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THE INDIAN SCHOOL, KINGDOM OF BAHRAIN
FIRST TERMINAL EXAMINATION – JULY 2010

STD : XII

MARKS : 100

SUBJECT : MATHEMATICS

TIME : 3 hrs

Please check that this question paper contains 3 printed pages.

Please check that this question paper contain 29 questions.

Section –A (each carry 1 mark)

- 1) Construct a 2×2 matrix, $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$.
- 2) Find the product of the matrices $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4]$
- 3) Define an equivalence relation .
- 4) The total number of binary operation on the set $\{a, b\}$ are ?
- 5) Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.
- 6) The value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to
- 7) Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.
- 8) Find the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.
- 9) Find the direction cosines of a line whose direction ratios are $-18, 12, -4$.
- 10) Find the intercepts cut off by the plane $2x + y - z = 5$.

Section –B (each carry 4 marks)

- 11) Find the inverse of the matrix by using elementary operations, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

(or)

If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,

show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

- 12) By using the properties of determinants prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$

13) By using the properties of determinants prove that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$$

14) Prove that $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}$, $x \in (0, \frac{\pi}{4})$.

(or)

Prove that $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $\frac{-1}{\sqrt{2}} \leq x \leq 1$.

15) Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$

16) Show that $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$

17) Let $A = N \times N$ and $*$ be the binary operation on A defined by

$(a,b) * (c,d) = (a+c, b+d)$, verify that $*$ is commutative and associative. Find the identity element for $*$ on A if any.

18) Verify that the relation R in the set $A = \{1,2,3,4,5,6\}$ as $R = \{(x,y) : y \text{ is divisible by } x\}$ is reflexive, symmetric and transitive.

(or)

Consider $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. show that f is invertible

$$\text{with } f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3} \right)$$

19) Verify that the vectors form the vertices of a right angled triangle

$$2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k} \text{ and } 3\hat{i} - 4\hat{j} - 4\hat{k}.$$

(or)

Find the area of the triangle with vertices $A(1,1,2)$, $B(2,3,5)$ and $C(1,5,5)$.

20) Three vectors a, b and c satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$ evaluate

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}, \text{ if } |\vec{a}|=1, |\vec{b}|=4 \text{ and } |\vec{c}|=2.$$

21) Probability of solving a specific problem by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If

both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

- 22) A pair of dice is thrown 4 times .If getting a doublet is considered a success, find the probability of (i) two successes . (ii) at least 3 successes.

Section –C (each carry 6 marks)

23) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, verify that $A^3 - 6A^2 + 7A + 2I = 0$.

(or)

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1} .

- 24) Solve the system of equations by matrix method $x - y + 2z = 7$, $3x + 4y - 5z = -5$ and $2x - y + 3z = 12$.
- 25) A man is known to speak truth 3 out of 4 times .He throws a die and reports that it is a six .Find the probability that it is actually a six.
- 26) Find the probability distribution of number of heads in three tosses of a fair coin also find its Expectation .

(or)

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls . one ball is transferred from Bag I to Bag II and then a ball is drawn from bag II .The ball so drawn is found to be Red in colour .Find the probability that the transferred ball is black.

27) Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

28) Find the equation of the line passing through the point (1,2, -4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

- 29) Find the image of the point (1,3,4) in the plane $x-y+z=5$.