2008_Delhi

1. For what value of k is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x+1, & x < 2 \\ k, & x = 2 \\ 3x-1, & x > 2 \end{cases}$$ [4 marks]

2. Find the equation of the tangent to the curve x = sin 3t, y = cos 2t; at t = π/4. [4 marks]

3. **OR** Differentiate $\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ w.r.t x [4 marks]

4. Show that the rectangle of maximum area that can be inscribed in a circle is a square. [6 marks]

5. **OR** Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height 'h' is $\frac{1}{3}$h. [6 marks]

2008 _ Foreign

6. Let $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & if \quad x < \dfrac{\pi}{2} \\ a, & if \quad x = \dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & if \quad x > \dfrac{\pi}{2} \end{cases}$ If f(x) be a continuous function at x = $\frac{\pi}{2}$, find a and b.

[4 marks]

7. If $y = \left[\log(x+\sqrt{x^2+1})\right]^2$ show that $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-2=0$ [4 marks]

8. **OR** If $y = x^x + (\sin x)^x$, find $\dfrac{dy}{dx}$ [4 marks]

9. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to y – coordinate of the point. [4 marks]

10. A point on the hypotenuse of a triangle is at distances *a* and *b* from the sides. Show that the minimum length of the hypotenuse is $\left(a^{2/3}+b^{2/3}\right)^{3/2}$ [6 marks]

11. **OR** Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere. [6 marks]

2009-Delhi

12. Differentiate w.r.t. x i) $x^{\sin x} + (\sin x)^{\cos x}$ (set 1) ii) $y = (\sin x)^x + \sin^{-1}\sqrt{x}$ (set 2)
iii) $(x)^{\cos x} + (\sin x)^{\tan x}$ [4 marks]

13. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line 4x – 2y + 5 = 0.

14. **OR** Find the intervals in which the function $f(x) = x^3 + \dfrac{1}{x^3}$ is i) increasing ii) decreasing. [4 marks]

15. Find the volume of the largest cylinder that can be inscribed in a sphere of radius *r* . [6 marks]

16. **OR** A tank with rectangular base and rectangular sides , open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs. 70 per sq. metre for the base and Rs. 45 per sq. metre for sides, what is the cost of the least expensive tank? [6 marks]

2009_AI

17. The length x of a rectangle is decreasing at the rate of 5cm/min and the width y is increasing at the rate 4cm/minute. When x = 8cm and y = 6cm, find the rate of change of (a) the perimeter, (b) the area of the rectangle. [4 marks]

18. **OR** Find the intervals in which the function f(x) = sinx + cosx , $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing. [4 marks]

19. If sin y = x. sin(a + y) , prove that $\frac{dy}{dx} = \frac{Sin^2(a+y)}{Sina}$ [4 marks]

20. **OR** If $(\cos x)^y = (\sin y)^x$, find dy/dx. [4 marks]

21. (Set 1) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

    (Set 2) If y = $3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

    (Set 3) If y = e^x(sinx + cosx), then show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ [4 marks]

22. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$. [6 marks]

23. **OR** A manufacturer can sell x items at a price of Rs. $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$. find the number of items he should sell to earn maximum profit. [6 marks]

2009_Foreign

24. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$ [4 marks]

25. **OR** If x = a(cos t + t sint) and y = a(sint – tcost) find $\frac{d^2y}{dx^2}$ [4 marks]

26. Find the equation of the tangent to the curve y = $\sqrt{4x-2}$ which is parallel to the line 4x – 2y + 5 = 0.

27. **OR** Using Differentials , find the approximate value of f(2.01), where f(x) = $4x^3 + 5x^2 + 2$ [4 marks]

28. (Set 1)Find dy/dx, if $x^y + y^x = a^b$, where a, b are constants.

    (Set 2) If y = acos(logx) + bsin(logx), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

    (Set 3) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, Prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ [4 marks]

29. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find its maximum volume. [6 marks]

30. **OR** Show that the total surface area of a closed cuboid with square base and given volume, is minimum, when it is a cube. [6 marks]

2010_Delhi

31. (Set 1)If $y = Sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$, 0 < x < 1 , find $\frac{dy}{dx}$

[Hint : Put $x = Sin\theta$ and $\sqrt{x} = Sin\phi$] [4 marks]

(Set 2) Find $\frac{dy}{dx}$**,** if y = $(\cos x)^x + (\sin x)^{1/x}$

(Set 3)

32. **OR** (Set 1)Show that the function f defined as follows, is continuous at x = 2, but not differentiable thereat: f(x) = $\begin{cases} 3x-2 & , \quad 0 < x \le 1 \\ 2x^2 - x & , \quad 1 < x \le 2 \\ 5x-4 & , \quad x > 2 \end{cases}$ [4 marks]

(Set 2) find all points of discontinuity of f, where f is defined as follows: f(x) = $\begin{cases} |x|+3 & , \quad x \le -3 \\ -2x & , \quad -3 < x < 3 \\ 6x+2 & , \quad x \ge 3 \end{cases}$

33. (Set 1) Find the points on the curve y = x^3 at which the slope of the tangent is equal to the y – coordinate of the point. [4 marks]
(Set 2)Find the equations of the normals to the curve y = x^3 + 2x + 6 which are parallel to the line x + 14y + 4 = 0.
(Set 3) Find the equation of the tangent to the curve y = $\frac{x-7}{(x-2)(x-3)}$, at the point, where it cuts the x – axis.

34. Show that the right circular cylinder open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base. [6 marks]

35. (Set 1 & 2)Find the values of x for which f(x) = $[x(x-2)]^2$, is an increasing function. Also, find the points on the curve , where the tangent is parallel to x – axis. [6 marks]
(Set 3) Find the intervals in which the function f given by f(x) = sin x – cos x , $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing. [6 marks]

2010_AI

36. If y = $e^{a\sin^{-1}x}$, $-1 \le x \le 1$, then show that (1 – x^2) $\frac{d^2y}{dx^2}$ – x $\frac{dy}{dx}$ – a^2y = 0 [4 marks]

37. If y = $\cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$, find $\frac{dy}{dx}$ Hint:let x = cosθ;3 = rcosα, 4 = rsinα ;ans: $-1/\sqrt{(1-x^2)}$ [4 marks]

38. If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium when it is maximum. [6 marks]

39. Find the intervals in which the following function is (a) strictly increasing (b) strictly decreasing: [question was asked incomplete in the examination] [6 marks]
$x^4 - 8x^3 + 22x^2 - 24x + 21$ or $\sin^4 x + \cos^4 x$ in $[0, \pi/2]$

40. (Set 2) Find the equations of the tangent and the normal to the curve:

$x = 1 - \cos\theta$; $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$ [6 marks]

ans $\left(\sqrt{2}-1\right)x - y + \frac{\pi}{4} - 2\sqrt{2} + 2 = 0$; $\left(\sqrt{2}+1\right)x + y - \frac{\pi}{4} = 0$

41. (Set 3) If $y = \operatorname{cosec}^{-1}x$, $x > 1$, then show that $x(x^2 - 1)\frac{d^2y}{dx^2} + (2x^2 - 1)\frac{dy}{dx} = 0$. [4 marks]

42. (Set 3) Show that the volume of the greatest cylinder that can be inscribed in a cone of height 'h' and semi-vertical angle 'α' is $\frac{4}{27}\pi h^3 \tan^2\alpha$ [6 marks]

2010_Foreign

43. If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx} = \sec x$. Also find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$

44. If $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, find $\frac{dy}{dx}$. [4 marks]

45. The lengths of the sides of an isosceles triangle are $9 + x^2$, $9 + x^2$ and $18 - 2x^2$ units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum. [6 marks]

46. (Set 2) Differentiate the following function w. r. t. x: $f(x) = Tan^{-1}\left[\frac{1-x}{1+x}\right] - Tan^{-1}\left[\frac{x+2}{1-2x}\right]$ [4 marks]

47. (Set 3) If $x = a(\cos t + \log\tan\frac{t}{2})$; $y = a(1 + \sin t)$, find $\frac{d^2y}{dx^2}$. [4 marks]

48. (Set 3)The sum of the perimeter of a circle and a square is K, where K is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle. [6 marks]

2010_Foreign_Comptmnt.

49. If $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at x = 1. Determine the values of a and b. [4 marks]

50. If $y = \cos^{-1}\left(\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}}\right)$, find $\frac{dy}{dx}$

51. **OR** If $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$ then find $\frac{dy}{dx}$ [4 marks]

52. Find the intervals in which the following function is (a) increasing (b) decreasing:

$f(x) = 2x^3 - 15x^2 + 36x + 17$ [4 marks]

53. **OR** Find the equation of the tangent to the curve $x = a\sin^3 t$, $y = b\cos^3 t$ at the point where $t = \frac{\pi}{4}$

54. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the total surface area is least when depth of the tank is half its width.

[6 marks]

55. (Set 3) Let f(x) = $\begin{cases} \dfrac{1-\cos 2x}{2x^2} & if \quad x<0 \\ k & if \quad x=0 \\ \dfrac{x}{|x|} & if \quad x>0 \end{cases}$ Be continuous at x = 0, find the value of k. [4 marks]

56. (Set 3) Two poles of heights 16 m and 22 m stand vertically on the ground 20 m apart. Find a point on the ground, in between the poles, such that the sum of the squares of the distances of this point from the tops of the poles is minimum. [6 marks]

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57. Find the value of k so that the function f defined by

f(x) = $\begin{cases} \dfrac{k\cos x}{\pi-2x} & if \quad x \neq \dfrac{\pi}{2} \\ 3 & if \quad x=\dfrac{\pi}{2} \end{cases}$ is continuous at x = $\dfrac{\pi}{2}$. [4 marks]

58. Find the intervals in which the function f given by f(x) = sin x + cos x , 0 ≤ x ≤ 2π, is strictly increasing or strictly decreasing. [4 marks]

59. **OR** Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y – coordinate of the point. [4 marks]

60. Prove that $\dfrac{d}{dx}\left[\dfrac{x\sqrt{a^2-x^2}}{2}+\dfrac{a^2}{2}Sin^{-1}\left(\dfrac{x}{a}\right)\right] = \sqrt{a^2-x^2}$ [4 marks]

61. **OR** If $y=\log\left(x+\sqrt{x^2+1}\right)$ prove that $\left(x^2+1\right)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=0$. [4 marks]

62. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening. [6 marks]

63. (Set 2) Find the value of k so that the function f defined by

f(x) = $\begin{cases} kx+1 & if \quad x\leq\pi \\ \cos x & if \quad x>\pi \end{cases}$ is continuous at x = π. [4 marks]

64. (Set 3) For what value of λ is the function f(x) = $\begin{cases} \lambda(x^2-2x) & if \quad x\leq 0 \\ 4x+1 & if \quad x>0 \end{cases}$ is continuous at x = 0.