Coordinate Geometry Formulae

2 – Dimension	**3 – Dimension**
Distance formula : for finding distance between two points $(x_1, y_1) \& (x_2, y_2)$ $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**Distance formula:** for finding distance between two points $(x_1, y_1, z_1) \& (x_2, y_2, z_2)$ $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Midpoint Formula: Midpoint of a line segment joining $(x_1, y_1) \& (x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	**Midpoint Formula:** Midpoint of a line segment joining $(x_1, y_1, z_1) \& (x_2, y_2, z_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Section Formula: The coordinates of the point dividing the line segment joining $(x_1, y_1) \& (x_2, y_2)$ in the ratio i) **m:n internally** is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$ ii)) **m:n externally** is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$	**Section Formula:** The coordinates of the point dividing the line segment joining $(x_1, y_1, z_1) \& (x_2, y_2, z_2)$ in the ratio i) **m:n internally** is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$ ii) **m:n externally** is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$
Slope (or gradient) Formula: Slope of a line joining the points $(x_1, y_1) \& (x_2, y_2)$is m = $\frac{y_2 - y_1}{x_2 - x_1}$	
Centroid Formula: Coordinates of the centroid of a triangle with vertices $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$	**Centroid Formula:** Coordinates of the centroid of a triangle with vertices $(x_1, y_1, z_1), (x_2, y_2, z_2) \& (x_3, y_3, z_3)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$
Area of a triangle with vertices $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$ is $\Delta = \frac{1}{2}\lvert x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\rvert$	

Remarks:

1. To prove that a triangle is equilateral , use distance formula; find the lengths of the three sides and show that they are equal.
2. To prove that a triangle is isosceles , use distance formula; show that any two of the sides are equal.
3. To prove that a triangle is right angled ;
   i) find the lengths of the three sides, using distance formula and show that square of the longest side is equal to sum of squares of the other two sides. OR
   ii) find the slopes of all the sides using slope formula and show that product of slopes of any two sides is equal to – 1. [*Two lines are perpendicular, if and only if the product of their slopes is -1*]

4. To prove that a quadrilateral is a parallelogram:
   i) using midpoint formula find the midpoint of both the diagonals. Show that they are same. This implies the diagonals bisect each other. Hence it is a parallelogram.
   ii) using slope formula, calculate the slopes of opposite sides and show they are equal. This implies opposite sides are parallel.[*Two lines are parallel, if and only if their slopes are equal*]
5. To prove three points A, B and C are collinear(i.e. points lying on the same line)
   i) use area formula and show that the area(ABC) = 0
   ii) use distance formula, calculate length of AB, BC and AC & show that sum of any two lengths is equal to the third length.
   iii) use slope formula and show that AB & BC have same slope. It implies they are parallel, but since one point is common in AB and BC; A, B and C are collinear.

Worked examples:

1. Find the distance between i) (- 3, 5) and (- 4, 2) ii) (- 3, 5, -2) and (- 4, 2, 0)

$$\text{Ans i) let } x_1 = -3, y_1 = 5$$
$$x_2 = -4, y_2 = 2$$
$$\sqrt{(-4-(-3))^2+(2-5)^2} = \sqrt{(-4+3)^2+(2-5)^2} = \sqrt{(-1)^2+(-3)^2} = \sqrt{10}$$
$$\text{it can also be calculated as } \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$\text{Ans ii) let } x_1 = -3, y_1 = 5, z_1 = -2$$
$$x_2 = -4, y_2 = 2, z_2 = 0$$
$$\sqrt{(-4-(-3))^2+(2-5)^2+(0-(-2))^2} = \sqrt{(-4+3)^2+(2-5)^2+(0+2)^2} = \sqrt{(-1)^2+(-3)^2+4} = \sqrt{14}$$

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2. Find the midpoint of line segment joining i) (- 3, 5) and (- 4, 2) ii) (- 3, 5, -2) and (- 4, 2, 0)

$$\text{Ans i) let } x_1 = -3, y_1 = 5$$
$$x_2 = -4, y_2 = 2$$
$$midpoint = \left(\frac{(-3)+(-4)}{2}, \frac{5+2}{2}\right) = \left(\frac{-7}{2}, \frac{7}{2}\right)$$
$$\text{Ans ii) let } x_1 = -3, y_1 = 5, z_1 = -2$$
$$x_2 = -4, y_2 = 2, z_2 = 0$$
$$midpoint = \left(\frac{-3+-4}{2}, \frac{5+2}{2}, \frac{-2+0}{2}\right) = \left(\frac{-7}{2}, \frac{7}{2}, -1\right)$$

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3. Find the coordinate of point R which divides PQ, internally in the ratio 2:1
   i) P(- 3, 5) and Q(- 4, 2), ii) P(- 3, 5, -2) and Q(- 4, 2, 0)

$$\text{Ans i) let } x_1 = -3, y_1 = 5$$
$$x_2 = -4, y_2 = 2$$
$$\text{let } m = 2 \text{ and } n = 1$$
$$R = \left(\frac{(2)(-4)+(1)(-3)}{2+1}, \frac{2(2)+1(5)}{2+1}\right) = \left(\frac{-11}{3}, 3\right)$$

$$\text{Ans ii) let } x_1 = -3, y_1 = 5, z_1 = -2$$
$$x_2 = -4, y_2 = 2, z_2 = 0$$
$$\text{let } m = 2 \text{ and } n = 1$$
$$R = \left(\frac{(2)(-4)+(1)(-3)}{2+1}, \frac{2(2)+1(5)}{2+1}, \frac{2(0)+1(-2)}{2+1} \right) = \left(\frac{-11}{3}, 3, \frac{-2}{3} \right)$$

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4. Find the coordinate of point R which divides the join of
   i) P(- 3, 4) and Q(- 4, 2), ii) P(- 3, 4, -2) and Q(- 4, 2, 0) externally in the ratio 2:1

$$\text{Ans i) let } x_1 = -3, y_1 = 5$$
$$x_2 = -4, y_2 = 2$$
$$\text{let } m = 2 \text{ and } n = 1$$
$$R = \left(\frac{(2)(-4)-(1)(-3)}{2-1}, \frac{2(2)-1(5)}{2-1} \right) = (-5, -1)$$
$$\text{Ans ii) let } x_1 = -3, y_1 = 5, z_1 = -2$$
$$x_2 = -4, y_2 = 2, z_2 = 0$$
$$\text{let } m = 2 \text{ and } n = 1$$
$$R = \left(\frac{(2)(-4)-(1)(-3)}{2-1}, \frac{2(2)-1(5)}{2-1}, \frac{2(0)-1(-2)}{2-1} \right) = (-5, -1, 2)$$

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5. Find the slope of the line segment joining (- 3, 5) and (- 4, 2).

$$\text{Ans let } x_1 = -3, y_1 = 5$$
$$x_2 = -4, y_2 = 2$$
$$m = \frac{2-5}{(-4)-(-3)} = 3$$

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6. Find the coordinates of the centroid of a triangle with vertices
   i) P(- 3, 5), Q(- 4, 2) & R(1, - 5) ii) P(- 3, 5, -2) , Q(- 4, 2, 0) and R(1, - 5, 4)

$$\text{Ans i) let } x_1 = -3, y_1 = 5,$$
$$x_2 = -4, y_2 = 2$$
$$x_3 = 1, y_3 = -5$$
$$centroid = \left(\frac{(-3)+(-4)+1}{3}, \frac{5+2+(-5)}{3} \right) = \left(-2, \frac{2}{3} \right)$$
$$\text{Ans ii) let } x_1 = -3, y_1 = 5, z_1 = -2$$
$$x_2 = -4, y_2 = 2, z_2 = 0$$
$$x_3 = 1, y_3 = -5, z_3 = 4$$
$$centroid = \left(\frac{(-3)+(-4)+1}{3}, \frac{5+2+(-5)}{3}, \frac{(-2)+0+4}{3} \right) = \left(-2, \frac{2}{3}, \frac{2}{3} \right)$$

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7. Find the area of the triangle with vertices P(- 3, 5), Q(- 4, 2) & R(1, - 5).

$$\text{Ans i) let } x_1 = -3, y_1 = 5,$$
$$x_2 = -4, y_2 = 2$$
$$x_3 = 1, y_3 = -5$$

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|(-3)(2-(-5)) + (-4)((-5)-5) + 1(5-2)\right|$$

$$= \frac{1}{2}\left|-21 + 40 + 3\right| = 11 \text{sq. units}$$
