Equating ① and ② $\Rightarrow$

$$\cancel{2} - 2\cos(x-y) = \cancel{2} - 2\cos x\cos y - 2\sin x\sin y$$
$$-2\cos(x-y) = -2(\cos x\cos y + \sin x\sin y)$$

$$\therefore \underline{\cos(x-y) = \cos x\cos y + \sin x\sin y}$$

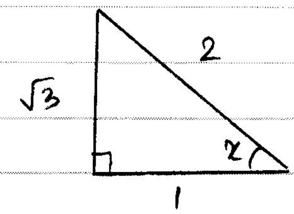
2. If $\cos x = \frac{-1}{2}$ and x in third quadrant, find other trigonometric functions.

Ans.

$\sin x = -\frac{\sqrt{3}}{2}$ $\quad$ $\operatorname{cosec} x = \frac{-2}{\sqrt{3}}$

$\cos x = \frac{-1}{2}$ $\quad$ $\sec x = -2$

$\tan x = \sqrt{3}$ $\quad$ $\cot x = \frac{1}{\sqrt{3}}$



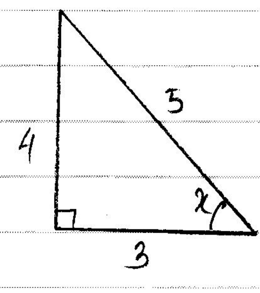
3. If $\tan x = \frac{-4}{3}$ and x in second quadrant, find the values of $\sin x/2$, $\cos x/2$ and $\tan x/2$

Ans.

$\cos x = -\frac{3}{5}$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{5+3}{5}\times\frac{1}{2}} = \sqrt{\frac{\cancel{8}^{4}\times 1}{5\times\cancel{2}}}$$

$$\underline{\sin\frac{x}{2} = \frac{2}{\sqrt{5}}}$$

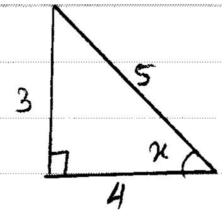


$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{5-3}{5}\times\frac{1}{2}} = \underline{\frac{1}{\sqrt{5}}}$$

$$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{1} = \underline{2}$$

4. If $\sin x = \frac{-3}{5}$ and x is in quadrant III, find the values of $\sin x/2$, $\cos x/2$ and $\tan x/2$

Ans. $\cos x = \frac{-4}{5}$



$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{5+4}{5}\times\frac{1}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{5-4}{5}\times\frac{1}{2}} = \sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}}$$

$$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{3}{\sqrt{10}} \times \frac{-\sqrt{10}}{1} = -3$$

5. Prove that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

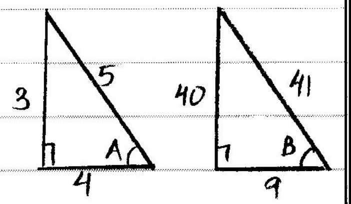
Ans.
$$\tan 3x = \tan(2x + x)$$
$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$
$$\tan 3x\,(1 - \tan x \tan 2x) = \tan 2x + \tan x$$
$$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$
$$\therefore \tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$$

6. Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Ans.
$$\cot 3x = \cot(2x + x)$$
$$\cot 3x = \frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}$$
$$\cot 3x \cot x + \cot 2x \cot 3x = \cot 2x \cot x - 1$$
$$\therefore \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

7. If $\sin A = \frac{3}{5}$ $\cos B = \frac{9}{41}$, A and B in quadrant II, then find $\sin(A-B)$

Ans. $\sin A = \dfrac{3}{5}$ $\cos A = \dfrac{-4}{5}$ $\sin B = \dfrac{40}{41}$ $\cos B = \dfrac{-9}{41}$



$$\sin(A-B) = \sin A\cos B - \cos A \sin B$$

$$\sin(A-B) = \left[\frac{3}{5} \times \frac{(-9)}{41}\right] - \left[\frac{-4}{5} \times \frac{40}{41}\right]$$

$$\sin(A-B) = \frac{-27 + 160}{205}$$

$$\therefore \sin(A-B) = \underline{\underline{\frac{133}{205}}}$$

8. Prove that $\dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin(A+B)}{\sin(A-B)}$

Ans. $LHS = \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B} \div \dfrac{\sin A}{\cos A} - \dfrac{\sin B}{\cos B}$

$$= \frac{\sin A\cos B + \sin B\cos A}{\cancel{\cos A\cos B}} \div \frac{\sin A\cos B - \cos A\sin B}{\cancel{\cos A\cos B}}$$

$$= \frac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B}$$

$$= \frac{\sin(A+B)}{\sin(A-B)} = \underline{\underline{RHS}}$$

9. Find the value of :

(i) $\tan 15°$

$$\tan 15° = \tan(45° - 30°)$$

$$\tan 15° = \frac{\tan 45° - \tan 30°}{1 + \tan 45 \tan 30°}$$

$$\tan 15° = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} \, \frac{(1 - 1/\sqrt{3})}{(1 - 1/\sqrt{3})}$$

or

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} =$$

$$\tan 15^\circ = \frac{1 - 2/\sqrt{3} + 1/3}{1 - 1/3}$$

$$\tan 15^\circ = \frac{3\sqrt{3} - 6 + \sqrt{3}}{\cancel{3}\sqrt{3}} \div \frac{3-1}{\cancel{3}}$$

$$\tan 15^\circ = \frac{4\sqrt{3} - 6}{2\sqrt{3}} = \frac{2\sqrt{3}(2-\sqrt{3})}{2\sqrt{3}}$$

$$\therefore \tan 15^\circ = 2 - \sqrt{3}$$

(ii) $\tan \frac{\pi}{8}$

Ans. $\frac{\pi}{8} \times \frac{180}{\pi} = \frac{\overset{45}{\cancel{180}}}{\cancel{8}\,2} = \frac{45^\circ}{2}$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\tan 45 = 2\tan A \div 1 - \tan^2 A$$

$$1 - \tan^2 A = 2\tan A$$

$$\tan^2 A + 2\tan A - 1 = 0$$

$$\tan A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$\tan A = -1 + \sqrt{2}$, $-1 - \sqrt{2}$ (rejected as $\tan\theta$ cannot be $-$ve in I quadrant)

$$\therefore \tan \frac{\pi}{8} = -1 + \sqrt{2}$$

10. Find the value of $\dfrac{\cos(2\pi + \theta)\operatorname{cosec}(2\pi + \theta)\tan(\pi/2 + \theta)}{\sec(\pi/2 + \theta)\cos\theta\cot(\pi + \theta)}$

Ans. $\dfrac{\cos(2\pi + \theta)\operatorname{cosec}(2\pi + \theta)\tan(\pi/2 + \theta)}{\sec(\pi/2 + \theta)\cos\theta\cot(\pi + \theta)}$

$$= \frac{\cos(360 + \theta)\operatorname{cosec}(360 + \theta)\tan(90 + \theta)}{\sec(90 + \theta)\cos\theta\cot(180 + \theta)}$$

$$= \frac{-\cos\theta \ \mathrm{cosec}\,\theta \ \cot\theta}{-\mathrm{cosec}\,\theta \ \cos\theta \ \cot\theta}$$

$$= \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} \div \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \underline{1}$$

11. Find the value of $\frac{\sin(180+\theta)\cos(360-\theta)\tan(270-\theta)}{\sec^2(90+\theta)\tan(-\theta)\sin(270+\theta)}$

Ans. $\frac{\sin(180+\theta)\cos(360-\theta)\tan(270-\theta)}{\sec^2(90+\theta)\tan(-\theta)\sin(270+\theta)}$

$[\sec(90+\theta)]^2 = (-\mathrm{cosec}\,\theta)^2 = \mathrm{cosec}^2\theta$

$$= \frac{-\sin\theta\cos\theta\cot\theta}{-\mathrm{cosec}^2\theta \cdot -\tan\theta - \cos\theta}$$

$$= \frac{\sin\theta\cos^2\theta}{\sin\theta} \div \frac{\sin\theta\cos\theta}{\cos\theta\sin^2\theta}$$

$$= \underline{-\cos^2\theta\sin\theta}$$

12. Find the value of $\frac{\tan(90-\theta)\sec(180-\theta)\sin(-\theta)}{\sin(180+\theta)\cot(360-\theta)\mathrm{cosec}(90-\theta)}$

Ans. $\frac{\tan(90-\theta)\sec(180-\theta)\sin(-\theta)}{\sin(180+\theta)\cot(360-\theta)\mathrm{cosec}(90-\theta)} = \frac{\cot\theta(-\sec\theta)(-\sin\theta)}{(-\sin\theta)(-\cot\theta)\sec\theta}$

$$= \cancel{\cos\theta}\ 1 \div \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \underline{1}$$

13. If $\tan A = k\tan B$ show that $\sin(A+B) = \frac{k+1}{k-1}(\sin A - B)$

Ans. $k = \frac{\tan A}{\tan B}$

Substituting value of k in RHS $\Rightarrow$

$$RHS = \frac{k+1}{k-1} \sin(A-B)$$

$$= \left(\frac{\tan A}{\tan B} + 1 \div \frac{\tan A}{\tan B} - 1 \right) \sin(A-B)$$

$$= \frac{\tan A + \tan B}{\tan A - \tan B} \times \sin(A-B)$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B} \times \sin(A-B)$$

$$= \frac{\sin(A+B)\, \cancel{\sin(A-B)}}{\cancel{\sin(A-B)}}$$

$$= \sin(A+B)$$

$$= \underline{LHS}$$

14. Prove that

(i) $$\tan 56^\circ = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

Ans.

$$\tan 56^\circ = \tan(45^\circ + 11^\circ)$$

$$\tan 56^\circ = \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ}$$

$$\tan 56^\circ = \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$\tan 56^\circ = 1 + \frac{\sin 11^\circ}{\cos 11^\circ} \div 1 - \frac{\sin 11^\circ}{\cos 11^\circ}$$

$$\tan 56^\circ = \frac{\cos 11^\circ + \sin 11^\circ}{\cancel{\cos 11^\circ}} \div \frac{\cos 11^\circ - \sin 11^\circ}{\cancel{\cos 11^\circ}}$$

$$\therefore \tan 56^\circ = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

(ii) $\tan 36^\circ = \dfrac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ}$

Ans.

$$\tan \cancel{35}36^\circ = \tan(45^\circ - 9^\circ)$$

$$\tan 36^\circ = \frac{\tan 45^\circ - \tan 9^\circ}{1 + \tan 45^\circ \tan 9^\circ}$$

$$\tan 36^\circ = \frac{1 - \tan 9^\circ}{1 + \tan 9^\circ}$$

$$\tan 36^\circ = 1 - \frac{\sin 9^\circ}{\cos 9^\circ} \div 1 + \frac{\sin 9^\circ}{\cos 9^\circ}$$

$$\tan 36^\circ = \frac{\cos 9^\circ - \sin 9^\circ}{\cancel{\cos} 9^\circ} \div \frac{\cos 9^\circ + \sin 9^\circ}{\cancel{\cos} 9^\circ}$$

$$\therefore \tan 36^\circ = \frac{\cos 9^\circ - \sin 9^\circ}{\cos 9^\circ + \sin 9^\circ}$$

(iii) $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

Ans.

$$\tan 70^\circ = \tan(50^\circ + 20^\circ)$$

$$\tan 70^\circ = \frac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \tan 20^\circ}$$

$$\tan 70^\circ - \tan 70^\circ \tan 50^\circ \tan 20^\circ = \tan 50^\circ + \tan 20^\circ$$

$$\tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \tan 70^\circ \tan 50^\circ \tan 20^\circ$$

$$\tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \cot(90 - 70)^\circ \tan 50^\circ \tan 20^\circ$$

$$\tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \frac{1}{\cancel{\tan 20^\circ}} \times \cancel{\tan 20^\circ} \cdot \tan 50^\circ$$

$$\tan 70^\circ = \tan 20^\circ + \tan 50^\circ + \tan 50^\circ$$

$$\therefore \tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$$

15. Prove that $\tan 7x - \tan 5x - \tan 2x = \tan 7x \tan 5x \tan 2x$

Ans.

$$\tan 7x = (\tan 2x + \tan 5x)$$

$$\tan 7x = \frac{\tan 2x + \tan 5x}{1 - \tan 2x \tan 5x}$$

$$\tan 7x - \tan 7x \tan 5x \tan 2x = \tan 2x + \tan 5x$$

$$\therefore \tan 7x - \tan 5x - \tan 2x = \tan 7x \tan 5x \tan 2x$$

16. Prove that :

(i) $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

Ans. $LHS = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$= \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2$$

$$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right) + 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$= 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$\therefore$ LHS = RHS

(ii) $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

$$LHS = \frac{\cos 10^\circ - \sqrt{3}\sin 10}{\sin 10^\circ \cos 10^\circ} = \frac{2\left[\cos 10^\circ - \sqrt{3}\sin 10^\circ\right]}{2\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2\left[\cos 10^\circ - \sqrt{3}\sin 10^\circ\right]}{\sin 20^\circ}$$ [using $2\sin x\cos x = \sin 2x$]

$$= \frac{4\left[\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right]}{\sin 20^\circ}$$ [multiplying & dividing numerator by 2]

$$= \frac{4\left[\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ\right]}{\sin 20^\circ}$$ [$\because \sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$]

$$= \frac{4\left[\sin(30^\circ - 10^\circ)\right]}{\sin 20^\circ}$$ [using $\sin x\cos y - \cos x\sin y = \sin(x-y)$]

$$= 4\frac{\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ}} = 4 = RHS$$

(iii) $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\cos x$

$$\cos 4x = \cos 2\cdot(2x)$$
$$\cos 4x = 2\cos^2 2x - 1$$

$$LHS = \sqrt{2+\sqrt{2+2\cos 4x}} = \sqrt{2+\sqrt{2+4\cos^2 2x-2}}$$
$$= \sqrt{2+\sqrt{4\cos^2 2x}}$$
$$= \sqrt{2+2\cos 2x}$$
$$= \sqrt{2+4\cos^2 x-2}$$
$$= \sqrt{4\cos^2 x}$$
$$= 2\cos x$$
$$\therefore \underline{LHS = RHS}$$

17. Find the value of :

(i) $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$

Ans. $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ = \sin(75+15^\circ)$
$$= \sin 90^\circ$$
$$= \underline{1}$$

(ii) $\cos 47^\circ \sin 17^\circ - \sin 47^\circ \cos 17^\circ$

Ans $\cos 47^\circ \sin 17^\circ - \sin 47^\circ \cos 17^\circ = \sin(47^\circ - 17^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

(iii) $\cos 42^\circ \cos 12^\circ + \sin 42^\circ \sin 12^\circ$

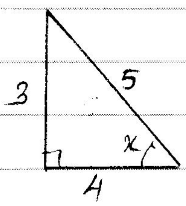
Ans. $\cos 42^\circ \cos 12^\circ + \sin 42^\circ \sin 12^\circ = \cos(42^\circ - 12^\circ)$

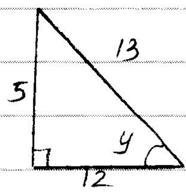
$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

18a) If $\sin x = 3/5$, $\cos y = -12/13$; x and y both lie in second quadrant, find:

(i) $\sin(x+y)$





Ans. $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \frac{3}{5} \times \left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right) \times \frac{5}{13}$

$= \frac{-36 - 20}{65}$

$\therefore \sin(x+y) = -\frac{56}{65}$

(ii) $\cos(x-y)$

Ans. $\cos(x-y) = \cos x \cos y \overset{+}{-} \sin x \sin y$

$= \left(-\frac{4}{5} \times -\frac{12}{13}\right) \underset{+}{-} \left(\frac{3}{5} \times \frac{5}{13}\right)$

$= \frac{48 \overset{+}{-} 15}{65} = \frac{33}{65}$ $\quad \frac{63}{65}$

$\therefore \cos(x-y) = \frac{33}{65}$

(iii) $\tan(x+y)$

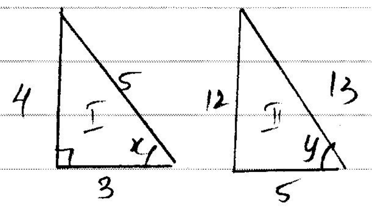
Ans. $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \left[\frac{-3}{4} + \left(\frac{-5}{12}\right)\right] \div 1 - \frac{15}{48}$

$= \frac{-14}{\cancel{12}} \div \frac{33}{\cancel{48}\,4}$

$\therefore \tan(x+y) = \frac{-56}{33}$

18b) If $\sin x = \frac{4}{5}$, $\cos y = \frac{-5}{13}$, $0 < x < \frac{\pi}{2}$; $\frac{\pi}{2} < y < \pi$; find:

(i) $\cos(x+y)$



$\sin x = \frac{4}{5}$ $\sin y = \frac{12}{13}$

$\cos x = \frac{3}{5}$ $\cos y = \frac{-5}{13}$

Ans. $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{3}{5} \times \frac{(-5)}{13}\right) - \left(\frac{4}{5} \times \frac{12}{13}\right)$

$= \frac{-15 - 48}{65}$

$\therefore \cos(x+y) = \frac{-63}{65}$

(ii) $\cos(x-y)$

Ans. $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \left[\frac{3}{5} \times \frac{(-5)}{13}\right] + \left(\frac{4 \times 12}{5 \times 13}\right)$

$\cos(x-y) = \dfrac{-15+48}{65}$

$\therefore$ $\cos(x-y) = \dfrac{33}{65}$

(iii) $\sin(x-y)$

Ans. $\sin(x-y) = \sin x \cos y - \cos x \sin y$

$= \dfrac{4}{5} \times \dfrac{(-5)}{13} - \left[\dfrac{3}{5} \times \dfrac{12}{13}\right]$

$= \dfrac{-20-36}{65}$

$\therefore$ $\sin(x-y) = \dfrac{-56}{65}$