3 – D Geometry Worksheet

1. Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) **(ii) (–3, 7, 2) and (2, 4, –1)**

(iii) (–1, 3, – 4) and (1, –3, 4) **(iv) (2, –1, 3) and (–2, 1, 3).**

Ans: i) $2\sqrt{5}$ ii) $\sqrt{43}$ iii) $2\sqrt{26}$ iv) $2\sqrt{5}$

2. Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

[refer previous worksheet Remark 5]

3. Verify the following:

(i) (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (– 4, 9, 6) are the vertices of a right angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

[refer previous worksheet Remark 2, 3, 4]

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

[Hint: let the point be P(x, y, z). given points be A(1, 2, 3) and B(3, 2, - 1). Then PA = PB] ans: $x - 2z = 0$

5. Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (– 4, 0, 0) is equal to 10.

[Hint: let the point be P(x, y, z). given points be A(4, 0, 0) and B(-4, 0, 0). Then PA + PB = 10]

6. Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio (i) 2 : 3 internally, (ii) 2 : 3 externally. Ans: i) (-4/5, 1/5, 27/5) ii) (-8, 17, 3)

7. Given that P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR. Ans : 1:2

8. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

[hint: any point on YZ plane is of the form (0, y, z)] ans: 2:3

9. Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and C (0, 1/3, 2) are collinear.

[refer Textbook page 275 , eg. 8, ch. 12]

10. Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).

[hint: points of trisection divides the line segment into three equal parts. So use the ratio 1 : 2 and 2 : 1]

Ans : (6, -4, -2) and (8, -10, 2)

11. Show that the points A (1, 2, 3), B (–1, –2, –1), C (2, 3, 2) and D (4, 7, 6) are the vertices of a parallelogram ABCD, but it is not a rectangle.

[hint: show AB = CD = 6, AD = BC = $\sqrt{43}$ but AC $\neq$ BD using distance formula]

12. Find the equation of the set of the points P such that its distances from the points A (3, 4, –5) and B (– 2, 1, 4) are equal.

[hint: using distance formula PA and PB and equate it.] ans: $10x + 6y - 18z - 29 = 0$.

13. The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, – 6), respectively, find the coordinates of the point C.

[hint: use centroid formula. Let the third vertex be C(p, q, r)] ans: (1, 1, 2)

14. Three vertices of a parallelogram ABCD are A(3, – 1, 2), B (1, 2, – 4) and C (– 1, 1, 2). Find the coordinates of the fourth vertex. Ans: (1, - 2, 8)

15. Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0,4, 0) and (6, 0, 0).

[hint: first find the midpoints of the three sides of the triangle using the midpoint formula . Median of a triangle joins the vertex of a triangle to the midpoint of the opposite side. Then use the distance formula] Ans: 7, $\sqrt{34}$ 7

16. If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (– 4, 3*b*, –10) and R(8, 14, 2*c*), then find the values of *a*, *b* and *c*.

[hint: use centroid formula] ans: a = - 2 , b = - 16/3, c = 2

17. Find the coordinates of a point on *y*-axis which are at a distance of $5\sqrt{2}$ from the point P (3, –2, 5).

[hint: let the point on y – axis be Q (0, y, 0). Then find PQ using distance formula and equate it to $5\sqrt{2}$]

Ans : (0, 2, 0) and (0, – 6, 0)

18. A point R with *x*-coordinate 4 lies on the line segment joining the points P(2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[hint: let coordinate of R = (4, y, z). let R divide PQ in the ratio k:1. Using section formula, find the coordinate of R and equate its x – coordinate to 4. Solve to find value of k. then using value of k , find y and z] ans: (4, -2, 6)

19. If A and B be the points (3, 4, 5) and (–1, 3, –7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where *k* is a constant.

[hint: let P = (x, y, z) . using distance formula find PA and PB and substitute in the given equation]

Ans : $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$

20. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, – 8) is divided by the XZ- plane. Also find the coordinates.

[hint: let the point on XZ – plane be P(x, 0, z) . let the given points be A (4, 8, 10) and B (6, 10, – 8). Let P divide AB in the ratio k:1. Using section formula find coordinates of P and equate the y coordinate to zero.]

Ans : k = -4/5(negative sign implies 4:5 externally) P = (-4, 0, 82)