1. **Direction Ratios** of a line joining two points P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) are
   $x_2 - x_1,\quad y_2 - y_1,\quad z_2 - z_1$
2. The **direction cosines** are $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$ where PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
3. The **direction cosines** are represented by l, m, n where $l = \cos\alpha = \frac{x_2 - x_1}{PQ}$; m = $\cos\beta = \frac{y_2 - y_1}{PQ}$ and

   n = $\cos\gamma = \frac{z_2 - z_1}{PQ}$
4. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ and $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
5. **Equation Of A Line:**
   a) The vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
   b) The Cartesian equation of a line through a point P(x_1, y_1, z_1) and having direction ratios a, b, c is
   $$\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$$
   c) If direction cosines are l, m and n then $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$
   d) Parametric equation of a line is x = $x_1 + \lambda$ a, y = $y_1 + \lambda$ b and z = $z_1 + \lambda$ c. any point on the line is of the form ($x_1 + \lambda$ a, $y_1 + \lambda$ b , $z_1 + \lambda$ c)
   e) Vector form of a line passing through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$
   f) Cartesian equation of a line passing through two points P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) is
   $$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

6. **Angle Between Two lines:**
   a) If θ is the angle between two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ then $\boxed{\cos\theta = \frac{\vec{b}_1 \bullet \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}}$

   b) If θ is the angle between two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$ then
   $$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1)^2 + (b_1)^2 + (c_1)^2}\sqrt{(a_2)^2 + (b_2)^2 + (c_2)^2}}$$
   c) condition for i) perpendicularity : $a_1a_2 + b_1b_2 + c_1c_2 = 0$
   ii) parallelism : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

7. **Shortest Distance between Two Lines:**
   a) Lines in space which are neither parallel nor intersecting are called skew lines.
   b) The shortest distance between two skew lines is the length of the line segment perpendicular to both lines.
   c) Shortest distance between two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ is $\boxed{d = \left|\frac{(\vec{a}_2 - \vec{a}_1) \bullet (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|}$
   d) two lines will intersect iff’ d = 0, i.e. $(\vec{a}_2 - \vec{a}_1) \bullet (\vec{b}_1 \times \vec{b}_2) = 0$
   e) two lines will be coplanar iff’ d = 0, i.e. $(\vec{a}_2 - \vec{a}_1) \bullet (\vec{b}_1 \times \vec{b}_2) = 0$
   f) distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is d = $\left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$ or $\left|\frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$

PLANE

1. **Equation of a plane:**

   i) Vector equation of a plane which is at a distance d from the origin and $\hat{n}$ is the unit vector normal to the plane through the origin is $\vec{r}\cdot\hat{n} = d$. [Normal Form]

   ii) Vector equation of a plane which is at a distance d from the origin and direction cosines of the normal to the plane l, m, n is $lx + my + nz = d$ [Normal Form]

   iii) Equation of a plane through a point with position vector $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $(\vec{r}-\vec{a})\cdot\vec{N} = 0$.

   iv) Cartesian equation of a plane passing through a given point (x_1, y_1, z_1) and perpendicular to a given line with direction ratios A, B, C is $\boxed{A(x-x_1)+B(y-y_1)+C(z-z_1)=0.}$

   v) Equation of a plane passing through three points (x_1, y_1, z_1) and (x_2, y_2, z_2) and (x_3, y_3, z_3) is

   $$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

   vi) Vector equation of a plane passing through three points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

   $$\boxed{(\vec{r}-\vec{a})\cdot\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0}$$

   vii) Equation of a plane cutting intercepts a, b and c at coordinate axes X, Y and Z axes respectively is

   $$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

   viii) Vector equation of a plane that passes through the intersection of the planes $\vec{r}\cdot\vec{n}_1 = d_1$ & $\vec{r}\cdot\vec{n}_2 = d_2$ is

   $$\boxed{\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2}$$

   ix) Cartesian equation of a plane that passes through the intersection of two given planes

   $$\boxed{(A_1x+B_1y+C_1z+D_1)+\lambda(A_2x+B_2y+C_2z+D_2)=0}$$

2. **Two planes** $A_1x+B_1y+C_1z+D_1=0$ & $A_2x+B_2y+C_2z+D_2=0$ **are coplanar if**

   $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = 0$$

3. **i) If θ is the angle between the planes** $\vec{r}\cdot\vec{n}_1 = d_1$ & $\vec{r}\cdot\vec{n}_2 = d_2$ then $\boxed{\cos\theta = \frac{\vec{n}_1\bullet\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}}$

   ii) **If θ is the angle between the planes** $A_1x+B_1y+C_1z+D_1=0$ & $A_2x+B_2y+C_2z+D_2=0$ then

   $$\cos\theta = \frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{(A_1)^2+(B_1)^2+(C_1)^2}\sqrt{(A_2)^2+(B_2)^2+(C_2)^2}}$$

   iii) If ϕ is the **angle between the line** $\vec{r}=\vec{a}+\lambda\vec{b}$ **and the plane** $\vec{r}\cdot\vec{n}=d$ then $\boxed{\sin\phi = \frac{\vec{b}\bullet\vec{n}}{|\vec{b}||\vec{n}|}}$

4. **Distance of a point** (x_1, y_1, z_1) from the plane Ax + By + Cz + D = 0 is d = $\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$

   ii) Distance between two parallel planes Ax + By + Cz + D_1 = 0 & Ax + By + Cz + D_2 = 0 is

   $$d = \left|\frac{D_2-D_1}{\sqrt{A^2+B^2+C^2}}\right|$$