① Eqn of plane passing thru' $(1,5,4),(2,4,4)$ & $(2,6,2)$ is

$$\begin{vmatrix} x-1 & y-5 & z-4 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix} = 0$$

expanding the determinant we get $x+y+z-10=0$ —①

Now substituting the point $(0,6,4)$ in the equation of plane ①

$$0+6+4-10=0$$
$$0=0$$

it satisfies the equation
$\Rightarrow$ all four points are coplanar.
Eq. of reqd. plane is

$$x+y+z-10=0$$

② Eqn. of plane passing thru' $(3,-1,2)$, $(5,2,4)$ & $(-1,-1,6)$ is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$\Rightarrow 3x-4y+3z-19=0$ —①

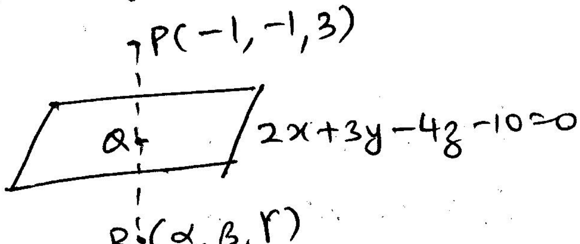
using $\perp$ distance formula distance of $(6,5,9)$ from Plane ① is

$$d = \left| \frac{3(6)-4(5)+3(9)-19}{\sqrt{3^2+4^2+3^2}} \right|$$

$$= \frac{6}{\sqrt{34}} = \frac{3\sqrt{34}}{17} \text{ units.}$$

(See eg. 28 page 495)

③



Eq. of PQ is $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-3}{-4} = \lambda$

$\therefore$
$$\left.\begin{aligned} x &= 2\lambda-1 \\ y &= 3\lambda-1 \\ z &= -4\lambda+3 \end{aligned}\right\} \text{coord. of } Q$$

Since Q is on the plane, it should satisfy the equation of plane. Sub. coordinates of Q in the equation of plane

$$2(2\lambda-1)+3(3\lambda-1)-4(-4\lambda+3)-10=0$$

solving $\lambda = 27/29$

$$\therefore \underbrace{x=\frac{25}{29},\ y=\frac{52}{29},\ z=\frac{-21}{29}}_{\text{coord. of } Q}$$

But Q is the mid point of PR

$$\therefore Q = \left(\frac{-1+\alpha}{2}, \frac{-1+\beta}{2}, \frac{3+\gamma}{2}\right)$$

$\Rightarrow \frac{25}{29} = \frac{-1+\alpha}{2}$

$\Rightarrow \alpha = \frac{79}{29}$,

Similarly $\beta = \frac{133}{29}$ & $\gamma = \frac{-129}{29}$

Hence image $= \left(\frac{79}{29}, \frac{133}{29}, \frac{-129}{29}\right)$

④ $\vec{a}_1 = -3\hat{i}+\hat{j}+5\hat{k}$

$\vec{a_2} = -\hat{i}+2\hat{j}+5\hat{k}$

$\vec{b}_1 = -3\hat{i}+\hat{j}+5\hat{k}$

$\vec{b_2} = -\hat{i}+2\hat{j}+5\hat{k}$

$\bar{a}_2-\bar{a}_1 = 2\hat{i}+\hat{j}$

$\bar{b}_1\times\bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= -5\hat{i}+10\hat{j}-5\hat{k}$

$(\bar{a_2}-\bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2)$

$= -10+10 = 0$

$\Rightarrow$ the lines are coplanar as shortest distance $= 0$

$\vec{n} = \vec{b}_1\times\vec{b}_2 = -5\hat{i}+10\hat{j}-5\hat{k}$

d.r. of normal $= -5, 10, -5$

$= -1, 2, -1$

Eq. of plane is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$-1(x+3)+2(y-1)-1(z-5)=0$

$\Rightarrow x-2y+z=0$

⑤ Method I:

Show that shortest dist $= 0$

Since the d.r.'s of the line are not proportional they are not parallel $\therefore$ intersecting

Method II

from line ①

$x = 3\lambda-1,\ y = 5\lambda-3,\ z = 7\lambda-5$

from line ②

$x = \mu+2,\ y = 3\mu+4,\ z = 5\mu+6$

If they intersect, if the two above points are same

$3\lambda-1 = \mu+2 \Rightarrow 3\lambda-\mu = 3$ — ①

$5\lambda-3 = 3\mu+4 \Rightarrow 5\lambda-3\mu = 7$ — ②

Solving ① & ② $\lambda = \frac{1}{2}$ & $\mu = -\frac{3}{2}$

Sub. λ & μ in z-coordinate of each line

$z = 7\lambda-5$	$z = 5\mu+6$
$= 7(\frac{1}{2})-5$	$= 5(-\frac{3}{2})+6$
$= -\frac{3}{2}$	$= -\frac{3}{2}$

Hence they intersect.

Point of intersection is

$x = \frac{1}{2},\ y = -\frac{1}{2},\ z = -\frac{3}{2}$

$(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2})$

⑥ Eq. of line is

$$\frac{x+1}{a} = \frac{y-2}{b} = \frac{z-3}{c} \quad \text{—①}$$

Since it is $\perp$ to other two lines $\vec{b} = \bar{b}_1 \times \bar{b}_2$

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & -1 \\ -1 & 2 & 3 \end{vmatrix}$$

$$= -10\hat{i} - 5\hat{j} + 0\hat{k}$$

$a = -10, \ b = -5, \ c = 0$

Sub. in ①

$$\frac{x+1}{-10} = \frac{y-2}{-5} = \frac{z-3}{0}$$

or

$$\frac{x+1}{-2} = \frac{y-2}{-1} = \frac{z-3}{0}$$

⑦ Eq. of plane is

$$a(x-1) + b(y+3) + c(z-4) = 0 \quad \text{—①}$$

Since the reqd. plane is $\perp$ to the other two planes, the reqd normal is $\perp$ to the other two normals

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 6 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= 16\hat{i} + 12\hat{j} - 12\hat{k}$$

d.r.'s of the normal are

$16, 12, -12$

i.e. $4, 3, -3$

$\therefore a = 4, \ b = 3, \ c = -3$

sub. in ①.

$$4(x-1) + 3(y+3) - 3(z-4) = 0$$

$$4x + 3y - 3z + 17 = 0$$

⑧ The normal to the plane is $\perp$ to both the lines

$$\cancel{\therefore \ \vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 2 \\ 2 & -1 & 1 \end{vmatrix}}$$

The point $(2, -1, 1)$ is on the plane

$$\therefore a(x-2) + b(y+1) + c(z-1) = 0 \quad \text{—①}$$

Also $(2, 4, 6)$ is a pt. on the plane

$$\therefore a(2-2) + b(4+1) + c(6-1) = 0$$

$$\Rightarrow \quad b + c = 0 \quad \text{or} \quad b = -c$$

Since the normal is $\perp$ to the lines on the plane

$$\vec{b} \cdot \vec{n} = 0$$

$$\Rightarrow \quad a(2) + b(-1) + c(1) = 0$$

$$\Rightarrow \quad 2a - b + c = 0$$

$$\Rightarrow \quad 2a + c + c = 0 \quad \Rightarrow a = -c$$

$[\because b = -c]$

the d.r.'s of the normal are

$a, b, c = -c, -c, c$

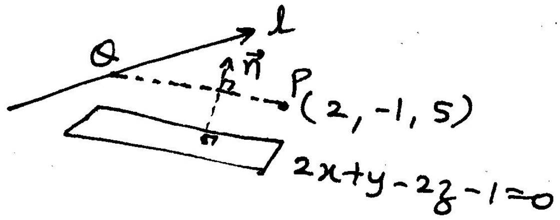
i.e. $-1, -1, 1$

Sub. in ①

$$-1(x-2) + -1(y+1) + 1(z-1) = 0$$

$$\Rightarrow x+y-z=0$$

(9)



from equation of line 'l'

$$x = 2\lambda+2,\ y = -\lambda+1,\ z = 2\lambda+1$$

$$\therefore Q = (2\lambda+2, -\lambda+1, 2\lambda+1)$$

$$\text{d.r. of } PQ = \{2\lambda+2-2,\ -\lambda+1+1,\ 2\lambda+1-5$$

$$= 2\lambda,\ -\lambda+2,\ 2\lambda-4$$

Since $\vec{n} \perp \vec{PQ}$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (2)(2\lambda) + (1)(-\lambda+2) - 2(2\lambda-4) = 0$$

$$\Rightarrow \cancel{4\lambda} - \lambda + 2 - \cancel{4\lambda} + 8 = 0$$

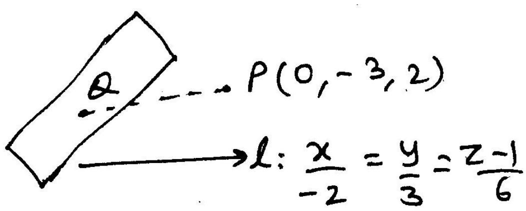
$$\lambda = 10$$

$$\therefore Q = (22, -9, 21)$$

$$PQ = \sqrt{(22-2)^2 + (-9+1)^2 + (21-5)^2}$$

$$= \sqrt{20^2 + 8^2 + 16^2} = 12\sqrt{5}$$

(10)



Since $PQ \parallel l$, their d.r's are proportional

Eq. of PQ is $\dfrac{x-0}{-2} = \dfrac{y+3}{3} = \dfrac{z-2}{6}$

$$\Rightarrow x = -2\lambda,\ y = 3\lambda-3,\ z = 6\lambda+2$$

$$\therefore \text{Coordinate of } Q = (-2\lambda, 3\lambda-3, 6\lambda+2)$$

But Q is on the plane so sub coord of Q in the eqn. of plane

$$2(-2\lambda) + (3\lambda-3) - (6\lambda+2) = 5$$

$$-4\lambda + 3\lambda - 6\lambda - 5 = 5$$

$$\lambda = -10/7$$

$$\therefore Q = \left(\frac{20}{7}, -\frac{51}{7}, -\frac{46}{7}\right)$$

$$PQ = \sqrt{\left(\frac{20}{7}\right)^2 + \left(\frac{30}{7}\right)^2 + \left(\frac{60}{7}\right)^2}$$

$$= \frac{10}{7}\sqrt{2^2+3^2+6^2}$$

$$= \frac{10}{\cancel{7}} \times \sqrt{\cancel{4+9+36}} = 10 \text{ units}$$

(11)

$$\bar{a}_2 = 3\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\bar{a}_1 = \hat{i} - 7\hat{j} - 2\hat{k}$$

$$\bar{b}_1 = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\bar{b}_2 = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\bar{a}_2 - \bar{a}_1 = 2\hat{i} + 11\hat{j}$$

$$\bar{b}_1 \times \bar{b}_2 = -\hat{i} - 3\hat{j} + 5\hat{k}$$

$$(\bar{a}_2 - \bar{a}_1)\cdot(\bar{b}_1 \times \bar{b}_2) = -35$$

$$S.D. = \left|\frac{-35}{\sqrt{35}}\right| = \sqrt{35}$$

(12) $\vec{n} = \vec{n_1} \times \vec{n_2}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -3 \\ 5 & -4 & 1 \end{vmatrix}$$

$$= -10\hat{i} - 18\hat{j} - 22\hat{k}$$

Eq. of plane is

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

$$-10(x-1) + (-18)(y+1) + (-22)(z-2) = 0$$

$$\Rightarrow 5x + 9y + 11z - 18 = 0$$

(13) Eq. of reqd plane is

$$(2x - 3y + 4z - 1) + \lambda(x - y + 4) = 0 \quad \text{(1)}$$

$$\Rightarrow (2+\lambda)x + (-3-\lambda)y \cancel{+(4+\ldots)} + 4z - 1 + 4\lambda = 0$$

d.r. of the normal

$$= 2+\lambda, -3-\lambda, 4$$

d.r. of the $\perp$ normal

$$= 2, -3, 1$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$2(2+\lambda) - 3(-3-\lambda) + 4 = 0$$

$$\lambda = -17/5$$

Substituting in eq. (1)

$$(2x - 3y + 4z - 1) - \frac{17}{5}(x - y + 4) = 0$$

$$10x - 15y + 20z - 5 - 17x + 17y - 68 = 0$$

$$-7x + 2y + 20z - 73 = 0$$

$$\Rightarrow 7x - 2y - 20z + 73 = 0$$

2