2011 Foreign

1. (Set 1) If f: R $\to$R is defined by f(x) = 3x + 2, define f[f(x)]. [1 mark]
2. (Set 2) Write fog, if f : R $\to$R and g : R $\to$R are given by f(x) = $|x|$ and g(x) = $|5x-2|$. [1 mark]
3. (Set 3) Write fog, if f : R $\to$R and g : R $\to$R are given by f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$. [1 mark]
4. Consider f : $R_+ \to [4,\infty]$ given by f(x) = x^2 + 4. Show that f is invertible with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. [4 marks]

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5. State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.
6. Consider the binary operation $*$ on the set {1, 2, 3, 4, 5} is defined by a $*$ b = min {a, b}. Write the operation table of the operation $*$. [4 marks]

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7. (Set 1) If 'f' is an invertible function, defined as f(x) = $\frac{3x-4}{5}$, write f^{-1}. [1 mark]
8. (Set 2) If f : R $\to$R and g : R $\to$R are given by f(x) = sinx and g(x) = $5x^2$, find $g \circ f(x)$. [1 mark]
9. (Set 3) If f(x) = $27x^3$ and g(x) = $x^{\frac{1}{3}}$, find $g \circ f(x)$. [1 mark]
10. Consider f : $R_+ \to [-5,\infty]$ given by f(x) = $9x^2$ + 6x - 5. Show that f is invertible with $f^{-1}(y)$ = $\left(\frac{\sqrt{y-6}-1}{3}\right)$. [4 marks]
11. OR Let A = N $\times$ N and $*$ be a binary operation on A defined by (a, b) $*$ (c,d) = (a + c, b + d). Show that $*$ is commutative and associative. Also find the identity element for $*$ on A, if any. [4 marks]

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12. If f : R $\to$R be defined by f(x) = $\left(3-x^3\right)^{1/3}$, then find fof(x). [1 mark]
13. (Set 1 & 2) Show that the relation S in the set A = $\{x \in Z : 0 \le x \le 12\}$ given by S = {(a, b): a, b ϵ Z, $|a-b|$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. [4 marks]
14. (Set 3) Show that the relation S defined on the set N $\times$ N by (a, b) S (c,d) $\Rightarrow$ a + d = b + c is an equivalence relation. [4 marks]

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15. What is the range of the function f(x) = $\frac{|x-1|}{(x-1)}$? [1 mark]
16. (Set 1) Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) : a, b ϵ Z and (a – b) is divisible by 5}. Prove that R is an equivalence relation. [4 marks]
17. (Set 2) Let * be a binary operation on Q, defined by a*b = $\frac{3ab}{5}$. Show that * is commutative as well as associative. Also find its identity, if it exists. [4 marks]

18. (Set 3) Show that the relation S in the set R of real numbers, defined as S = {(a, b) : a, b ϵ R and a ≤ b^3} is neither reflexive, nor symmetric nor transitive. [4 marks]

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19. If the function f : R → R, defined by f(x) = 3x – 4, is invertible, find f^{-1}. [1 mark]

20. Let f : X → Y be a function. Define a relation R on X given by R = {(a, b) : f(a) = f(b)}. Show that R is an equivalence relation on X. [4 marks]

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21. If the binary operation *, defined on Q, is defined as a * b = 2a + b – ab, for all a, b ϵ Q, find the value of 3 * 4. [1 mark]

22. Show that the relation R in the set of real numbers, defined as R = {(a, b) : $a \leq b^2$} is neither reflexive, nor symmetric nor transitive. [4 marks]

2009 AI

23. Let * be a binary operation on N given by a * b = HCF(a, b), a, b ϵ N, write the value of 22 * 4.

24. Let f : N → N be defined by f(n) = $\begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$, for all $n \in N$

Find whether the function f is bijective. [4 marks]

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25. If the binary operation *on the set of integers Z, is defined by a * b = a + $3b^2$, then find the value of 2 * 4. [1 mark]

26. Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): $|a-b|$ is even}, is an equivalence relation. [4 marks]

2008 Foreign

27. Let * be a binary operation, defined by a * b = 3a + 4b – 2, find 4 * 5. [1 mark]

28. Show that the relation R defined by R = {(a, b) : (a – b) is divisible by 3; a, b ϵ N }is an equivalence relation. [4 marks]

2008 AI

29. Show that the relation R defined by (a, b) R (c, d) ⇔ a + d = b + c on the set N X N is an equivalence relation. [4 marks]

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30. If f(x) = x + 7 and g(x) = x – 7, x ϵ R, find (fog)(7). [1 mark]

31. (i) Is the binary operation *, defined on the set N, given by a*b = $\frac{a+b}{2}$ for all a, b ϵ N, commutative?

(ii) Is the above binary operation * associative? [4 marks]