Notes Ch.4 Determinants

1. **Determinant:** If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then ad – bc is known as the determinant of A, denoted by det(A) or Δ or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.
2. **Minor:** Minor (M_{ij}) of an element a_{ij} is the determinant obtained by deleting its i^{th} row and j^{th} column and it is denoted by M_{ij}.
3. **Cofactor :** Cofactor (A_{ij}) of an element a_{ij} is calculated by $A_{ij} = (-1)^{i+j} M_{ij}$.
4. Value of determinant of a matrix A is obtained by the sum of products of elements of a row (or a column) with corresponding co-factors. For example $|A| = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$.
5. If elements of a row (or column) are multiplied with co-factors of elements of any other row (or column), then their sum is zero. For example, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$.
6. **Adjoint :** Adjoint of a square matrix is obtained by interchanging the rows and columns (transpose) of the matrix containing cofactors. It is represented by adjA.
7. The adjoint of a square matrix of order 2 can be obtained by interchanging the principal diagonal elements and changing the sign of elements on the skew diagonal.
8. If A & B are square matrices of order n, then i) A (adjA) = $|A|I$ = (adjA) A ii) adj(AB) = (adj B) (adj A) iii) adj(A^T) = (adj A)^T iv) $|adj\ A| = |A|^{n-1}$ v) adj(adj A) = $|A|^{n-2}A$
9. **Singular matrix:** A square matrix is said to be singular if its determinant is zero (|A| = 0). Otherwise its said to be a non singular matrix.
10. A square matrix is invertible iff its non – singular.
11. **Inverse of a matrix:** If A is a non singular matrix then $A^{-1} = \frac{1}{|A|} adjA$
12. If A and B are square matrices of the same order then $|AB| = |A||B|$. The determinant of the product of matrices is equal to product of their respective determinants.
13. If A is a square matrix of order n, then $|kA| = k^n|A|$.
14. If value of determinant 'Δ' becomes zero by substituting x = α, then x – α is a factor of 'Δ'.
15. A (adj A) = (adj A) A = |A| I, where A is square matrix of order n.
16. If A is a square matrix of order n, then $|adj\ A| = |A|^{n-1}$.
17. If A is an invertible matrix of order 2, then det(A^{-1}) = $\frac{1}{|A|}$
18. If A and B are non-singular matrices of the same order, then AB and BA are also nonsingular matrices of the same order.
19. If AB = BA = I, where A and B are square matrices, then B is called inverse of A and is written as $B = A^{-1}$. Also $B^{-1} = A$. for e.g. If AB = 8I then $A^{-1} = (1/8)B$ and $B^{-1} = (1/8)A$.
20. **Consistentency of a system of equations:** A system of equations is consistent or inconsistent according as its solution exists or not.
21. **Solving a system of equations by Matrix Method:** For a square matrix A in matrix equation AX = B
(a) If |A| ≠ 0, then there exists unique solution. $X = A^{-1}B$
(b) If |A| = 0 and (adj A) B ≠ 0, then there exists no solution.
(c) If |A| = 0 and (adj A) B = 0, then system may or may not be consistent.