35. $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$

$\vec{b} = \hat{i}-3\hat{j}+4\hat{k}$

Area of ||gm (with diagonals)

$= \frac{1}{2}|\vec{a}\times\vec{b}|$

$= \frac{1}{2}|-2\hat{i}-14\hat{j}-10\hat{k}|$

$= \frac{1}{2}\times 10\sqrt{3} = 5\sqrt{3}$ sq. units

36. $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$

$\vec{b} = \hat{i}-3\hat{j}-5\hat{k}$

$\vec{c} = 3\hat{i}-4\hat{j}-4\hat{k}$

$\vec{a}\cdot\vec{b} = 2(1)+(-1)(-3)+1(-5)$
$= 0$

$\Rightarrow \vec{a}\perp\vec{b}$ $\therefore$ right $\angle^d\Delta$.

$\boxed{\text{OR}}$ $|\vec{a}|=\sqrt{6}$, $|\vec{b}|=\sqrt{35}$

$|\vec{c}|=\sqrt{41}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2$

hence right $\Delta$.

37. $\vec{a}+\vec{b} = -\vec{c}$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \cancel{\vec{c}\cdot\vec{c}}\ (-\vec{c})\cdot(-\vec{c})$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\therefore 9+25+2\vec{a}\cdot\vec{b} = 49$

$\Rightarrow \vec{a}\cdot\vec{b} = 15/2$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{15/2}{3\times5} = \frac{1}{2}$

$\therefore \theta = 60^\circ$

38. $\left.\begin{aligned}\bar{a}\cdot(\bar{b}+\bar{c}) &= 0\\ \bar{b}\cdot(\bar{c}+\bar{a}) &= 0\\ \bar{c}\cdot(\bar{a}+\bar{b}) &= 0\end{aligned}\right\}$ adding all of them

$2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}) = 0$ — (1)

$|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

$= |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})$

$= 9+16+25+0$ [from (1)]

$= 50$

$\therefore |\bar{a}+\bar{b}+\bar{c}| = \sqrt{50} = 5\sqrt{2}$

39. $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow (\bar{a}+\bar{b}+\bar{c})\cdot(\bar{a}+\bar{b}+\bar{c}) = 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}) = 0$

$\Rightarrow 9+16+25+2(\quad " \quad) = 0$

$\therefore \bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a} = -25$

40. Given i) $|\vec{a}|=|\vec{b}|=|\vec{c}|$

ii) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$ (mutually $\perp$)

angle between $(\bar{a}+\bar{b}+\bar{c})$ & $\vec{a}$ is

$\cos\alpha = \frac{(\bar{a}+\bar{b}+\bar{c})\cdot\vec{a}}{|\bar{a}+\bar{b}+\bar{c}||\vec{a}|} = \frac{\bar{a}\cdot\bar{a}+\bar{b}\cdot\bar{a}+\bar{c}\cdot\bar{a}}{|\bar{a}+\bar{b}+\bar{c}||\vec{a}|}$

$= \frac{|\vec{a}|^2+0+0}{|\bar{a}+\bar{b}+\bar{c}||\vec{a}|} = \frac{|\vec{a}|^{\cancel{2}}}{|\bar{a}+\bar{b}+\bar{c}|\cancel{|\vec{a}|}}$ — (1)

Similarly $\cos\beta = \frac{|\vec{b}|}{|\bar{a}+\bar{b}+\bar{c}|}$ — (2) & $\cos\gamma = \frac{|\vec{c}|}{|\bar{a}+\bar{b}+\bar{c}|}$ — (3)

from (1), (2) & (3) $\cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \alpha = \beta = \gamma$

41. refer Q. 38

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2}$

42. let $\vec{a} = \vec{a}_1+\vec{a}_2$ where $\bar{a}_1$ is $||^{el}$ to $\vec{b}$ & $\bar{a}_2$ is $\perp$ to $\vec{b}$

$\therefore \vec{a}_1 = \lambda(3\hat{i}+\hat{k})$

$\Rightarrow \vec{a}_2 = \bar{a}-\bar{a}_1 = (5\hat{i}-2\hat{j}+5\hat{k})-(3\lambda\hat{i}+\lambda\hat{k})$

$= (5-3\lambda)\hat{i}-2\hat{j}+(5-\lambda)\hat{k}$

Since $a_2$ is $\perp$ b

$\bar{a}_2 \cdot \bar{b} = 0$

$\Rightarrow (5-3\lambda)(3) + (-2)(0) + (5-\lambda) 1 = 0$

Solving we get $\lambda = 2$

$\therefore \vec{a}_1 = 2(3\hat{i} + \hat{k}) = 6\hat{i} + 2\hat{k}$

$\vec{a}_2 = -\hat{i} - 2\hat{j} + 3\hat{k}$

(43) $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow (\bar{a} + \bar{b}) \times \bar{c} = -\vec{c} \times \vec{c}$

$\Rightarrow \bar{a} \times \bar{c} + \bar{b} \times \bar{c} = 0$

$\Rightarrow \bar{b} \times \bar{c} = -\bar{a} \times \bar{c}$

$\bar{b} \times \bar{c} = \bar{c} \times \bar{a}$ — (1)

Also $\bar{a} + \bar{c} = -\bar{b}$

$(\bar{a} + \bar{c}) \times \vec{b} = -\bar{b} \times \bar{b}$

$\bar{a} \times \bar{b} + \bar{c} \times \bar{b} = 0$

$\bar{a} \times \bar{b} = -\bar{c} \times \bar{b}$

$\bar{a} \times \bar{b} = \bar{b} \times \bar{c}$ — (2)

from (1) & (2)

$\bar{a} \times \bar{b} = \bar{b} \times \bar{c} = \bar{c} \times \bar{a}$

(44) $\bar{AB} = \bar{b} - \bar{a}$

$\bar{AC} = \bar{c} - \bar{a}$

Area of $\Delta = \frac{1}{2} |\bar{AB} \times \bar{AC}|$

$= \frac{1}{2} |(\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})|$

$= \frac{1}{2} |\bar{b} \times \bar{c} - \bar{b} \times \bar{a} - \bar{a} \times \bar{c} + \bar{a} \times \bar{a}|$

$= \frac{1}{2} |\bar{b} \times \bar{c} + \bar{a} \times \vec{b} + \bar{c} \times \bar{a} + 0|$

$= \frac{1}{2} |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$

(45) let $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

then $\vec{a} \cdot \hat{i} = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot \hat{i}$

$= a$ [as $\hat{j} \cdot \hat{i} = \hat{k} \cdot \hat{i} = 0$] — (1)

$\vec{a} \cdot \hat{j} = b$ — (2)

$\vec{a} \cdot \hat{k} = c$ — (3)

$\therefore (a \cdot \hat{i})\hat{i} + (a \cdot \hat{j})\hat{j} + (\hat{a} \cdot \hat{k})\hat{k}$

$= a\hat{i} + b\hat{j} + c\hat{k}$ [using (1), (2), (3)]

$= \vec{a}$

(46) $\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c} \perp \bar{a}$ & $\vec{c} \perp \vec{b}$ — (i)

Also $\bar{b} \times \bar{c} = \vec{a} \Rightarrow \vec{a} \perp \vec{b}$ & $\vec{a} \perp \vec{c}$ — (ii)

from (i) & (ii) they are mutually $\perp$.

Now, $\bar{a} \times \bar{b} = \vec{c}$ & $\bar{b} \times \bar{c} = \vec{a}$ (given)

$\Rightarrow |\bar{a} \times \bar{b}| = |\bar{c}|$ & $|\bar{b} \times \bar{c}| = |\vec{a}|$

$\Rightarrow |\vec{a}||\bar{b}| \sin\theta = |\vec{c}|$ & $|\bar{b}||\vec{c}| \sin\phi = |\bar{a}|$

Since they are mutually $\perp$, $\theta = \phi = 90°$

$\Rightarrow |\vec{a}||\bar{b}| = |\vec{c}|$ — (iii) & $|\vec{b}||\vec{c}| = |\vec{a}|$ — (iv)

Sub. (iii) in (iv) $|\vec{b}| \cdot |\vec{a}||\bar{b}| = |\vec{a}|$

$\Rightarrow |\vec{b}|^2 = 1 \Rightarrow |\vec{b}| = 1$

Sub. this in (iii) $|\vec{a}| = |\vec{c}|$

(47) $\bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{c} \Rightarrow \bar{a} \cdot \bar{b} - \bar{a} \cdot \bar{c} = 0$

$\Rightarrow \bar{a} \cdot (\bar{b} - \bar{c}) = 0$

$\Rightarrow \bar{a} \perp (\bar{b} - \bar{c})$ or $\bar{b} - \bar{c} = 0$ (as $\bar{a} \neq 0$) — (i)

Also $\bar{a} \times \bar{b} = \bar{a} \times \bar{c}$

$\Rightarrow \bar{a} \times \bar{b} - \bar{a} \times \bar{c} = 0 \Rightarrow \bar{a} \times (\bar{b} - \bar{c}) = 0$

$\Rightarrow \vec{a} \parallel (\bar{b} - \bar{c})$ or $\bar{b} - \bar{c} = 0$ — (ii) (as $\bar{a} \neq 0$)

from (i) & (ii) $\bar{b} - \bar{c} = 0 \Rightarrow \vec{b} = \vec{c}$

(48) Given: i) $\bar{a}\times\bar{b} = \bar{c}\times\bar{d}$

ii) $\bar{a}\times\bar{c} = \bar{b}\times\bar{d}$

To prove: $(\bar{a}-\bar{d}) \parallel (\bar{b}-\bar{c})$

Proof $(\bar{a}-\bar{d})\times(\bar{b}-\bar{c})$

$= \bar{a}\times\bar{b} - \bar{a}\times\bar{c} - \bar{d}\times\bar{b} + \bar{d}\times\bar{c}$

$= \bar{c}\times\bar{d} - \bar{b}\times\bar{d} - \bar{d}\times\bar{b} + \bar{d}\times\bar{c}$

(using given)

$= \cancel{\bar{c}\times\bar{d}} - \cancel{\bar{b}\times\bar{d}} + \cancel{\bar{b}\times\bar{d}} - \cancel{\bar{c}\times\bar{d}}$

$= 0$

$\Rightarrow (\bar{a}-\bar{d}) \parallel (\bar{b}-\bar{c})$

(49) $|\bar{a}\times\bar{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \sin^2\theta$

$= |\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)$

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$

$= |\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2$

$= |\bar{a}|^2|\bar{b}|^2 - (\bar{a}.\bar{b})^2$

$= a^2b^2 - (\bar{a}.\bar{b})^2$

(50) let $\hat{a}+\hat{b} = \hat{c}$ $\quad |\hat{a}| = |\hat{b}| = |\hat{c}| = 1$

$\Rightarrow |\vec{a}+\bar{b}| = 1$

$\Rightarrow |\bar{a}+\bar{b}|^2 = (\bar{a}+\bar{b}).(\bar{a}+\bar{b}) = 1$

$\Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}.\bar{b} = 1$

$\therefore \quad \bar{a}.\bar{b} = -1/2$

$|\bar{a}-\bar{b}|^2 = (\bar{a}-\bar{b}).(\bar{a}-\bar{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 - 2\bar{a}.\bar{b}$

$= 1 + 1 - 2\times -\frac{1}{2}$

$= 3$

$\therefore |\bar{a}-\bar{b}| = \sqrt{3}$