

S. No.	DIFFERENTIATION		INTEGRATION	
1	$\frac{d}{dx}(x^n)$	$nx^{n-1}$	$\int x^n dx$	$\frac{x^{n+1}}{n+1} + C$
2	$\frac{d}{dx}(ax+b)^n$	$n(ax+b)^{n-1}a$	$\int (ax+b)^n dx$	$\frac{(ax+b)^{n+1}}{(n+1)a} + C$
3	$\frac{d}{dx}(k)$	0	$\int o dx$	C
4	$\frac{d}{dx}(ku)$	$k \frac{du}{dx} = ku'$	$\int k dx$	$kx + C$
5	$\frac{d}{dx}(u \pm v)$	$u' \pm v'$		
6	$\frac{d}{dx}(uv)$	$u'v + uv'$		
7	$\frac{d}{dx}\left(\frac{u}{v}\right)$	$\frac{u'v - uv'}{v^2}$		
8	$\frac{d}{dx}(\sin x)$	$\cos x$	$\int \cos x dx$	$\sin x + C$
9	$\frac{d}{dx}(\cos x)$	$-\sin x$	$\int \sin x dx$	$-\cos x + C$
10	$\frac{d}{dx}(\tan x)$	$\sec^2 x$	$\int \sec^2 x dx$	$\tan x + C$
11	$\frac{d}{dx}(\cot x)$	$-\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx$	$-\cot x + C$
12	$\frac{d}{dx}(\sec x)$	$\sec x \tan x$	$\int \sec x \tan x dx$	$\sec x + C$
13	$\frac{d}{dx}(\operatorname{cosec} x)$	$-\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x dx$	$-\operatorname{cosec} x + C$
14	$\frac{d}{dx}(\sin^{-1} x)$	$\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx$	$\sin^{-1} x + C$
15	$\frac{d}{dx}(\cos^{-1} x)$	$\frac{-1}{\sqrt{1-x^2}}$		
16	$\frac{d}{dx}(\tan^{-1} x)$	$\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx$	$\tan^{-1} x + C$
17	$\frac{d}{dx}(\cot^{-1} x)$	$\frac{-1}{1+x^2}$		
18	$\frac{d}{dx}(\sec^{-1} x)$	$\frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx$	$\sec^{-1} x + C$
19	$\frac{d}{dx}(\operatorname{cosec}^{-1} x)$	$\frac{-1}{x\sqrt{x^2-1}}$		
20	$\frac{d}{dx}(e^x)$	$e^x$	$\int e^x dx$	$e^x + C$
21	$\frac{d}{dx}(\log_a x)$	$\frac{1}{x} \log_e a$	$\int \frac{1}{ax+b} dx$	$\frac{\log ax+b }{a} + C$
22	$\frac{d}{dx}(\log_e x)$	$\frac{1}{x}$	$\int \frac{1}{x} dx$	$\log x  + C$
23	$\frac{d}{dx}(a^x)$	$a^x \log a, \quad a > 0$	$\int a^x dx$	$\frac{a^x}{\log a} + C$
24	$\frac{d}{dx}(u^v)$	$u^v \left[ \frac{v}{u} \frac{du}{dx} + \log u \frac{dv}{dx} \right]$		
25	Chain rule	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$		
26	$\frac{d}{dx}(u^n)$	$nu^{n-1} \frac{du}{dx}$		

a, b, C and k are constants; x, y, z, u & v are variables

LIST OF SUBSTITUTIONS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS:

1. If  $f(x)$  involves  $\sqrt{a^2 - x^2}$  put  $x = a \sin\theta$  ( or a  $\cos\theta$  )
2. If  $f(x)$  involves  $\sqrt{a^2 + x^2}$  put  $x = a \tan\theta$  ( or a  $\cot\theta$  )
3. If  $f(x)$  involves  $\sqrt{x^2 - a^2}$  put  $x = a \sec\theta$  ( or a  $\csc\theta$  )
4. If  $f(x)$  involves both  $\sqrt{a^2 - x^2}$  and  $\sqrt{a^2 + x^2}$  put  $x^2 = a^2 \cos 2\theta$
5. If  $f(x)$  involves both  $\sqrt{a - x}$  and  $\sqrt{a + x}$  put  $x = a \cos 2\theta$

Worksheet on Continuity		Class : 12
01.	<p>Check the continuity of the following functions at the given points</p> <p>i) <math>f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, &amp; \text{if } x \neq 3 \\ 5 &amp; \text{if } x = 3 \end{cases}</math> at the point <math>x = 3</math>.</p> <p>ii) <math>f(x) = \begin{cases}  x - a , &amp; \text{if } x \neq a \\ 1 &amp; \text{if } x = a \end{cases}</math> at the point <math>x = a</math></p> <p>iii) <math>f(x) = \begin{cases} \frac{1}{2} - x, &amp; \text{if } 0 \leq x &lt; \frac{1}{2} \\ 1, &amp; \text{if } x = \frac{1}{2} \\ \frac{3}{2} - x, &amp; \text{if } \frac{1}{2} &lt; x \leq \frac{1}{2} \end{cases}</math> at the point <math>x = \frac{1}{2}</math></p>	
<b>Find the points of discontinuity if any of the following functions:</b>		
02.	$f(x) = \begin{cases} x^3 - x^2 + 2x - 2 & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$	
03.	$f(x) = \begin{cases} \frac{x^4 - 16}{x - 4}, & \text{if } x \neq 4 \\ 16 & \text{if } x = 4 \end{cases}$	
04.	$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$	

05.	$\text{If } f(x) = \begin{cases} \frac{1 - \cos ax}{x \cdot \sin x}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$ <p>is continuous at <math>x = 0</math>. Find the value of <math>a</math>.</p>
07.	$\text{If } f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ <p>is continuous at <math>x = 0</math>. Find the value of <math>k</math>.</p>
08.	$\text{If } f(x) = \begin{cases} \frac{2x + 3 \sin x}{3x + 2 \sin x}, & \text{if } x \neq 0 \\ 4k, & \text{if } x = 0 \end{cases}$ <p>is continuous at <math>x = 0</math>. Find the value of <math>k</math>.</p>
09.	$\text{If } f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ <p>is continuous at <math>x = 1</math>. Determine the values of <math>a</math> and <math>b</math>.</p>
10.	$\text{Let } f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x} & \text{if } x < \frac{\pi}{2} \\ a & \text{if } x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} & \text{if } x > \frac{\pi}{2} \end{cases}$ <p>Be continuous at <math>x = \frac{\pi}{2}</math>, find <math>a</math> and <math>b</math></p>
11.	$\text{Let } f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{if } x > 0 \end{cases}$ <p>Be continuous at <math>x = 0</math>, find <math>a</math> and <math>b</math></p>
12.	<p>Find the values of <math>a</math> and <math>b</math> so that the function</p> $f(x) = \begin{cases} x + a\sqrt{2} \sin x, & \text{if } 0 \leq x < \frac{\pi}{4} \\ 2x \cot x + b, & \text{if } \frac{\pi}{4} \leq x < \frac{\pi}{2} \\ a \cos 2x - b \sin x, & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases}$ <p>is continuous.</p>

13.	$\text{Let } f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x} & \text{if } x < \frac{\pi}{2} \\ a & \text{if } x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} & \text{if } x > \frac{\pi}{2} \end{cases}$ <p>Be continuous at <math>x = \frac{\pi}{2}</math>, find a and b</p>
14.	<p>If the function <math>f(x) = \begin{cases} 3ax + b &amp; \text{if } x &gt; 1 \\ 11 &amp; \text{if } x = 1 \\ 5ax - 2b &amp; \text{if } x &lt; 1 \end{cases}</math> is continuous at <math>x = 1</math> find the values of a and b</p>
15.	$\text{Let } f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{if } x > 0 \end{cases}$ <p>be continuous find a</p>
16.	<p>Determine a, b, c for which the function</p> $F(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \frac{\sqrt{x^2 + bx^2} - \sqrt{x}}{b\sqrt{x^3}} & x > 0 \end{cases}$ <p>is continuous at <math>x = 0</math></p>
10.	<p>i) Discuss the continuity of <math>f(x) = \frac{ x-2 }{x-2}</math>, <math>x \neq 2</math> and <math>f(2) = 0</math> at the point <math>x = 2</math>.</p> <p>ii) Examine the continuity of the following function at <math>x = 0</math>.</p> $f(x) = \begin{cases} \frac{2 x  + x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ <p>iii) A function <math>f(x)</math> is defined by <math>f(x) = x^2 + A</math>, <math>x \geq 0</math>; <math>f(x) = -x^2 - A</math>, <math>x &lt; 0</math>. What should be A so that <math>f(x)</math> is continuous at <math>x = 0</math>? [Ans: <math>A = 0</math>]</p> <p>iv) Is the function defined by <math>f(x) = \frac{3x + 4 \tan x}{x}</math> continuous at <math>x = 0</math>? If not how may this function be defined to make it continuous at this point?</p> <p>v) A function <math>f</math> is defined by <math>f(x) = \frac{1 - \cos x}{x^2}</math>, <math>x \neq 0</math>; <math>f(x) = A</math>, when <math>x = 0</math>. Find A so that <math>f</math> is continuous at <math>x = 0</math>.</p> <p>vi) Let <math>f(x) = \frac{\sin x}{x} + \cos x</math>, <math>x \neq 0</math>; <math>f(x) = 2</math>, if <math>x = 0</math>. Show that <math>f</math> is continuous at <math>x = 0</math>.</p> <p>vii) Discuss the continuity of the function <math>f(x) = \frac{x -  x }{x}</math>, <math>x \neq 0</math> and <math>f(x) = 2</math> if <math>x = 0</math>.</p> <p>viii) Show that the function <math>f(x) =  x </math> is continuous at <math>x = 0</math>.</p> <p>ix) Find k so that <math>f(x) = \frac{\sin kx}{x}</math>, <math>x \neq 0</math>; <math>f(x) = 4 + x</math>, if <math>x = 0</math> is continuous at <math>x = 0</math>. Ans : <math>k = 4</math></p> <p>x) If <math>f(x) = \begin{cases} 1, &amp; \text{if } x \leq 3 \\ ax + b, &amp; \text{if } 3 &lt; x &lt; 5 \\ 7, &amp; \text{if } 5 \leq x \end{cases}</math> determine a and b so that <math>f(x)</math> is continuous. Ans: <math>a = 3</math>, <math>b = -8</math></p>

### Practice Questions

1. If  $x = a(\cos \theta - \sin \theta)$  and  $y = a(1 - \cos \theta)$  find  $\frac{dy}{dx}$
2. If  $x = (\cos \theta + \log \tan \frac{\theta}{2})$   $y = \sin \theta$  find  $\frac{dy}{dx}$
3. If  $x = a(\cos t + \log \tan \frac{t}{2})$ ;  $y = a(1 + \sin t)$ , find  $\frac{d^2y}{dx^2}$ .
4. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$  find  $\frac{d^2y}{dx^2}$
5. If  $x = a \cos 2t (1 + \cos 2t)$  and  $y = b \cos 2t (1 - \cos 2t)$  find  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$
6. Find  $\frac{d^2y}{dx^2}$  if  $x = a \sin 2t(1 + \cos 2t)$  and  $y = b \cos 2t(1 - \cos 2t)$
7. Find  $\frac{d^2y}{dx^2}$ , if  $x = a \sec^2 \theta$  and  $y = a \tan^2 \theta$
8. Find  $\frac{d^2y}{dx^2}$ , if  $x = \sec \theta - \cos \theta$  and  $y = \sec^2 \theta - \cos^2 \theta$
9. If  $y = \sec x + \tan x$  then prove that  $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$
10. If  $y = \log \tan \left( \frac{\pi}{4} + \frac{x}{2} \right)$ , show that  $\frac{dy}{dx} = \sec x$ . Also find the value of  $\frac{d^2y}{dx^2}$  at  $x = \frac{\pi}{4}$
11. If  $y = (\tan^{-1} x)^2$  Show that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$
12. If  $y = \operatorname{cosec}^{-1} x$ ,  $x > 1$ , then show that  $x(x^2 - 1) \frac{d^2y}{dx^2} + (2x^2 - 1) \frac{dy}{dx} = 0$ .
13. If  $y = (\cos^{-1} x)^2$  prove that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$ .
14. If  $y = (\cot^{-1} x)^2$ , show that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$
15. If  $y = e^{a \cos^{-1} x}$   $-1 \leq x \leq 1$  Prove that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$
16. If  $y = e^{m \cos^{-1} x}$  prove that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$ .
17. If  $y = e^{a \sin^{-1} x}$ ,  $-1 \leq x \leq 1$ , then show that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$  (2010\_AI)
18. If  $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$ , show that  $(1 - x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$
19. If  $y = \sin(m \sin^{-1} x)$  prove that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$
20. If  $y = \sin(\sin x)$ , show that  $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + (\cos^2 x) y = 0$

21. If  $y = 3e^{2x} + 2e^{3x}$ , prove that  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
22. If  $y = e^x(\sin x + \cos x)$ , then show that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
23. If  $y = 2\sin x + 3\cos x$  prove that  $y + y_2 = 0$
24. If  $y = 3 \cos(\log x) + 4 \sin(\log x)$  show that  $x^2y_2 + xy_1 + y = 0$
25. If  $y = a\cos(\log x) + b\sin(\log x)$ , prove that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .
26. If  $y = b\sin mx + a\cos mx$  prove that  $m^2 y + y_2 = 0$
27. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$  then, prove that  $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

OR If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$  find  $\frac{dy}{dx}$

28. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , Prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
29. If  $\sqrt{y+x} + \sqrt{y-x} = a$ , show that  $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$
30. If  $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ , Prove that  $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$
31. If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$ , Prove that  $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$
32. If  $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$ , then show that  $\frac{dy}{dx} = \frac{x+y}{x-y}$
33. If  $y = \log\left(x + \sqrt{x^2 + 1}\right)$  prove that  $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ .
34. If  $y = \left[\log\left(x + \sqrt{x^2 + 1}\right)\right]^2$  show that  $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2 = 0$
35. If  $y = \log\left[x + \sqrt{x^2 + a^2}\right]$ , prove that  $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$
36. If  $y = \left[x + \sqrt{x^2 + 1}\right]^n$ , prove that  $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$
37. If  $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$  find  $\frac{dy}{dx}$
38. If  $x^p y^q = (x+y)^{p+q}$  Prove that  $\frac{dy}{dx} = \frac{y}{x}$  and  $\frac{d^2y}{dx^2} = 0$
39. If  $x^y = e^{x-y}$  prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$
40. If  $x^m \cdot y^n = (x+y)^{m+n}$  then show that  $\frac{dy}{dx} = \frac{y}{x}$
41. If  $e^x + e^y = e^{x+y}$  then show that  $\frac{dy}{dx} = -e^{y-x}$
42. Find  $\frac{dy}{dx}$       a)  $(\log x)^x + x^{\log x}$       b)  $x^{\cos x} = \cos x^{\sin x}$       c)  $y = \log(\sqrt{\sin x - \cos x})$

43. Differentiate w.r.t. x
- i)  $x^{\sin x} + (\sin x)^{\cos x}$
  - ii)  $y = (\sin x)^x + \sin^{-1} \sqrt{x}$
  - iii)  $(x)^{\cos x} + (\sin x)^{\tan x}$
  - iv)  $(\cos x)^y = (\sin y)^x$
  - v)  $(\cos x)^x + (\sin x)^{1/x}$
  - vi)  $\cos^{-1} x + \cos^{-1} \sqrt{1-x^2}$
  - vii)  $\cos(x^2 + y^2) = \log xy$
  - viii)  $(x)^{\sqrt{x}} + (\sqrt{x})^x$
  - ix)  $y = x e^x + e^{-x} + e^{x^e}$
  - x)  $x^y + y^x = a^b$
  - xi)  $x^y + y^x = a^b$

44. If  $y = (\sin x - \cos x)^{(\sin x - \cos x)}$ ,  $\frac{\pi}{4} < x < \frac{3\pi}{4}$  then find  $\frac{dy}{dx}$

45.

46. If  $y = x^{a^x}$  then show that  $\frac{dy}{dx} = x^{a^x} a^x \left( \frac{1}{x} + \log a \log x \right)$

47. If  $y = a^{x^x}$  then show that  $\frac{dy}{dx} = a^{x^x} x^x \log a (1 + \log x)$

48.

49. If  $y = x \sin x$  prove that  $\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} + \cot x$

50.

51. If  $y = (1+x)(1+x^2)(1+x^4) \dots (1+x^{2^n})$  find  $\frac{dy}{dx}$  at  $x=0$

52. If  $y = x^{x^{x^{\dots \infty}}}$ , prove that  $\frac{dy}{dx} = \frac{y^2}{x(1-y \log x)}$

53. If  $y = \sin x^{\sin x^{\sin x^{\dots \infty}}}$ , prove that  $\frac{dy}{dx} = \frac{y^2 \cot x}{(1-y \log x)}$

54. If  $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}}$  then provethat  $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

55. If  $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$  then provethat  $\frac{dy}{dx} = \frac{1}{2y-1}$

56. If  $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$ , then prove that  $\frac{dy}{dx} = \frac{1}{x(2y-1)}$

57. If  $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$  then provethat  $\frac{dy}{dx} = \frac{\sin x}{2y-1}$

58. If  $x = \sqrt{a \sin^{-1} t}$  and  $y = \sqrt{a \cos^{-1} t}$  Provethat  $\frac{dy}{dx} = -\frac{y}{x}$

59. a) Differentiate  $\tan^{-1} \left( \frac{\cos x}{1 + \sin x} \right)$  w.r.t  $\sec^{-1} x$       b) If  $x^y + y^x = 2$  find  $\frac{dy}{dx}$

60. If  $\text{Cosy} = x, \cos(a+y)$ , prove that  $\frac{dy}{dx} = \frac{\text{Cos}^2(a+y)}{\text{Sina}}$

61. If  $y = x \cdot \text{Siny}$ , prove that  $\frac{dy}{dx} = \frac{y}{x(1-x \text{Cosy})}$

62. If  $\text{siny} = x \cos(a+y)$  prove that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$

63. If  $\sin y = x \cdot \sin(a + y)$ , prove that  $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

64. Differentiate  $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$  w.r.t  $\tan^{-1}\frac{\sin x}{1+\cos x}$

65. If  $y = \cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ , find  $\frac{dy}{dx}$ .

66. Find  $y = a^{t+\frac{1}{t}}$  and  $x = \left(t + \frac{1}{t}\right)^a$  find  $\frac{dy}{dx}$

67. If  $(x-a)^2 + (y-b)^2 = c^2$ , prove that  $\left[\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^{\frac{3}{2}}$  is a constant independent of  $a$  and  $b$

68. Find  $\frac{dy}{dx}$  if  $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$   $0 < x < \pi/2$

69. Differentiate  $\tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$  w.r.t  $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

70. If  $y = \sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$  show that  $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$

71. If  $y = \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$ ,  $0 < x < 1$ , find  $\frac{dy}{dx}$  [Hint : Put  $x = \sin\theta$  and  $\sqrt{x} = \sin\phi$ ]

72. Differentiate  $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$  with respect to  $x$

73. Show that  $\frac{d}{dx}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2-x^2}$

74. If  $y = \sin^{-1}[x^2\sqrt{1-x^2} - x\sqrt{1-x^4}]$ , prove that  $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} - \frac{1}{\sqrt{1-x^2}}$

75. If  $y = \sin^{-1}[x^2\sqrt{1-x^2} + x\sqrt{1-x^4}]$ , prove that  $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$

76. If  $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}$ , prove that  $\frac{dy}{dx} = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$ ,  $|x| < 1$

77. If  $y = \cos^{-1}\left(\frac{x}{k}\right) - \frac{\sqrt{k^2-x^2}}{x}$ , prove that  $\frac{dy}{dx} = \frac{\sqrt{k^2-x^2}}{x^2}$

78. If  $y = \sin[2\sin^{-1}x]$ , Prove that  $\frac{dy}{dx} = 2\sqrt{\frac{1-y^2}{1-x^2}}$

79. If  $y = \tan^{-1}\left[\frac{4x}{1+5x^2}\right] + \tan^{-1}\left[\frac{2+3x}{3-2x}\right]$ , prove that  $\frac{dy}{dx} = \frac{5}{1+25x^2}$

80. Differentiate the following function w. r. t.  $x$ :  $f(x) = \tan^{-1}\left[\frac{1-x}{1+x}\right] - \tan^{-1}\left[\frac{x+2}{1-2x}\right]$



81. If  $y = \tan^{-1} \left[ \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$ , prove that  $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^4}}$
82. If  $y = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1} \left[ \sqrt{\frac{a-b}{a+b}} \tan\left(\frac{x}{2}\right) \right]$ , prove that  $\frac{dy}{dx} = \frac{1}{a+b \cos x}$ .
83. If  $y = \cos^{-1} \left( \frac{3x + 4\sqrt{1-x^2}}{5} \right)$ , find  $\frac{dy}{dx}$  Hint: let  $x = \cos\theta$ ;  $3 = r\cos\alpha$ ,  $4 = r\sin\alpha$ ; ans:  $-1/\sqrt{1-x^2}$
84. If  $y = \cos^{-1} \left( \frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}} \right)$ , find  $\frac{dy}{dx}$
- 85.
86. If  $y = \sin^{-1} \left( \frac{5x + 12\sqrt{1-x^2}}{13} \right)$ , find  $\frac{dy}{dx}$
87. If  $y = \sin^{-1} \left( \frac{3\sin x + 4\cos x}{5} \right)$ , find  $\frac{dy}{dx}$
88. If  $y = \sqrt{\frac{1-x}{1+x}}$ , prove that  $(1-x^2) \frac{dy}{dx} + y = 0$
89. If  $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ , prove that  $\frac{dy}{dx} = 1 - y^2$
90. If  $xy \cdot \log(x+y) = 1$ , prove that  $\frac{dy}{dx} = \frac{y(x^2y + x + y)}{x(xy^2 + x + y)}$
91. If  $y = e^{2x}(ax + b)$ , prove that  $y_2 - 4y_1 + 4y = 0$
92. If  $x = a\cos\theta + b\sin\theta$ ,  $y = a\sin\theta - b\cos\theta$ , prove that  $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$
93. Differentiate  $\tan^{-1} \left[ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right]$  w.r.to  $\frac{1}{2} \cos^{-1} x^2$
94. Find  $\frac{dy}{dx}$  if  $x^2 + y^2 = t - \frac{1}{t}$ ;  $x^4 + y^4 = t^2 + \frac{1}{t^2}$
95. Differentiate  $\tan^{-1} \left[ \frac{3x - x^3}{1 - 3x^2} \right]$  w.r.to  $\tan^{-1} x$ .
96. Differentiate  $\tan^{-1} \left[ \frac{3x - x^3}{1 - 3x^2} \right]$  w.r.to  $\tan^{-1} \left[ \frac{x}{\sqrt{1-x^2}} \right]$ .
97. Differentiate  $\tan^{-1} \left[ \frac{\sqrt{1+x^2} - 1}{x} \right]$  w.r.to  $\tan^{-1} x$ .
98. Find the derivative of  $f(x)$  with respect to  $g(x)$
- a.  $f(x) = \tan^{-1} \left( \frac{\sqrt{1+x^4} - 1}{x^2} \right)$ ;  $g(x) = \tan^{-1} \left( \frac{x^2}{\sqrt{1-x^4}} \right)$
- b.  $f(x) = x^{\sin^{-1} x}$ ;  $g(x) = \sin^{-1} x$

c.  $f(x) = \cot^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$ ,  $g(x) = \sin^{-1} x$

99. Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  if

a.  $x = \frac{3at}{1+t^2}$  ;  $y = \frac{3at^2}{1+t^2}$

b.  $x = 3 \text{Cost} - \text{Cos}^3t$  ;  $y = 3 \text{Sint} - \text{Sin}^3t$

c.  $x = a(\text{Cost} + t\text{Sint})$  ;  $y = a(\text{Sint} - t\text{Cost})$

d.  $x = a(3\text{Cost} + \text{Cos}3t)$  ;  $y = a(3\text{Sint} + \text{Sin}3t)$

e.  $x = \frac{\text{Sin}^3t}{\sqrt{\text{Cos}2t}}$  ;  $y = \frac{\text{Cos}^3t}{\sqrt{\text{Cos}2t}}$

100. Find  $\frac{dy}{dx}$  if,

i)  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

vi)  $xy = \tan(xy)$

ii)  $xy + y^2 = \tan x + y$

vii)  $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 6$

iii)  $x^2 + xy + y^2 = 100$

viii)  $x^2 + 2xy + y^3 = 42$

iv)  $x^3 + x^2y + xy^2 + y^3 = 81$

v)  $\text{Sin}(xy) + \frac{x}{y} = x^2 - y \text{Sin}^2x + \text{Sin}(xy) + x^2y = 1$

101. Verify Rolle's theorem for the function  $f(x) = x^2 + 2x - 8$  in the interval  $[-4, 2]$

102. Verify Rolles theorem for the following function  $f(x) = x^3 - 7x^2 + 16x - 12$  in the interval  $[2, 3]$

103. Verify Rolle's theorem for the function  $f(x) = e^{a^2 - x^2}$  in  $[-a, a]$

104. Verify Rolle's Theorem for  $f(x) = \sin x + \cos x + 1$  on  $[0, \pi/2]$

105. Using Rolle's theorem, find the point(s) on the curve of  $f(x) = 16 - x^2$  in the interval  $[-1, 1]$ , through which the tangent is parallel to x-axis.

106. Verify LMVT for  $f(x) = (x-1)(x-2)(x-3)$  in the interval  $[0, 4]$ .

107. Verify L.M.V theorem for  $f(x) = x(x-2)$  on  $[1, 3]$

108. Verify LMVT for  $f(x) = (x-1)(x-2)(x-3)$  in the interval  $[0, 4]$ .

109. Verify LMV theorem for  $f(x) = x^3 + 5x^2 - 3x$  in  $[1, 3]$

110. Verify LMVT for the function  $f(x) = 2\text{Sin}x + \text{Sin}2x$  in  $[0, \pi]$

Find the derivative of the following w. r. t. x:

1. $\log x^x$	2. $\log_{10}x + \log_x 10 + \log_x x + \log_{10} 10$	3. $x^n \log x + x(\log x)^n$
4. $\log \left( \frac{1+\sqrt{x}}{1-\sqrt{x}} \right)$	5. $\frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \log(x + \sqrt{x^2+a^2})$	6. $\frac{x\sqrt{x+x^2}}{\sqrt{x+1}}$
7. $\log(x^x + \sec^2 x)$	8. $\sin \sqrt{\sin \sqrt{x}}$	9. $x^y \cdot y^x = 16$
10. $\frac{1}{\sqrt{x^2+a^2} + \sqrt{x^2+b^2}}$	11. If $y = \cos^{-1} \left( \frac{5 \cos x - 12 \sin x}{13} \right)$ ,	12. $\tan^{-1} \left( \frac{5ax}{a^2-6x^2} \right)$
13. $\tan^{-1} \left( \frac{3a^2x-x^3}{a(a^2-3x^2)} \right)$	14. $\tan^{-1} \left( \frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right)$	15. $y = \tan^{-1} \sqrt{\frac{x+1}{x-1}}$
16. $\frac{x \sin^{-1} x}{\sqrt{1-x^2}}$	17. $\tan^{-1} \left( \frac{4x}{1+5x^2} \right) + \tan^{-1} \left( \frac{2+3x}{3-2x} \right)$	18. $y = \tan^{-1} \left( \frac{1-x}{1+x} \right)$
19. $2 \tan^{-1} x + \sin^{-1} \frac{2x}{1+x^2}$	20. $\cos^{-1} \left( \frac{x^2-y^2}{x^2+y^2} \right) = \tan^{-1} \alpha$	21. $\cos^{-1} (2x^2 \sqrt{1-x^4})$
22. $y = \tan^{-1} \left( \frac{3ax-a^3x^3}{1-3a^2x^2} \right)$	23. $x = \sin^{-1} \left( \frac{2t}{1+t^2} \right)$ ; $y = \tan^{-1} \left( \frac{2t}{1-t^2} \right)$	24. $e^{x+y} = e^x + e^y$
25. $x\sqrt{1+y} + y\sqrt{1+x}$	26. $y y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$	27. $x^2 \sqrt{x} + x^3 \log x$
28. $\frac{x \sin 2x}{x + \sin 2x}$	29. $3^x + 2^{x+\sin^{-1} 2x}$	30. $\cos^{-1} \sqrt{2x^3-1}$
31. $(2x^2-5x+3)^{\frac{3}{2}}$	32. $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}}$	33. $x^{\sin x} (\sin x)^x$
34. $(\sin x)^{x+\sin x}$	35. $y = \sqrt{x + \sqrt{x\sqrt{x\sqrt{x} + \dots \infty}}}$	36. $x^{x^2}$
37. $x^{\log x} + (\log x)^x$	38. $x = \sqrt{a^{\sin^{-1} t}}$ ; $y = \sqrt{a^{\cos^{-1} t}}$	39. $\sin(x^x)$
40. $y = \cot^{-1} \left( \frac{a-b \tan x}{b+a \tan x} \right)$	41. $y = \tan^{-1} \left( \frac{\sqrt{1+x^4} + \sqrt{1-x^4}}{\sqrt{1+x^4} - \sqrt{1-x^4}} \right)$	42. $y = \tan^{-1} \left( \frac{1}{1+x+x^2} \right)$
43. $x = a \sec^3 t$ ; $y = a \tan^3 t$	44. $\tan^{-1} \left[ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right]$	45. $y = x^x + \sin(x^x)$
46. $x^{\sin x} = \sin x^x$	47. $\sin^{-1} \{ x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \}$	48. $x = e^{\cos 2t}$ ; $y = e^{\sin 2t}$

49. $e^{\sin x} + (\sin x)^x$	50. $x = a \left( \frac{1-t^2}{1+t^2} \right), y = \frac{2bt}{1+t^2}$	51. $y \log x + x \log y = e^y$
52. $(\log x)^x + x^{\log x}$	53. $x = \cos \theta + \cos 2\theta, y = \sin \theta + \sin 2\theta$	54. $y = x^{x^x}$
55. $x^{\cos x} = \cos x^{\sin x}$	56. $x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right), y = a \sin \theta$	57. $x^y = y^x$
58. $y = (\tan x)^x + (\cos x)^x$	59. $\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$	60. $y^{\sqrt{x}} = \sqrt{x}^{-\log x}$
61. $y = (x^{\sin x})(\cos x)^x$	62. $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots}}}$	63.
64. $y = \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2, x \neq 0$	65. $\frac{(x-2)(x+1)}{\sqrt{x}}$	66. $\frac{(x+3)(x^2+1)}{x}$
67. $y = \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2, x \neq 0$	68. $\frac{(x-2)(x+1)}{\sqrt{x}}$	69. $\frac{(x+3)(x^2+1)}{x}$
70. $\left( x - \frac{1}{x} \right) \left( x^2 - \frac{1}{x^2} \right)$	71. $\frac{(2x^2 - 4)}{3x^2 + 7}$	72. $\frac{1}{x^4 \sec x}$
73. $\frac{x + \sin x}{x + \cos x}$	74. $\frac{x}{1 + \tan x}$	75. $\frac{\log x}{x}$
76. $\frac{e^x}{x}$	77. $\frac{\sin x + \cos x}{\sin x - \cos x}$	78. $\frac{\sec x - 1}{\sec x + 1}$
79. $\frac{x^2 + \sec x}{1 + \tan x}$	80. $\frac{x^2 + \sin x}{x \cos x}$	81. $x^4 \tan x$
82. $e^x \log x$	83. $x^3 e^x \sin x$	84. $\frac{1 - \tan x}{\sec x}$
85. $e^{\cos x}$	86. $e^{x^2}$	87. $\cos x^2$
88. $\log \sin x$	89. $\tan \sqrt{x}$	90. $\operatorname{cosec}(1 + x^2)$
91. $\sec(\tan x)$	92. $(x^2 + \cos x)^4$	93. $\cos(1 - x^2)^2$
94. $\cos(\sin x^2)$	95. $\cot^2 x^2$	96. $\sqrt{\sin x^3}$
97. $\sin(3x^{\frac{1}{3}}), x > 0$	98. $[\sin(3x + 2)]^{\frac{7}{2}}$	99. $\cot^3 x - \tan \sqrt{x}$
100. $\tan(x^2 + x + 2)$	101. $\sec^2 \left( \frac{x}{a} \right)$	102. $x^4 \sin 2x$
103. $(1 + \sin^2 x)^3$	104. $\cos^2 5x$	105. $\tan^4 x^2$
106. $\sec^n(\sqrt{x})$	107. $x^2 \sin \frac{1}{x}$	108. $(\sin x + \cos x)^2$
109. $\sin(\cos(\sin x))$	110. $\sec \sqrt{1 + x^2}$	111. $\frac{1}{1 + \sin^2 x}$
112. $\sin^5 x \sin x^5$	113. $\sin^m x \cos^n x$	114. $(3 \sin^2 x - \cos^5 x + 2)^3$
115. $(x \sin x - \cos \sqrt{x})^5$	116. $\left( \frac{2 \tan x}{\tan x + \cos x} \right)^2$	117. $\sin x^2 \tan x^3$
118. $(\tan \sqrt{x} + x^2 - \sin x)^3$	119. $\log x e^{\tan x + x^2}$	120. $e^{2x} \sin 3x$
121. $\cos(\sin \sqrt{ax + b})$	122. $\frac{\sin x}{1 + \cos x}$	123. $\sin \left( \frac{1 + x^2}{1 - x^2} \right)$
124. $\sqrt{\frac{1 + \tan x}{1 - \tan x}}$	125. $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$	126. $\frac{1 - \cos x}{1 + \cos x}$
127. $\frac{\sec x + \tan x}{\sec x - \tan x}$	128. $\frac{1 + \tan x}{1 - \tan x}$	129. $\frac{1 - \tan x}{1 + \tan x}$
130. $\frac{\tan x - 1}{\sec x}$	131. $\frac{\tan x - \cot x}{\tan x + \cot x}$	132. $\sqrt{\frac{\sec x + \tan x}{\sec x - \tan x}}$
133. $\frac{\cot^2 x - 1}{\cot^2 x + 1}$	134. $\sqrt{\sec^2 x + \operatorname{cosec}^2 x}$	135. $(\tan \sqrt{1 + x^2})^{\frac{1}{2}}$
136. $\sqrt{\cos(1 + x)^2}$	137. $\sin \sqrt{\sin x + \cos x}$	138. $\tan^2 \sqrt{x^4 + 3}$

139. $\frac{\sin x}{\sqrt{\cos x}}$	140. $e^{\sqrt{1-x^2}} \tan x$	141. $\frac{e^{ax}}{\sin (bx+c)}$
142. $\frac{e^x \log x}{x^2}$	143. $\frac{e^{2x} \cos x}{\sin x}$	144. $\log [x + \sqrt{x^2 + a^2}]$
145. $5^x$	146. $5^{\sin x}$	147. $a^{2x} \sin^2 x$
148. $e^{\sqrt{\cot x}}$	149. $e^{5x^2} \sqrt{1-x^2}$	150. $\sin x e^{\cos x}$
151. $(\sin 2x + \cos 3x)e^{-5x}$	152. $e^{\sqrt[3]{ax}}$	153. $a^{\sqrt{x}}$
154. $e^{x \sec^2 x - \tan x}$	155. $e^{3x} \cot x$	156. $x e^{\sqrt{\sin x}}$
157. $e^{-ax^2} \sin (\log x)$	158. $\log x . e^{(\tan x + x^2)}$	159. $e^x \log (\sin 2x)$
160. $\sqrt{\frac{1+e^x}{1-e^x}}$	161. $\log [x + e^{\sqrt{x}}]$	162. $e^{\sin x} . \sin (e^x)$
163. $e^x \log (1 + x^2)$	164. $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$	165. $\log \sin x$
166. $\log (\sec x + \tan x)$	167. $\log \tan \frac{x}{2}$	168. $\log (\operatorname{cosec} x - \cot x)$
169. $\log \tan \left(\frac{\pi}{4} + \frac{x}{2}\right)$	170. $\log (\sqrt{\sec x})$	171. $\sin (\log \cos x)$
172. $\log \left(\sec \frac{x}{2} + \tan \frac{x}{2}\right)$	173. $\log [\sin (\log x)]$	174. $\log [\log (\log x)]$
175. $\frac{\log x}{1+x \log x}$	176. $\tan (ax + b)$	177. $\sec (7 - 4x)$
178. $\operatorname{cosec} (x^2 + 1)$	179. $\sin 5x \cos 3x$	180. $\tan x^2$
181. $\sqrt{\sec x}$	182. $\cos \sqrt{x}$	183. $\sin^2 x \cos^3 x$
184. $\sec^2 3x + \operatorname{cosec}^2 2x$	185. $\sqrt{a \sin^2 x + b \cos^2 x}$	186. $x^4 \sin (ax + b)^2$
187. $\cot (\sin \sqrt{x})$	188. $\sin \left(\frac{1+x^2}{1+x^3}\right)$	189. $\left(\frac{x}{1+\tan x}\right)^{1/2}$
190. $\sqrt{\sin \sqrt{x}}$	191. $\operatorname{cosec}^2 3x$	192. $\sqrt{\sec (ax + b)}$
193. $\sec \sqrt{a + bx}$	194. $\sec (\operatorname{cosec} x)$	195. $\operatorname{cosec} x^2 - 3 \cos \sqrt{x} + \frac{1}{x^2} \cot x^3$
196. $\sqrt{\frac{1-\sin x}{1+\sin x}}$	197. $\sqrt{\frac{1+\cos x}{1-\cos x}}$	198. $\frac{\sin^2 x}{1+\cos^2 x}$
199. $e^{ax} \cos (bx + c)$	200. $\sin (x^2 e^x)$	201. $x^2 e^x \sin x$
202. $e^{(x \sin x + \cos x)}$	203. $\tan (1 + e^x)$	204. $\log \cos x$
205. $\log (\sin x - \cos x)$	206. $e^{\cos x} . \sin (2x + 1)$	207. $e^{x^2} . \sin (x^2 + 1)$
208. $\sin (e^x + \log x)$	209. $x^3 \sin (\log x)$	210. $3^{\log \sin x}$
211. $\log [x + \sqrt{x^2 + 1}]$	212. $\log (2 \sec x + 3 \tan x)$	213. $\log \left(\frac{1+\sin x}{1-\sin x}\right)$
214. $\log \left(\frac{x + \sqrt{x^2 + 1}}{x + \sqrt{x^2 - 1}}\right)$	215. $\sin (\log (1 + x^2))$	216. $\log \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$
217. $e^{mx+c} . \log (ax + b)$	218. $\cos \{\sin (\log x)\}$	219. $\cos \left(\frac{1-x^2}{1+x^2}\right)$
220. $\tan [\sin (\sec x)]$	221. $\log (x^4 \sec x)$	222. $\sin (e^x) \log x$
223. $x^n e^x \log x$	224. $e^{5x} \log \sqrt{1 + 2x}$	225. $[\log (\tan x)]^2$
226. $x \sqrt{x^2 + c^2} + c^2 \log [x + \sqrt{x^2 + c^2}]$	227. $x^x$	228. $x^{x^x}$
229. $x^{1/x}$	230. $x^{\sqrt{x}}$	231. $(\sin x)^x$
232. $x^{\sin x}$	233. $(1 + x)^{\log x}, x > 0$	234. $(\log x)^{\tan x}$
235. $(x^2 \sin x)^{1/x}, x \neq 0$	236. $(\sin x)^{\log x}$	237. $x^{\sin x + \cos x}$
238. $e^x \cos^3 x \sin^2 x$	239. $(x + 1)^2 (x + 2)^3 (x + 3)^4$	240. $\frac{8^x}{x^8}$
241. $(2x + 3)^{x-5}$	242. $\sqrt{(x-1)(x-2)(x-3)(x-4)}$	243. $x^x + x^{1/x}$
244. $\tan x^{\sin x} + \sin x^{\tan x}$	245. $x^{\sin x} + (\sin x)^x$	246. $x^{\log x}$

247. $x^{\tan x}$	248. $x^{\cos x}$	249. $(\tan x)^x$
250. $(\cos x)^{\log x}$	251. $(\sin x)^{\cos x}$	252. $x^x + x^{\sin x}$
253. $x^{\tan x} + (\sin x)^{\cos x}$	254. $x^x + (\tan x)^{\log x}$	255. $x^{\sin 2x}$
256. $\frac{2^x \cot x}{\sqrt{x}}$	257. $(\tan x)^x + x^{\cot x}$	258. $\frac{\sqrt{x} \log \tan x}{x e^{2x}}$
259. $\cot x^2 \cdot \sec x^x$	260. $(1 + \cos x)^x$	261. $x^x (1 + \log x)$
262. $\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$	263. $\tan^{-1} \left[ \frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right]$	264. $\cos^{-1} \left[ \frac{3 \cos x - 4 \sin x}{5} \right]$
265. $\tan^{-1} \left[ \frac{3a^2 x - x^3}{a^3 - 3ax^2} \right]$	266. $\cos^{-1} \left[ \frac{x - \sqrt{1-x^2}}{\sqrt{2}} \right]$	267. $\tan^{-1} \left[ \frac{\sqrt{1+a^2 x^2} - 1}{ax} \right]$
268. $\cot^{-1} \left( \frac{1-x^2}{2x} \right)$	269. $\sin^{-1} \left[ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right]$	270. $\sec^{-1} \left[ \frac{\sqrt{x+1}}{\sqrt{x-1}} \right] + \sin^{-1} \left[ \frac{\sqrt{x-1}}{\sqrt{x+1}} \right]$
271. $y = \sin^{-1} x + \sin^{-1} \sqrt{1-x^2}$	272. $\tan^{-1} \left[ \frac{\cos x - \sin x}{\cos x + \sin x} \right]$	273. $\tan^{-1} \left[ \frac{\cos x}{1 + \sin x} \right]$
274. $\cot^{-1} \sqrt{\frac{1 + \cos 3x}{1 - \cos 3x}}$	275. $\sec^{-1} \left[ \frac{1}{1-2x^2} \right]$	276. $\tan^{-1} \left[ \sqrt{1+x^2} + x \right]$
277. $\frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}}$	278. $(x \cdot \sin x)^{1/x} + (x \cdot \cos x)^x$	279. $x^{x \cdot \cos x} + \frac{x^2 - 1}{x^2 + 1}$
280. $\cos x \cdot \cos 2x \cdot \cos 3x \cdot \cos 4x$	281. $x^y + y^x = 1$	282. $\frac{(x+2)^{5/2}}{(x+6)^{1/2} (x+7)^{7/2}}$

## Chapter 5

## CONTINUITY AND DIFFERENTIABILITY

HOTS by Kendriya Vidyalaya Sangathan

### 1 Mark Questions

1. Check the continuity of the function  $f(x) = \begin{cases} 1, & \text{if } x \leq 0 \\ 2, & \text{if } x > 0 \end{cases}$
2. Check the continuity of the function  $f(x) = |x|$  at  $x = 0$
3. Check the continuity of the function  $f(x) = \sin x + x^2$  at  $x = \pi$
4. Examine the continuity of the function  $f(x) = |x|$
5. Write the points of discontinuity of the function  $f(x) = [x]$  where  $[x]$  denotes the greatest integer function.

6. Write the points of discontinuity of the function  $f(x) = \frac{x^2 - 25}{x - 5}$ .
7. Give an example of a function which is continuous but not differentiable.
8. Find the derivative of  $\sin(\cos^2(\sqrt{x}))$ .
9. Find the points in the open interval  $(0, 3)$  where the greatest integer function  $f(x) = [x]$  is not differentiable.
10. Write the derivative of the function  $f(x) = \tan^{-1} \sqrt{\sin x}$  w. r. to  $x$ .
11. Is it true that  $\log(x^{\sin x} + \cos^{\sin x} x) = \sin x \log x + \sin x \log \cos x$ ?
12. If  $x = f(t)$  and  $y = g(t)$ , then is  $\frac{d^2 y}{dx^2} = \frac{d^2 y / dt^2}{d^2 x / dt^2}$ ?
13. Check the applicability of Rolle's theorem for  $f(x) = [x]$  on  $[1, 5]$ .
14. Discuss the continuity of the function  $f(x) = \sin|x|$
15. Discuss the continuity of the function  $f(x) = |x| - |x-1|$

#### 4 Marks Questions

16. Differentiate  $\log(x + \sqrt{1+x^2})$
17. Differentiate  $\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}$
18. Find  $\frac{dy}{dx}$  for  $\sin(xy) + \frac{x}{y} = x^2 - y$
19. If  $2^x + 2^y = 2^{x+y}$ , show that  $\frac{dy}{dx} = -2^{y-x}$
20. If  $\tan^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$ , show that  $\frac{dy}{dx} = \frac{x(1 - \tan a)}{y(1 + \tan a)}$
21. Differentiate  $\tan^{-1}\left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$
22. Differentiate  $x^x + x^a + a^x + a^a$
23. Differentiate  $\left(x + \frac{1}{x}\right)^x + x^{\left(\frac{x+1}{x}\right)}$
24. Find  $\frac{dy}{dx}$  for  $x = \frac{\sin^3 \theta}{\sqrt{\cos 2\theta}}$ ,  $y = \frac{\cos^3 \theta}{\sqrt{\cos 2\theta}}$
25. Find  $\frac{dy}{dx}$  for  $x = \sqrt{a^{\sin^{-1} t}}$ ,  $y = \sqrt{a^{\cos^{-1} t}}$
26. If  $y = (\tan^{-1} x)^2$  show that  $(1+x^2)^2 \frac{d^2 y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} - 2 = 0$
27. If  $\sin y = x \cos(a+y)$ , show that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$

28. If  $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ , show that  $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

29. If  $x \sin(a+y) + \sin a \cos(a+y) = 0$ , show that  $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

30. Check the continuity of the function  $f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$  at  $x = 0$

### 6 Marks Questions

31. If  $y = (\sin^{-1} x)^2$  show that  $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$

32. Find  $\frac{d^2y}{dx^2}$  for the function  $x = a(\theta + \sin \theta)$ ,  $y = a(1 + \cos \theta)$  at  $\theta = \frac{\pi}{2}$

33. Find  $\frac{dy}{dx}$  for  $\tan^{-1}(\sqrt{1+x^2} + x)$

34. Find  $\frac{dy}{dx}$  for  $\tan(x+y) + \tan(x-y) = 1$

35. Show that the function defined by  $g(x) = x - [x]$  is discontinuous at all integral points.

Answers: Continuity and Differentiability	
<u>1 Mark Questions</u>	
Q1	{Not Continuous}
Q2	{Continuous}
Q3	{Continuous}
Q4	{Continuous}
Q5	{All integers}
Q6	{x = 5}
Q7	{f(x) =  x }
Q8	$\frac{-\cos(\cos^2 \sqrt{x}) \sin \sqrt{x} \cos \sqrt{x}}{\sqrt{x}}$
Q9	{1, 2}
Q10	$\left\{ \frac{\cos x}{2\sqrt{\sin x} (1 + \sin x)} \right\}$
Q11	{No}



Q12	{No}
Q13	{ Not applicable}
Q15	{Continuous}
Q14	{Continuous}

4 Marks Questions

Q1	$\left\{ \frac{1}{\sqrt{1+x^2}} \right\}$
Q2	$\left\{ \frac{a}{\sqrt{a^2-x^2}(a+\sqrt{a^2-x^2})} \right\}$
Q3	$\left\{ \frac{2xy^2 - y - y^3 \cos(xy)}{y^2 x \cos(xy) - x + y^2} \right\}$
Q4	$\{-2^{y-x}\}$
Q6	{-1}
Q7	$\{x^x(1+\log x) + a x^{a-1} + a^x \log a\}$
Q8	$\left\{ \left(x + \frac{1}{x}\right)^x \left[ \frac{x^2-1}{x^2+1} + \log\left(x + \frac{1}{x}\right) \right] + x^{1+\frac{1}{x}} \left( \frac{x+1-\log x}{x^2} \right) \right\}$
Q9	$\{-\cot 3\theta\}$
Q10	$\left\{ -\frac{y}{x} \right\}$
Q15	{Not Continuous}

6 Mark Questions

Q2	$\left\{ -\frac{1}{a} \right\}$
Q3	$\left\{ \frac{1}{\sqrt{1+x^2}(2x^2+1+2x\sqrt{1+x^2})} \right\}$
Q4	$\left\{ -\frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x+y) - \sec^2(x-y)} \right\}$

No	y	$\frac{dy}{dx}$	No	y	$\frac{dy}{dx}$
1	k	0	19	$\text{Cot}^{-1}x$	$\frac{-1}{1+x^2}$
2	kx	k	20	$\text{Sec}^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
3	$x^n$	$nx^{n-1}$	21	$\text{Cosec}^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
4	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	22	f(x)	f'(x)
5	$\frac{1}{x}$	$-\frac{1}{x^2}$	23	kf(x)	k f'(x)
6	$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	24	$[f(x)]^n$	$n[f(x)]^{n-1}$
7	$e^x$	$e^x$	25	$\sqrt{f(x)}$	$\frac{1}{2\sqrt{f(x)}} f'(x)$
8	$a^x$	$a^x \log a$	26	$\frac{1}{f\{x\}}$	$-\frac{1}{[f\{x\}]^2} f'(x)$
9	logx	$\frac{1}{x}$	27	$\frac{1}{[f\{x\}]^n}$	$-\frac{n}{[f\{x\}]^{n+1}} f'(x)$

1.

10	$\sin x$	$\cos x$	28	$e^{f(x)}$	$e^{f(x)} f'(x)$
11	$\cos x$	$-\sin x$	29	$a^{f(x)}$	$a^{f(x)} f'(x) \log a$
12	$\tan x$	$\sec^2 x$	30	$\log f(x)$	$\frac{1}{f(x)} f'(x)$
13	$\cot x$	$-\operatorname{cosec}^2 x$	31	$\operatorname{Sin} f(x)$	$\operatorname{Cos} f(x) \cdot f'(x)$
14	$\sec x$	$\sec x \tan x$	32	$\operatorname{Cos} f(x)$	$-\operatorname{Sin} f(x) \cdot f'(x)$
15	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$	33	$\tan \cdot f(x)$	$\operatorname{Sec}^2 f(x) \cdot f'(x)$
16	$\operatorname{Sin}^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	34	$\cot \cdot f(x)$	$-\operatorname{cosec}^2 f(x) \cdot f'(x)$
17	$\operatorname{Cos}^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	35	$\sec \cdot f(x)$	$\operatorname{Sec} \cdot f(x) \tan \cdot f(x) \cdot f'(x)$
18	$\operatorname{Tan}^{-1} x$	$\frac{1}{1+x^2}$	36	$\operatorname{cosec} \cdot f(x)$	$-\operatorname{cosec} \cdot f(x) \cot \cdot f(x) \cdot f'(x)$