THE INDIAN SCHOOL, BAHRAIN

FIRST TERMINAL EXAMINATION, JUNE 2014

STD: XI **MAX. MARKS: 100**

SUB: MATHEMATICS **TIME : 3 HOURS**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 26 questions divided into three sections A, B and C.*
3. *Question numbers 1 to 6 are of 1 mark each. Question numbers 7 to 19 are of 4 marks each and Question numbers 20 to 26 are of 6 marks each.*
4. *This paper contains two printed pages.*

Section A

1. Find the value of $\sin(\frac{-19\pi}{3})$
2. Simplify: $i^{10} + i^{11} + i^{12} + i^{13}$.
3. Find the principal solution of $\tan\theta = \sqrt{3}$
4. Solve: $x^2 - 2x + \frac{3}{2} = 0$
5. Evaluate $\sin75^o\cos15^o - \cos75^o\sin15^o$.
6. Find the multiplicative inverse of $2 - 2i$.

Section B

7. Prove the following by using the principle of mathematical induction for all $n \in N$

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

OR

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

8. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

9. Prove the following by using the principle of mathematical induction for all $n \in N$

$$1.2 + 2.2^2 + 3.2^3 + ... + n.2^n = (n-1)\,2^{n+1} + 2$$

10. If $x + iy = (u + iv)^3$, then show that $4(u^2 - v^2) = \frac{x}{u} + \frac{y}{v}$

11. Prove that $\cot 5x \cot 6x - \cot 6x \cot 11x - \cot 11x \cot 5x = 1$

12. Prove that $\cos^2 x + \cos^2(x + 120^\circ) + \cos^2(x - 120^\circ) = \frac{3}{2}$

OR

Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

13. Find the square root of $-7-24i$.

14. In triangle ABC, prove that $a(b\cos C - c\cos B) = b^2 - c^2$.

15. Find the general solution of the following equation:
$\cos x + \cos 2x + \cos 3x = 0$ **OR** $2\sin^2 x + 3\cos x = 0$

16. Prove the following by using the principle of mathematical induction for all n $\in$ N
$1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

17. If $x - iy = \sqrt{\dfrac{a-ib}{c-id}}$, prove that $(x^2+y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}$

18. Solve the following system of inequalities graphically :
$5x + 4y \le 20$ and $y \ge 2$

19. Find $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$, if $z_1 = 2 - i$ *and* $z_2 = 1 + i$.

OR

Find real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6-24i$.

Section C

20. Using a unit circle, derive the identity $\cos(x + y) = \cos x \cos y - \sin x \sin y$.

21. Find $\sin\left(\dfrac{x}{2}\right)$, $\cos\left(\dfrac{x}{2}\right)$ and $\tan\left(\dfrac{x}{2}\right)$, if $\tan x = \dfrac{4}{3}$; $\pi < x < \dfrac{3}{2}\pi$

OR

In triangle ABC, prove that $\dfrac{b^2-c^2}{a^2}\sin 2A + \dfrac{c^2-a^2}{b^2}\sin 2B + \dfrac{a^2-b^2}{c^2}\sin 2C = 0$

22. Prove the following by using the principle of mathematical induction for all n $\in$ N
$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots\ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

23. Prove the following by using the principle of mathematical induction for all n $\in$ N
$3^{2n+2} - 8n - 9$ is divisible by 8.

24. Express the following complex number in the polar form. $z = \dfrac{i+1}{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}}$

25. Find real θ such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is i) purely real ii) purely imaginary.

OR

If α and β are different complex numbers such that $|\beta| = 1$, then find the value of $\left|\dfrac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

26. Solve graphically $x + 2y \le 8$, $2x + y \ge 2$, $x - y < 0$, $x \ge 0$, and $y \ge 0$.