1. A particle moves along the curve y = $\frac{4}{3}x^3 + 5$. Find the points on the curve at which y – coordinate changes as fast as x – coordinate. Ans: (1/2, 31/6) ; (- ½, 29/6)
2. Side of an equilateral triangle is increasing at the rate of 2cm/sec. At what rate is the area increasing when side is 15 cm? Ans: $15\sqrt{3}$ cm^2/sec
3. A man 1.6m tall walks ate the rate of 0.5m/sec away from a lamp post, 8 metres high. Find the rate at which his shadow is increasing and the rate at which the tip of the shadow is moving away from the pole. Ans: 0.125m/sec; 0.625m/sec
4. Find the intervals in which the following functions are increasing or decreasing:

   a) $x^3 + 12x^2 + 36x$ b) $6 + 12x + 3x^2 - 2x^3$ c) $x^3 + 3x^2 - 4$

   d) $2x^3 - 15x^2 + 36x + 17$ e) $2x^3 - 9x^2 - 24x - 5$ f) $2x^3 - 9x^2 + 12x + 15$

   g) $x^4 - 4x^3 + 4x^2 + 15$ h) sinx – cosx , 0 < x < 2π i) $\log(1 + x) - \frac{x}{1+x}$

   j) $\frac{x}{1+x^2}$ k) $\frac{4x^2+1}{x}$ l) $\log(1 + x) - \frac{2x}{2+x}$, x > - 1 m) sinx + cosx in [0, 2π]

   m_1) x + cosx in [0, 2π] n) $\frac{x}{2} + \frac{2}{x}$, x ≠ 0 o) $x^3 + \frac{1}{x^3}$, x ≠ 0

   p) sin2x in [0, π] q) $\sin^2 x$ in [0, π] r) $\frac{4\sin x - 2x - x\cos x}{2 + \cos x}$

   s) $x^4 - 8x^3 + 22x^2 - 24x + 21$ t) $\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

   u) sin3x, x∈ [0, π/2] v) $(x – 1)(x + 2)^2$ w) $\sin^4 x + \cos^4 x$ in [0, π/2]

   x) $5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, \quad x > 0$ y) e^{x^2} z) $\frac{x}{\log x}$

5. Show that the function f(x) = $\tan^{-1}$[sinx + cosx], x > 0 is strictly decreasing on the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
6. Show that the function f(x) = $\tan^{-1}$[sinx + cosx], x > 0 is always increasing in the interval $\left(0, \frac{\pi}{4}\right)$
7. Show that the function f(x) = $3x^5 + 40x^3 + 240x$ is always increasing on R.
8. Show that the function f(x) = $\frac{-1}{x}$ is increasing, for x > 0.
9. Show that the function y = $\frac{3}{x} + 7$ is strictly decreasing for x∈ R, x ≠ 0
10. Prove that the following function is always increasing on R:

    a) f(x) = $x^3 - 6x^2 + 12x - 16$ b) f(x) = $x^3 - 3x^2 + 3x + 107$
11. Show that the function f(x) = $x^2 – 5x + 1$ is neither increasing nor decreasing on [0, 5].
12. Find whether the function f(x) = $\cos\left(2x + \frac{\pi}{4}\right)$; $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ is increasing or decreasing.
13. For what value of a, the function f(x) = a(x + sinx) + a, is increasing on R.
14. For what value of m, the function f(x) = mx + c, is decreasing for x∈ R
15. Prove that the tangents to the curve y = $x^2 – 5x + 6$ at the points (2, 0) and (3, 0) are at right angles.
16. If the tangent to the curve y = $x^3 + ax + b$ at P(1, -6) is parallel to the line y – x = 5, find the values of a and b. ans a = -2, b = -5
17. The slope of the curve $2y^2 = ax^2 + b$ at (1, - 1) is – 1. Find a and b. Ans a = 2; b = 0
18. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.

    Hint: dy/dx = 1 or – 1 Ans: (4, 8/3) and (4, -8/3)

19. Show that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-x/a}$ at the point where it crosses the y – axis.
20. Find the equations of the tangent and the normal to the curve y(x – 2)(x – 3) – x + 7 = 0 at the point where it cuts the x – axis. Ans: (7, 0) x – 20y – 7 = 0; 20x + y – 140 = 0
21. Find the equation of the tangent to the curve y = $(x^3-1)(x-2)$ at the points where the curve cuts the x – axis. Ans: 3x + y – 3 = 0 and 7x – y – 14 = 0
22. Find the equation of the tangent line to the curve y = $\sqrt{5x-3}-2$ which is parallel to the line 4x – 2y + 3 = 0. Ans (73/80, -3/4) 80x – 40y – 103 = 0
23. Find the equation of the normal to the curve x^2 = 4y which passes through the point (1, 2).
Ans; (2, 1); x + y – 3 = 0
24. Find the coordinates of the points on the curve y = x^2 + 3x + 4 , the tangents at which pass through the origin. Ans : (2, 14) and (-2, 2).
25. Find the equation of the normal to the curve y = xlogx which is parallel to the line 2x – 2y + 3 = 0.
Ans : x – y = $3e^{-2}$
26. Find the angle of intersection of the curves xy = 6 and x^2y = 12.
ans : use tan θ = (m1 – m2)/ (1 + m1m2) p (2, 3) tan θ = 3/11
27. Find the equation of the tangent and the normal to the curve $x^{2/3}$ + $y^{2/3}$ = 2 at (1, 1).
ans x + y – 2 = 0 and x – y = 0.
28. Show that the curves 4x = y^2 and 4xy = k cut at right angles if k^2 = 512.
29. Show that the curves 2x = y^2 and 2xy = k cut at right angles if k^2 = 8.
30. Show that the curves x^3 – 3xy^2 + 2 = 0 and 3x^2y – y^3 – 2 = 0 cut orthogonally.
31. Find the equation of the normal to the curve x = a$\cos^3$θ, y = a$\sin^3$θ at θ = $\frac{\pi}{4}$
32. Find the slope of the normal to the curve x = 1 – asinθ, y = b$\cos^2$θ at θ = $\frac{\pi}{2}$ ans : - a/2b
33. If y = $x^{10}-10$ and if x changes from 2 to 1.99, what is the corresponding approximate change in y?
ans -0.32

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34. Find the maximum and minimum value of each of the following functions without using derivatives:
i) $-(x-1)^2+10$ ii) $-|x+1|+3$ iii) $|\sin 4x+3|$ iv) sin2x + 5
v) $9x^2+12x+2$ vi) $(x+1)^2+5$ vii) $|x-1|+3$
35. Find the points of local maxima and local minima, if any, of the following functions. Also find the local maximum and local minimum values.
i) – x + 2sinx x϶ [0, 2π] ii) x + 2cosx, x϶ [0, π] iii) sinx + cosx , 0 < x < π/2
iv) sinx – cosx , 0 < x < 2π v) $\frac{x}{2}+\frac{2}{x}$, x > 0 vi) $3x^4+4x^3-12x^2+12$
vii) sinx + $\frac{1}{2}$ cos2x , x϶ [0, π/2] viii)sin2x – x, $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ ix) $3x^4-2x^3-6x^2+6x+1$
36. Prove that the function f(x) = x^3+x^2+x+1 does not have a maxima or minima.
37. Find the absolute minimum and absolute maximum values of the following functions:
i) 2cosx + x, x϶ [0, π] ii) 12$x^{\frac{4}{3}}$ – 6$x^{\frac{1}{3}}$, $x\in[-1,1]$ iii) sinx + cosx, x϶ [0, π]
38. Divide the number 4 into two parts such that the sum of the square of one and the cube of the other is a minimum. Ans : 8/3 and 4/3
39. Show that the area of a rectangle of given perimeter is maximum, when the rectangle is a square.
40. Show that of all the rectangles of given area, the square has the least perimeter.
41. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

42. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum. [Ans : $2P/(\pi + 8)$ & $P/(\pi + 8)$]
43. A rectangle is inscribed in a semi-circle of radius r with one of its sides on the diameter of the circle. Find the dimensions of the rectangle, so that its area is maximum. Also find maximum area.
[Ans $r/\sqrt{2}$, $\sqrt{2}r$, r^2 sq units]
44. The combined resistance R of two resistances R_1 and R_2 is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1 + R_2 = C$, a constant , show that the maximum resistance is obtained by choosing $R_1 = R_2$.
45. Two sides of a triangle are given. Find the angle between the sides such that the area shall be maximum. Also find the maximum area. Ans : 90^{o}
46. Prove that the area of right angled triangle of given hypotenuse is maximum when the triangle is isosceles.
47. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long. Ans 25/4 sq. un.
48. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
49. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.
50. A point on the hypotenuse of a triangle is at distances a and b from the sides. Show that the minimum length of the hypotenuse is $\left(a^{2/3} + b^{2/3}\right)^{3/2}$
51. The lengths of the sides of an isosceles triangle are $9 + x^2$, $9 + x^2$ and $18 - 2x^2$ units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum.
52. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.
53. Find the sides of a rectangle of greatest area that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.
[Ans $4\sqrt{2}$, $2\sqrt{2}$]
54. The sum of the perimeter of a circle and a square is K, where K is some constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.
55. If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium when it is maximum.
56. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?
57. A wire of length 26 m is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the area of the two be minimum. Ans $224/(4\sqrt{3} + 9)$m, $144\sqrt{3}/(4\sqrt{3} + 9)$m
58. Let AP and BQ be two vertical poles at points A and B respectively. If AP = 16 m. BQ = 22 m and AB = 20 m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.
59. A straight line AB of length 8 cm is divided into two parts AP and PB by a point P. Find the position of P if $AP^2 + BP^2$ is minimum. Ans : AP = BP = 4 cm
60. Find the point on the curve $y^2 = 4x$ which is nearest to the point i) (2, 1).Ans :(1, 2)(ii) (2, -8) ans (4, -4)
61. Find the shortest distance between the line $y - x = 1$ and the curve $x = y^2$. Ans : $3\sqrt{2}/8$
62. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the line $y = 3x - 3$.
Ans : (- 2, - 8)
63. An enemy jet is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest point between the soldier and the jet.
64. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier placed at (3, 7) wants to shoot down the helicopter when it is nearest to him. Find the nearest distance. Ans $\sqrt{5}$

65. A particle is moving in a straight line such that its distances at any time t is given by s = $\frac{t^4}{4} - 2t^3 + 4t^2 - 1$. Find when its velocity is minimum and acceleration is maximum.
66. Two poles of heights 16 m and 22 m stand vertically on the ground 20 m apart. Find a point on the ground, in between the poles, such that the sum of the squares of the distances of this point from the tops of the poles is minimum.
67. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.
68. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.
69. A window is in the form of a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16 m, find the width of the window so that the maximum amount of light may enter.
70. A window is in the form of a rectangle surmounted by a semi-circular opening. If the perimeter of the window is p metres show that the window will allow maximum possible light only when the radius of the semicircle is $\frac{p}{\pi + 4}$ cm .
71. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible. ans 3 cm
72. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?
73. An open box is to be constructed by removing equal squares from the corners of a 3 metre by 8 metre regular sheet of aluminum and folding up the flaps. Find the volume of the largest such box. Ans 200/27 m^3
74. If 40 sq. ft of sheet metal is to be used in the construction of an open tank with a square base, find the dimensions so that the capacity is greatest possible.
75. A tank with rectangular base and rectangular sides , open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs. 70 per sq. metre for the base and Rs. 45 per sq. metre for sides, what is the cost of the least expensive tank?
76. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the total surface area is least when depth of the tank is half its width.
77. Show that the total surface area of a closed cuboid with square base and given volume, is minimum, when it is a cube.
78. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$
79. A right circular cylinder is to be made so that the sum of its radius and its height is 6 m. Find the maximum volume of the cylinder. Ans :32π cu. m; r = 4 m
80. A right circular cylinder is inscribed in a given cone. Show that the curved surface area of the cylinder is maximum when diameter of cylinder is equal to radius of base of cone.
81. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic cm, find the dimensions of the can which has the minimum surface area.
82. A closed circular cylinder has a volume of 2156 cu. cm. What will be the radius of its base so that its total surface area is minimum? [Use π = 22/7] ans 7cm
83. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find its maximum volume.

84. Show that the right circular cylinder open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base. (OR) Show that a right circular cylinder, which is open at the top and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.
85. Show that the right circular cylinder open at the top, and of given surface area and maximum volume is such that its height is equal to the diameter of the base.
86. Show that the right circular cylinder open at the top and given volume has the minimum total surface area, provided its height is equal to radius of its base.
87. Prove that a conical tent with given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.
88. Show that the semi vertical angle of the cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.
89. Show that the semi vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}(\sqrt{2})$.
90. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30°, is $\frac{4}{81}\pi h^3$
91. Show that the volume of the greatest cylinder that can be inscribed in a cone of height 'h' and semi-vertical angle 'α' is $\frac{4}{27}\pi h^3 \tan^2\alpha$
92. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r .
93. Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.
94. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.
95. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is 4r/3.
96. The sum of the surfaces of a sphere and a cube is given. Show that when the sum of their volumes is least the diameter of the sphere is equal to the edge of the cube.
97. A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is π : (π + 2)
98. A cylinder is inscribed in a sphere of radius a, show that if the volume of the cylinder is V, then $V = 2\pi r^2\sqrt{a^2 - r^2}$, where r is the radius of the base of the cylinder. Hence, find the height of the cylinder when V is maximum and also find the maximum volume. Ans 2a/√3; $4\pi a^3/3\sqrt{3}$
99. Show that the height h of a right circular cylinder of maximum total surface area , including the two ends that can be inscribed in a sphere of radius r is given by $h^2 = 2r^2\left(1-\frac{1}{\sqrt{5}}\right)$
100. A manufacturer can sell x items at a price of Rs. $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs. $\left(\frac{x}{5}+500\right)$. find the number of items he should sell to earn maximum profit.

ANSWERS:

4. a) inc: x < - 6 or x > - 2; dec: (- 6, - 2) b) inc: [-1, 2]; dec: x < - 1 or x > 2
c) inc: x < - 2 or x > 0; dec: (- 2, 0) d) inc: x < 2 or x > 3; dec: (2, 3)

e) inc: $(-\infty, -1) \cup (4, \infty)$; dec: $(-1, 4)$ f) inc: $(-\infty, 1) \cup (2, \infty)$; dec: $(1, 2)$

g) inc: $(0, 1) \cup (2, \infty)$; dec: $(-\infty, 0) \cup (1,2)$ h) inc: $0 < x < 3\pi/4$ or $7\pi/4 < x < 2\pi$; dec: $3\pi/4 < x < 7\pi/4$

i) inc: $x > 0$; dec: $-1 < x < 0$ j) inc: $-1 \le x \le 1$; dec: $x \ge 1$ or $x \le -1$

k) inc: $(-\infty, -1/2) \cup (1/2, \infty)$; dec: $(-1/2, 1/2) - \{0\}$ l) inc in R

m) T.B. eg. 13 m_1) inc for all $x \in R$ n) inc: $x > 2$ or $x < -2$; dec: $(-2, 2) - \{0\}$

o) inc: $(-\infty, -1) \cup (1, \infty)$; dec: $(-1, 1)$ p)

q) inc: $0 < x < \pi/2$; dec: $\pi/2 < x < \pi$ r) inc: $0 < x < \pi/2$ or $3\pi/2 < x < 2\pi$; dec: $\pi/2 < x < 3\pi/2$

s) inc: $1 < x < 2$ or $x > 3$; dec: $x < 1$ or $2 < x < 3$ t) inc: $(-2, 1) \cup (3, \infty)$; dec: $(-\infty, -2) \cup (1,3)$

u) inc: $(0, \pi/6)$; dec: $(\pi/6, \pi/2)$ v) inc: $x > 0$ or $x < -2$; dec: $(-2, 0)$

w)inc: $(\pi/4, \pi/2)$; dec: $(0, \pi/4)$ x) inc: $(0, 1)$ dec: $x > 1$

y) inc: for all $x \ (\neq -3) \in R$ z) inc: (e, ∞); dec: $(0, 1) \cup (1, e)$

12. increasing fn.

13. $a > 0$ 14. $m < 0$ 34. i) 10, nil; ii) 3, nil iii) 4, 2 iv) 6, 4 v) 5, nil vi) min = -2

35. i) local max value = $\sqrt{3} - \pi/3$ at $x = \pi/3$; local min value = $-(5\pi/3 + \sqrt{3})$ at $x = 5\pi/3$

ii) local max value = $\sqrt{3} + \pi/6$ at $x = \pi/6$; local min value = $5\pi/6 - \sqrt{3}$ at $x = 5\pi/6$

iii) local max value = $\sqrt{2}$ at $x = \pi/4$

iv) local max value = $\sqrt{2}$ at $x = 3\pi/4$; local min value = $-\sqrt{2}$ at $x = 7\pi/4$

v) local min value = 2 at $x = 2$

vi) local max value = 12 at $x = 0$; local min value = 7 at $x = 1$; local min value = -20 at $x = -2$

vii) local max value = 3/4 at $x = \pi/6$

viii) local max value = $\sqrt{3}/2 - \pi/6$ at $x = \pi/6$; local min value = $\pi/6 - \sqrt{3}/2$ at $x = -\pi/6$

ix) local max value = 39/16 at $x = 1/2$; local min value = -6 and 200 at $x = -1$ and 1

37. i) abs. max value = $\pi/6 + \sqrt{3}$ at $x = \pi/6$; abs. min. value = $5\pi/6 - \sqrt{3}$ at $x = 5\pi/6$

ii) abs. max value = 18 at $x = -1$; abs. min. value = $-9/4$ at $x = 1/8$

iii) abs. max value = $\sqrt{2}$ at $x = \pi/4$; abs. min. value = -1 at $x = \pi$