f = I_A$ and $f \circ g = I_B$ where B=$R - \left\{\frac{3}{5}\right\}$ and A=R-$\left\{\frac{7}{5}\right\}$.

19. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to Range$ of f is invertible. Find the inverse of f.

20. Let A = **R** – {3} and B = **R** – {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is bijective.

21. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y - 4}$, where R_+ is the set of all non-negative real numbers.

22. Prove that $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n} = \frac{\pi}{4}$

23. Prove that $\tan^{-1}3/4 + \tan^{-1}3/5 - \tan^{-1} 8/9 = \pi/4$

24. Prove that $4\tan^{-1}\frac{1}{5} - \tan^{-1}\left(\frac{1}{70}\right) + \tan^{-1}\left(\frac{1}{99}\right) = \pi/4$

25. If $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$ then prove that $a + b + c = abc$

26. Prove that $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

27. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$

28. Prove that $\tan^{-1}1 + \tan^{-1} 1/2 + \tan^{-1} 1/3 = \pi/2$

29. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

30. Prove that $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] = \frac{2b}{a}$

31. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{65} = \pi$

32. Prove that $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

33. Simplify: $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

34. Simplify: $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

35. Simplify : $\tan^{-1}\left(\dfrac{\cos x-\sin x}{\cos x+\sin x}\right)$

36. Simplify a) $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$ b) $\tan^{-1}\dfrac{\sqrt{1+x^2}}{x}$ c) $\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)$

37. Solve $\tan^{-1}\left(\dfrac{x-1}{x-2}\right)+\tan^{-1}\left(\dfrac{x+1}{x+2}\right)=\pi/4$

38. Solve $\cos^{-1}x+\sin^{-1}x/2=\pi/6$

39. Solve $\tan^{-1}(x-1)+\tan^{-1}(x+1)+\tan^{-1}x=\tan^{-1}3x$ (ii)Solve for x $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)-\dfrac{1}{2}\tan^{-1}x=0\ if\ x.>0$

40. Solve $\tan^{-1}2x+\tan^{-1}3x=\dfrac{\pi}{4}$ ii) Solve $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

41. Find the value of $\sec^2\left(\tan^{-1}2\right)+\operatorname{cosec}^2\left(\cot^{-1}3\right)=15$

42. Prove that $\tan^{-1}\sqrt{x}=\dfrac{1}{2}\cos^{-1}\dfrac{1-x}{1+x}$ b) Prove that $\cot^{-1}\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}=\dfrac{x}{2}$

43. Prove that $\tan^{-1}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}=\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x$

44. Prove that $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]=\dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x^2$

45. Prove that $2\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right]=\cos^{-1}\dfrac{b+a\cos\theta}{a+b\cos\theta}$

46. Prove that $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}=\dfrac{9}{4}\sin^{-1}\dfrac{2\sqrt{2}}{3}$

47. Prove that $\tan^{-1}x+\tan^{-1}\dfrac{2x}{1-x^2}=\tan^{-1}\dfrac{3x-x^3}{1-3x^2}$ ii) Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x\}]=\sqrt{\dfrac{1+x^2}{2+x^2}}$

48. If $\cos^{-1}a+\cos^{-1}b+\cos^{1}c=\pi$ then prove that $a^2+b^2+c^2+2abc=1$

49. If $\cos^{-1}x/a+\cos^{-1}y/b=A$ then prove that $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{2xy}{ab}\cos A=\sin^2 A$

50. Prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=0$

51. Find the value of $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=15$

52. Find the value of $\sin^{-1}(\sin\frac{2\pi}{3})+\cos^{-1}(\cos\frac{2\pi}{3})$

53. Find the principal value of $\tan^{-1}(\tan\frac{3\pi}{4})$

54. Prove that $\sin(2\sin^{-1}x)=2x\sqrt{1-x^2}$.

55. Prove that $\cos^{-1}x=2\sin^{-1}\sqrt{\frac{1-x}{2}}=2\cos^{-1}\sqrt{\frac{1+x}{2}}$.

56. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}$

57. Prove that $\tan^{-1}\frac{2}{11}+\cot^{-1}\frac{24}{7}=\tan^{-1}\frac{1}{2}$

58. Prove that $\cos\left[2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]+x=0.$

59. Prove that $\cos^{-1}\frac{12}{13}=\tan^{-1}\frac{5}{12}$

60. Prove that $\sec^{-1}\left[\frac{x^2+1}{x^2-1}\right]+2\tan^{-1}x=\pi$

61. Prove that $\cot^{-1}\left[\frac{1+x}{1-x}\right]+\tan^{-1}x=\frac{\pi}{4}$

62. Prove that $\cot^{-1}[\operatorname{cosec}x+\cot x]=\frac{x}{2}$

63. Prove that $\cot^{-1}(\tan x)+\tan^{-1}(\cot x)=\pi-2x$

64. Prove that $\sin(2\sin^{-1}0.8)=0.96$