THE INDIAN SCHOOL

KINGDOM OF BAHRAIN

SECOND TERMINAL EXAMINATION – NOVEMBER 2012

STD: XII **MARKS: 100**

SUBJET: MATHEMATICS **TIME: 3 HOURS**

General Instruction:

(i) All questions are compulsory.

(ii) The questions paper consists of 29 questions divided into three section A, B and C. Section A comprises of 10 questions of one marks each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 Questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

(vi) You can use any logical method to answer the questions.

SECTION – A

(Question numbers 1 to 10 carry 1 mark each)

1. Show that if f: A→B and g: B→C are one - one, then $g \circ f$: A→C is also one – one.
2. Find the principal value of $\csc^{-1}(-\sqrt{2})$
3. Find the number of all possible matrices of order 3×3 with each entry 0 or 1
4. If X, Y, Z, W and P are matrices or order 2× n, 3× k, 2× p, n×3 and p× k respectively. Also if n = p then find the order of the matrix 7X – 5Z.
5. Using the suitable property of determinants if a,b,c are in A.P, find the value of
$$\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$$
6. Find unit vector in the direction of vector $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + \hat{k}$
7. Find the equation of a line parallel to x axis and passing through the origin.
8. Find the projection of the vector $\hat{\imath} + 3\,\hat{\jmath} + 7\,\hat{k}$ on the vector $7\,\hat{\imath} - \hat{\jmath} + 8\,\hat{k}$

9. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

10. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

SECTION B

11. Consider $f: \{1,2,3\} \to \{a, b, c\}$ given by $f(1) = a, f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$

OR

Let $*$ be the binary operation on N given by $a * b = LCM$ of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $*$ *is* commutative

(iii) Is $*$ is associative

(iv) Find the identity of $*$ in N

12. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

OR

Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

13. By using properties of determinants.

Show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

14. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} |x| + 3 \text{ if } x \leq -3 \\ -2x \text{ if } -3 < x < 3 \\ 6x + 2 \text{ if } x \geq 3 \end{cases}$

OR

If the functions f(x), defined below is continuous at $x = 0$, find the value of k

$$f(x) = \begin{cases} \frac{1-\cos 2x}{2x^2}, x < 0 \\ k, \ x = 0 \\ \frac{x}{|x|}, \ x > 0 \end{cases}$$

15. If $y = e^{a\cos^{-1}x}, -1 \leq x \leq 1$. Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

OR

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

16. If $\cos y = x \cos(a + y)$ with $\cos a \neq \pm 1$. Prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

OR

If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that $\frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ is a constant independent a and b.

17. Verify Mean Value theorem if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$ where a= 1 and b= 4

18. Using differentials, find the approximate value of $(0.0037)^{\frac{1}{2}}$

19. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0)

20. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

21. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{-y-1}{6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{-5+y}{-2} = \frac{7-z}{-1}$

22. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdles is 5/6. What is the probability that he will knock down fear than 2 hurdles.

SECTION-C

23. If A= $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and using the product of AB, solve the following system of linear equations $x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$.

24. Find the equation of the plane which contains the of intersection of the planes $\vec{r}.(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0, \vec{r}.(2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$.

OR

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

25. Find the mean of the Binomial distribution $B\left(4, \frac{1}{3}\right)$

26. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $x = 0$ if he opposed, and $x = 1$ if he is in favour. Find E (x) and var (x).

27. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$. Then prove that $\frac{dy}{dx} = -\cot 3t$.

28. Differentiate $(x + \frac{1}{x})^x + x^{(1+\frac{1}{x})}$

29. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

OR

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

a) Parallel to the line $2x - y + 9 = 0$

b) Perpendicular to the line $5y - 15x = 13$
