

Chapter 13 – Limits and Derivatives

Maths

# Exercise 13.1

**Question 1:** 

Evaluate the Given limit: 
$$x \rightarrow 3$$

Answer

 $\lim_{x \to 3} x + 3 = 3 + 3 = 6$ 

**Question 2:** 

Evaluate the Given limit: 
$$\lim_{x \to \pi} \left( x - \frac{22}{7} \right)$$

Answer

$$\lim_{x \to \pi} \left( x - \frac{22}{7} \right) = \left( \pi - \frac{22}{7} \right)$$

**Question 3:** 

Evaluate the Given limit:  $\lim_{r \to 1} r^2$ 

Answer

$$\lim_{r\to 1}\pi r^2 = \pi \left(1\right)^2 = \pi$$

**Question 4:** 

Evaluate the Given limit:  $\lim_{x \to 4} \frac{4x+3}{x-2}$ 

Answer

$$\lim_{x \to 4} \frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$$

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**Question 5:** 

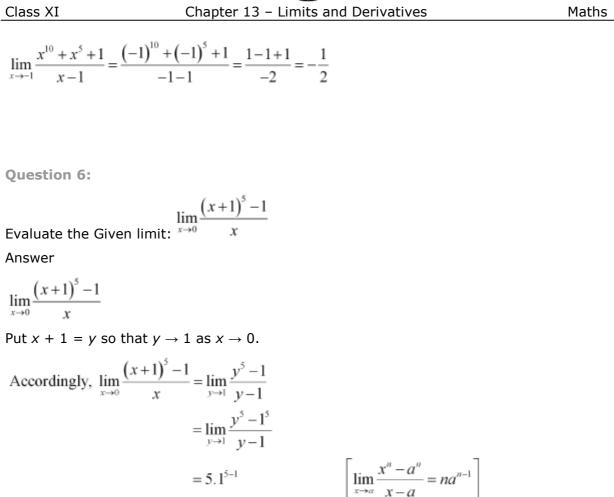
Evaluate the Given limit: 
$$\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$$

Answer

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 $= 5.1^{5-1}$ 

$$\therefore \lim_{x \to 0} \frac{(x+5)^5 - 1}{x} = 5$$

**Question 7:** 

$$\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

Evaluate the Given limit: Answer

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0

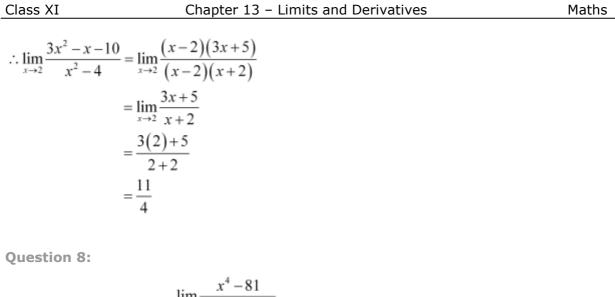
At x = 2, the value of the given rational function takes the form 0.

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Evaluate the Given limit:  $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$ Answer

0

At x = 2, the value of the given rational function takes the form 0.

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x \to 3} \frac{(x - 3)(x + 3)(x^2 + 9)}{(x - 3)(2x + 1)}$$

$$= \lim_{x \to 3} \frac{(x + 3)(x^2 + 9)}{2x + 1}$$

$$= \frac{(3 + 3)(3^2 + 9)}{2(3) + 1}$$

$$= \frac{6 \times 18}{7}$$

$$= \frac{108}{7}$$

**Question 9:** 

Evaluate the Given limit: 
$$\lim_{x\to 0} \frac{dx+b}{cx+1}$$

Answer

$$\lim_{x \to 0} \frac{ax+b}{cx+1} = \frac{a(0)+b}{c(0)+1} = b$$

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**Question 10:** 

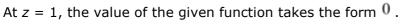
$$\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

Evaluate the Given limit:

### Answer

 $\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$ 

0



Put 
$$z^{\frac{1}{6}} = x$$
 so that  $z \to 1$  as  $x \to 1$ .  
Accordingly,  $\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$   
 $= \lim_{x \to 1} \frac{x^2 - 1^2}{x - 1}$   
 $= 2 \cdot 1^{2 - 1}$   
 $\left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$   
 $= 2$ 

$$\therefore \lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} = 2$$

**Question 11:** 

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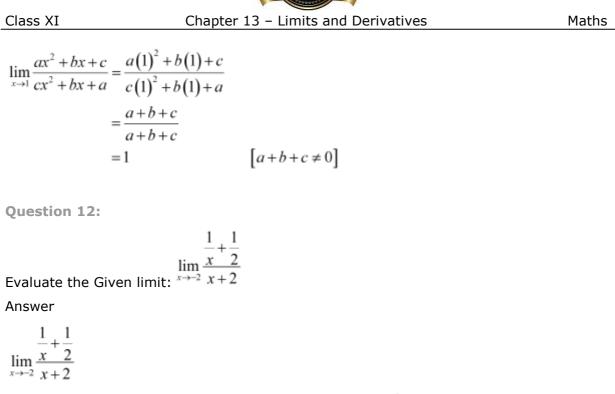
Evaluate the Given limit:  $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$ Answer

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0

At x = -2, the value of the given function takes the form 0.

Now, 
$$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} = \lim_{x \to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2}$$
$$= \lim_{x \to -2} \frac{1}{2x}$$
$$= \frac{1}{2(-2)} = \frac{-1}{4}$$

**Question 13:** 

$$\lim_{x\to 0}\frac{\sin ax}{bx}$$

Evaluate the Given limit:  $x \rightarrow 0$ 

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### Answer

 $\lim_{x\to 0} \frac{\sin ax}{bx}$ 

0

At x = 0, the value of the given function takes the form 0.

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Now, 
$$\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx}$$
  
 $= \lim_{x \to 0} \left( \frac{\sin ax}{ax} \right) \times \left( \frac{a}{b} \right)$   
 $= \frac{a}{b} \lim_{ax \to 0} \left( \frac{\sin ax}{ax} \right) \qquad [x \to 0 \Rightarrow ax \to 0]$   
 $= \frac{a}{b} \times 1 \qquad \left[ \lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$   
 $= \frac{a}{b}$ 

**Question 14:** 

Evaluate the Given limit:  $\frac{\sin ax}{\sin bx}$ ,  $a, b \neq 0$ 

#### Answer

 $\lim_{x\to 0}\frac{\sin ax}{\sin bx},\ a,\ b\neq 0$ 

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0

At x = 0, the value of the given function takes the form  $\overline{0}$ .

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**Question 15:** 

Evaluate the Given limit: 
$$\frac{\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}}{\pi(\pi - x)}$$

Answer

 $\lim_{x\to\pi}\frac{\sin\left(\pi-x\right)}{\pi\left(\pi-x\right)}$ 

It is seen that  $x \to \pi \Rightarrow (\pi - x) \to 0$ 

$$\therefore \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \lim_{(\pi - x) \to 0} \frac{\sin(\pi - x)}{(\pi - x)}$$
$$= \frac{1}{\pi} \times 1 \qquad \qquad \left[ \lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$$
$$= \frac{1}{\pi}$$

**Question 16:** 

Evaluate the given limit:  $\lim_{x\to 0} \frac{\cos x}{\pi - x}$ 

Answer

$$\lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$$

**Question 17:** 

Evaluate the Given limit: 
$$\frac{\lim_{x\to 0} \frac{\cos 2x - 1}{\cos x - 1}}{\cos x - 1}$$

#### Answer

 $\lim_{x\to 0} \frac{\cos 2x - 1}{\cos x - 1}$ 

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0

At x = 0, the value of the given function takes the form 0. Now,

$$\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \to 0} \frac{1 - 2\sin^2 x - 1}{1 - 2\sin^2 \frac{x}{2} - 1} \qquad \left[ \cos x = 1 - 2\sin^2 \frac{x}{2} \right]$$
$$= \lim_{x \to 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} = \lim_{x \to 0} \frac{\left(\frac{\sin^2 x}{x^2}\right) \times x^2}{\left(\frac{x}{2}\right)^2} \right]$$
$$= 4 \frac{\lim_{x \to 0} \left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x \to 0} \left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$
$$= 4 \frac{\left(\frac{\lim_{x \to 0} \frac{\sin x}{x}}{x}\right)^2}{\left(\frac{\lim_{x \to 0} \frac{\sin x}{2}}{\frac{x}{2}}\right)^2} \qquad \left[x \to 0 \Rightarrow \frac{x}{2} \to 0\right]$$
$$= 4 \frac{1^2}{1^2} \qquad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1\right]$$
$$= 4$$

**Question 18:** 

Evaluate the Given limit: 
$$\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x}$$

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Answer

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 $\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x}$ 

0

At x = 0, the value of the given function takes the form 0.

Now,

$$\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x} = \frac{1}{b} \lim_{x \to 0} \frac{x(a + \cos x)}{\sin x}$$
$$= \frac{1}{b} \lim_{x \to 0} \left( \frac{x}{\sin x} \right) \times \lim_{x \to 0} (a + \cos x)$$
$$= \frac{1}{b} \times \frac{1}{\left( \lim_{x \to 0} \frac{\sin x}{x} \right)} \times \lim_{x \to 0} (a + \cos x)$$
$$= \frac{1}{b} \times (a + \cos 0) \qquad \left[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$
$$= \frac{a + 1}{b}$$

**Question 19:** 

Evaluate the Given limit:  $x \to 0$ 

Answer

$$\lim_{x \to 0} x \sec x = \lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$$

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**Question 20:** 

Evaluate the Given limit:  $\frac{\sin ax + bx}{ax + \sin bx} a, b, a + b \neq 0$ 

Answer

At x = 0, the value of the given function takes the form 0. Now,

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$$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin ax}{ax}\right)ax + bx}{ax + bx\left(\frac{\sin bx}{bx}\right)}$$

$$= \frac{\left(\lim_{a \to 0} \frac{\sin ax}{ax}\right) \times \lim_{x \to 0} (ax) + \lim_{x \to 0} bx}{\lim_{x \to 0} ax + \lim_{x \to 0} bx\left(\lim_{b \to 0} \frac{\sin bx}{bx}\right)} \qquad [As \ x \to 0 \Rightarrow ax \to 0 \text{ and } bx \to 0]$$

$$= \frac{\lim_{x \to 0} (ax) + \lim_{x \to 0} bx}{\lim_{x \to 0} ax + \lim_{x \to 0} bx} \qquad \left[\lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{\lim_{x \to 0} (ax + bx)}{\lim_{x \to 0} (ax + bx)}$$

$$= \lim_{x \to 0} (1)$$

**Question 21:** 

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Evaluate the Given limit:  $\lim_{x\to 0} (\operatorname{cosec} x - \cot x)$ 

Answer

At x = 0, the value of the given function takes the form  $\infty - \infty$ . Now,

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$$\lim_{x \to 0} (\operatorname{cosec} x - \operatorname{cot} x)$$

$$= \lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left( \frac{1 - \cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \frac{\left( \frac{1 - \cos x}{x} \right)}{\left( \frac{\sin x}{x} \right)}$$

$$= \frac{\lim_{x \to 0} \frac{1 - \cos x}{x}}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{0}{1} \qquad \left[ \lim_{x \to 0} \frac{1 - \cos x}{x} = 0 \text{ and } \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= 0$$

**Question 22:** 

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

Answer

 $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$ At  $x = \frac{\pi}{2}$ , the value of the given function takes the form  $\frac{0}{0}$ .

Now, put 
$$x - \frac{\pi}{2} = y$$
 so that  $x \to \frac{\pi}{2}, y \to 0$ 

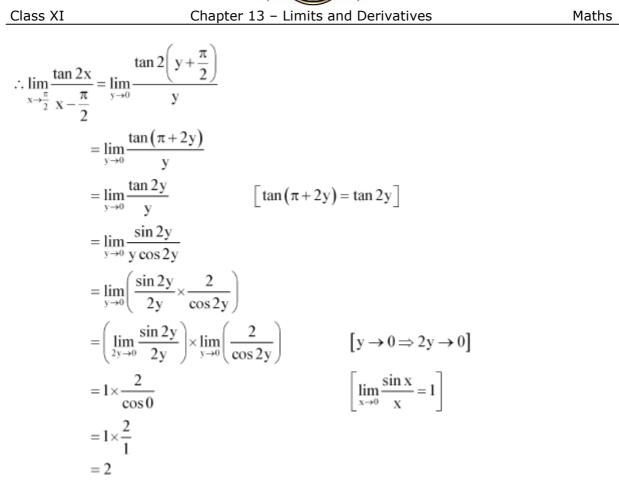
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**Question 23:** 

Find  $\lim_{x \to 0} f(x)$  and  $\lim_{x \to 1} f(x)$ , where  $f(x) = \begin{cases} 2x+3, & x \le 0\\ 3(x+1), & x > 0 \end{cases}$ 

#### Answer

The given function is

$$f(x) = \begin{cases} 2x+3, & x \le 0\\ 3(x+1), & x > 0 \end{cases}$$
$$\lim_{x \to 0^{+}} f(x) = \lim_{x \to 0} [2x+3] = 2(0) + 3 = 3$$
$$\lim_{x \to 0^{+}} f(x) = \lim_{x \to 0} 3(x+1) = 3(0+1) = 3$$
$$\therefore \lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{+}} f(x) = \lim_{x \to 0} f(x) = 3$$

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$$\lim_{x \to 1^{-}} f(x) = \lim_{x \to 1} 3(x+1) = 3(1+1) = 6$$
$$\lim_{x \to 1^{+}} f(x) = \lim_{x \to 1} 3(x+1) = 3(1+1) = 6$$
$$\therefore \lim_{x \to 1^{-}} f(x) = \lim_{x \to 1^{+}} f(x) = \lim_{x \to 1^{+}} f(x) = 6$$

**Question 24:** 

Find  $\lim_{x \to 1} f(x)$ , where  $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ -x^2 - 1, & x > 1 \end{cases}$ Answer

The given function is

$$f(x) = \begin{cases} x^2 - 1, \ x \le 1 \\ -x^2 - 1, \ x > 1 \end{cases}$$

$$\lim_{x \to 1^{-}} f(x) = \lim_{x \to 1} \left[ x^{2} - 1 \right] = 1^{2} - 1 = 1 - 1 = 0$$
$$\lim_{x \to 1^{+}} f(x) = \lim_{x \to 1} \left[ -x^{2} - 1 \right] = -1^{2} - 1 = -1 - 1 = -2$$
It is observed that 
$$\lim_{x \to 1^{-}} f(x) \neq \lim_{x \to 1^{+}} f(x).$$
Hence, 
$$\lim_{x \to 1} f(x)$$
 does not exist.

**Question 25:** 

 $\lim_{x \to 0} f(x), \text{ where } f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0\\ 0, & x = 0 \end{cases}$ Answer
The given function is

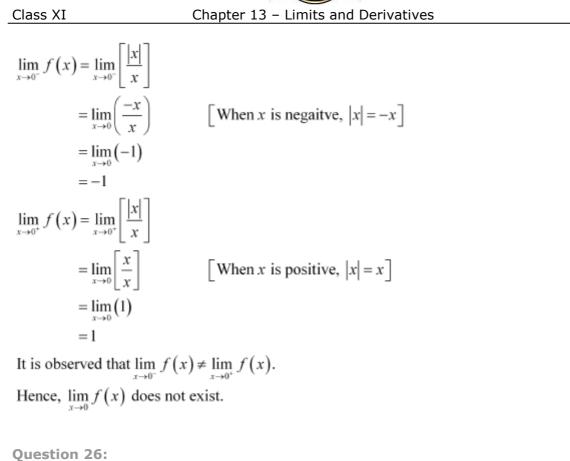
$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0\\ 0, & x = 0 \end{cases}$$

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 $\lim_{x \to 0} f(x), \text{ where } f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0\\ 0, & x = 0 \end{cases}$ 

Answer

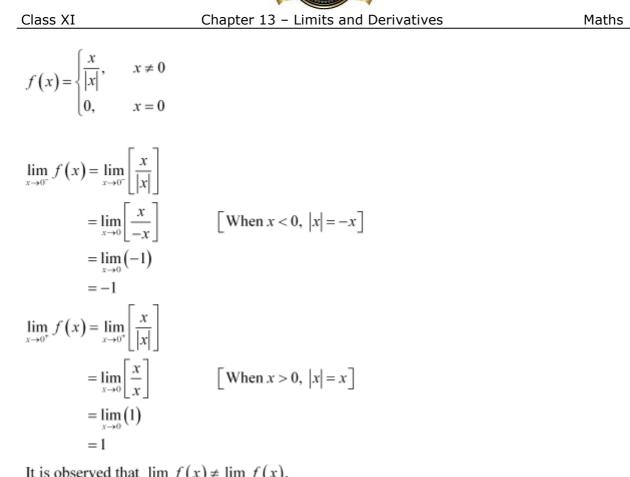
The given function is

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It is observed that  $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$ . Hence,  $\lim_{x\to 0} f(x)$  does not exist.

**Question 27:** 

Find  $\lim_{x\to 5} f(x)$ , where f(x) = |x| - 5Answer

The given function is f(x) = |x| - 5.

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$$\lim_{x \to 5^{-}} f(x) = \lim_{x \to 5^{-}} [|x|-5]$$

$$= \lim_{x \to 5} (x-5) \qquad [When x > 0, |x| = x]$$

$$= 5-5$$

$$= 0$$

$$\lim_{x \to 5^{+}} f(x) = \lim_{x \to 5^{+}} (|x|-5)$$

$$= \lim_{x \to 5} (x-5) \qquad [When x > 0, |x| = x]$$

$$= 5-5$$

$$= 0$$

$$\therefore \lim_{x \to 5^{-}} f(x) = \lim_{x \to 5^{+}} f(x) = 0$$
Hence, 
$$\lim_{x \to 5} f(x) = 0$$

**Question 28:** 

Suppose  $f(x) = \begin{cases} a+bx, \ x < 1 \\ 4, \ x = 1 \\ b-ax \ x > 1 \end{cases}$  and if  $x \to 1 \\ x \to 1 \\ and if x \to 1 \\ f(x) = f(1)$  what are possible values of a and b? Answer

The given function is

$$f(x) = \begin{cases} a+bx, \ x < 1\\ 4, \ x = 1\\ b-ax \ x > 1 \end{cases}$$

$$\lim_{x \to 1^{-}} f(x) = \lim_{x \to 1} (a + bx) = a + b$$
  

$$\lim_{x \to 1^{+}} f(x) = \lim_{x \to 1} (b - ax) = b - a$$
  

$$f(1) = 4$$
  
It is given that  $\lim_{x \to 1} f(x) = f(1)$ .  

$$\therefore \lim_{x \to 1^{-}} f(x) = \lim_{x \to 1^{+}} f(x) = \lim_{x \to 1} f(x) = f(1)$$

$$\Rightarrow a+b=4 \text{ and } b-a=4$$

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On solving these two equations, we obtain a = 0 and b = 4.

Thus, the respective possible values of *a* and *b* are 0 and 4.

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**Question 29:** 

Let  $a_1, a_2, \dots, a_n$  be fixed real numbers and define a function  $f(x) = (x - a_1)(x - a_2)\dots(x - a_n)$ 

 $\lim_{x \to a_1} f(x)$ ? For some  $a \neq a_1, a_2..., a_n$ , compute  $\lim_{x \to a} f(x)$ . Answer

The given function is  $f(x) = (x - a_1)(x - a_2)...(x - a_n)$  $\lim_{x \to a_1} f(x) = \lim_{x \to a_1} \left[ (x - a_1)(x - a_2)...(x - a_n) \right]$   $= \left[ \lim_{x \to a_1} (x - a_1) \right] \left[ \lim_{x \to a_1} (x - a_2) \right] ... \left[ \lim_{x \to a_1} (x - a_n) \right]$   $= (a_1 - a_1)(a_1 - a_2)...(a_1 - a_n) = 0$   $\therefore \lim_{x \to a_1} f(x) = 0$ Now,  $\lim_{x \to a} f(x) = \lim_{x \to a} \left[ (x - a_1)(x - a_2)...(x - a_n) \right]$   $= \left[ \lim_{x \to a} (x - a_1) \right] \left[ \lim_{x \to a} (x - a_2) \right] ... \left[ \lim_{x \to a} (x - a_n) \right]$   $= (a - a_1)(a - a_2)....(a - a_n)$ 

**Question 30:** 

If 
$$f(x) = \begin{cases} |x|+1, & x < 0\\ 0, & x = 0\\ |x|-1, & x > 0 \end{cases}$$
.

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For what value (s) of a does  $\lim_{x\to a} f(x)$  exists? Answer The given function is

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$$f(x) = \begin{cases} |x|+1, & x < 0\\ 0, & x = 0\\ |x|-1, & x > 0 \end{cases}$$

When 
$$a = 0$$
,

$$\lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{-}} (|x|+1)$$
  
=  $\lim_{x \to 0^{-}} (-x+1)$  [If  $x < 0$ ,  $|x| = -x$ ]  
=  $-0+1$   
= 1  
$$\lim_{x \to 0^{+}} f(x) = \lim_{x \to 0^{+}} (|x|-1)$$
  
=  $\lim_{x \to 0} (x-1)$  [If  $x > 0$ ,  $|x| = x$ ]  
=  $0-1$   
=  $-1$ 

Here, it is observed that  $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$ .

 $\therefore \lim_{x \to 0} f(x) \text{ does not exist.}$ 

When a < 0,

$$\lim_{x \to a^{-}} f(x) = \lim_{x \to a^{-}} (|x|+1)$$

$$= \lim_{x \to a} (-x+1) \qquad [x < a < 0 \Rightarrow |x| = -x]$$

$$= -a+1$$

$$\lim_{x \to a^{+}} f(x) = \lim_{x \to a^{+}} (|x|+1)$$

$$= \lim_{x \to a} (-x+1) \qquad [a < x < 0 \Rightarrow |x| = -x]$$

$$= -a+1$$

$$\therefore \lim_{x \to a^{-}} f(x) = \lim_{x \to a^{+}} f(x) = -a+1$$
Thus, limit of  $f(x)$  exists at  $x = a$ , where  $a < 0$ .

When a > 0

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$$\lim_{x \to a^{n}} f(x) = \lim_{x \to a^{n}} (|x|-1) \qquad [0 < x < a \Rightarrow |x| = x] = a - 1$$

$$\lim_{x \to a^{n}} f(x) = \lim_{x \to a^{n}} (|x|-1) \qquad [0 < a < x \Rightarrow |x| = x] = a - 1$$

$$\lim_{x \to a^{n}} f(x) = \lim_{x \to a^{n}} (|x|-1) \qquad [0 < a < x \Rightarrow |x| = x] = a - 1$$
Thus, limit of  $f(x)$  exists at  $x = a$ , where  $a > 0$ .  
Thus, limit of  $f(x)$  exists for all  $a \neq 0$ .  
Question 31:  
If the function  $f(x)$  satisfies  $\frac{\sin \frac{f(x)-2}{x^{2}-1} = \pi}{x^{2}-1}$ , evaluate  $\frac{\lim_{x \to 1} f(x)}{x^{2}-1}$ .  
Answer  

$$\lim_{x \to 1} \frac{\frac{f(x)-2}{x^{2}-1} = \pi}{=x^{2} - \frac{\lim_{x \to 1} (x^{2}-1)}{\lim_{x \to 1} (x^{2}-1)} = \pi}$$

$$\Rightarrow \lim_{x \to 1} (f(x)-2) = \pi \lim_{x \to 1} (x^{2}-1)$$

$$\Rightarrow \lim_{x \to 1} (f(x)-2) = 0$$

 $\Rightarrow \lim_{x \to 1} f(x) - \lim_{x \to 1} 2 = 0$  $\Rightarrow \lim_{x \to 1} f(x) - 2 = 0$ 

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 $\therefore \lim_{x \to 1} f(x) = 2$ 

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## Exercise 13.2

**Question 1:** 

Find the derivative of  $x^2 - 2$  at x = 10.

Answer

Let  $f(x) = x^2 - 2$ . Accordingly,

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h}$$
$$= \lim_{h \to 0} \frac{\left[ (10+h)^2 - 2 \right] - (10^2 - 2)}{h}$$
$$= \lim_{h \to 0} \frac{10^2 + 2.10 \cdot h + h^2 - 2 - 10^2 + 2}{h}$$
$$= \lim_{h \to 0} \frac{20h + h^2}{h}$$
$$= \lim_{h \to 0} (20+h) = (20+0) = 20$$

Thus, the derivative of  $x^2 - 2$  at x = 10 is 20.

**Question 2:** 

Find the derivative of 99x at x = 100.

Answer

Let f(x) = 99x. Accordingly,

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$
$$= \lim_{h \to 0} \frac{99(100+h) - 99(100)}{h}$$
$$= \lim_{h \to 0} \frac{99 \times 100 + 99h - 99 \times 100}{h}$$
$$= \lim_{h \to 0} \frac{99h}{h}$$
$$= \lim_{h \to 0} (99) = 99$$

Thus, the derivative of 99x at x = 100 is 99.

**Question 3:** 

Find the derivative of x at x = 1.

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Answer

Let f(x) = x. Accordingly,

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{(1+h) - 1}{h}$$
$$= \lim_{h \to 0} \frac{h}{h}$$
$$= \lim_{h \to 0} (1)$$
$$= 1$$

Thus, the derivative of x at x = 1 is 1.

**Question 4:** 

Find the derivative of the following functions from first principle.

(i) 
$$x^3 - 27$$
 (ii)  $(x - 1) (x - 2)$   
(ii)  $\frac{1}{x^2} \frac{x+1}{(iv)} \frac{x+1}{x-1}$ 

Answer

(i) Let  $f(x) = x^3 - 27$ . Accordingly, from the first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\left[ (x+h)^3 - 27 \right] - (x^3 - 27) \right]}{h}$$
$$= \lim_{h \to 0} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - x^3}{h}$$
$$= \lim_{h \to 0} \frac{h^3 + 3x^2h + 3xh^2}{h}$$
$$= \lim_{h \to 0} (h^2 + 3x^2 + 3xh)$$
$$= 0 + 3x^2 + 0 = 3x^2$$

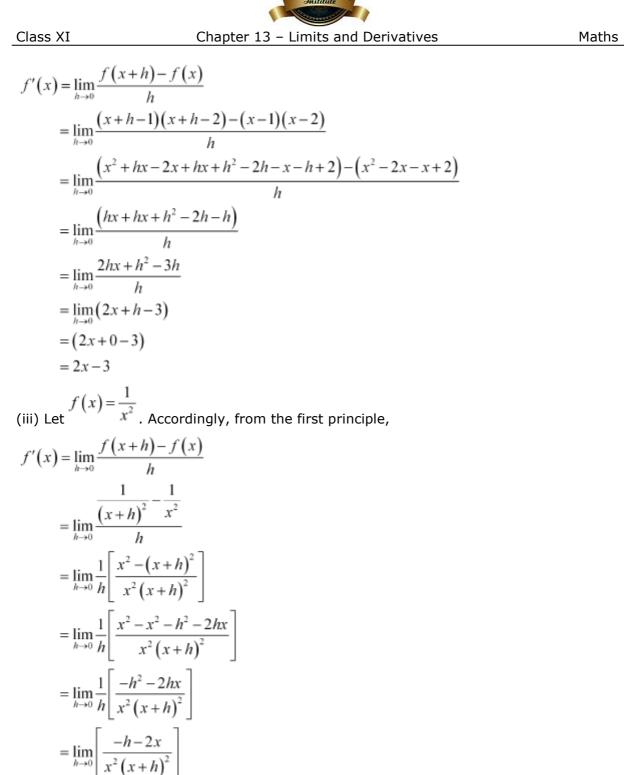
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(ii) Let f(x) = (x - 1) (x - 2). Accordingly, from the first principle,

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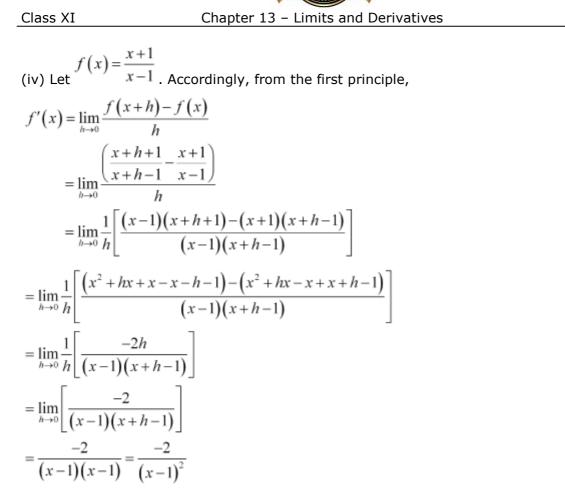
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 $=\frac{0-2x}{x^2(x+0)^2}=\frac{-2}{x^3}$ 

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Question 5:

For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

Prove that 
$$f'(1) = 100f'(0)$$

Answer

The given function is

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$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \left[ \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1 \right]$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \left[ \frac{x^{100}}{100} \right] + \frac{d}{dx} \left( \frac{x^{99}}{99} \right) + \dots + \frac{d}{dx} \left( \frac{x^2}{2} \right) + \frac{d}{dx} (x) + \frac{d}{dx} (1)$$
On using theorem  $\frac{d}{dx} (x^n) = nx^{n-1}$ , we obtain
$$\frac{d}{dx} f(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \dots + \frac{2x}{2} + 1 + 0$$

$$= x^{99} + x^{98} + \dots + x + 1$$
At  $x = 0$ ,
$$f'(0) = 1$$
At  $x = 1$ ,
$$f'(1) = 1^{99} + 1^{98} + \dots + 1 + 1 = [1 + 1 + \dots + 1 + 1]_{100 \text{ terms}} = 1 \times 100 = 100$$
Thus,  $f'(1) = 100 \times f^1(0)$ 

**Question 6:** 

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Find the derivative of  $x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$  for some fixed real number *a*. Answer

Let 
$$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$$
  
 $\therefore f'(x) = \frac{d}{dx}(x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n)$   
 $= \frac{d}{dx}(x^n) + a\frac{d}{dx}(x^{n-1}) + a^2\frac{d}{dx}(x^{n-2}) + ... + a^{n-1}\frac{d}{dx}(x) + a^n\frac{d}{dx}(1)$   
On using theorem  $\frac{d}{dx}x^n = nx^{n-1}$ , we obtain  
 $f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1} + a^n(0)$   
 $= nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1}$ 

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Chapter 13 – Limits and Derivatives

**Question 7:** 

For some constants *a* and *b*, find the derivative of

(i) 
$$(x - a) (x - b)$$
 (ii)  $(ax^2 + b)^2$  (iii)  $\frac{x-a}{x-b}$   
Answer  
(i) Let  $f(x) = (x - a) (x - b)$   
 $\Rightarrow f(x) = x^2 - (a+b)x + ab$   
 $\therefore f'(x) = \frac{d}{dx}(x^2 - (a+b)x + ab)$   
 $= \frac{d}{dx}(x^2) - (a+b)\frac{d}{dx}(x) + \frac{d}{dx}(ab)$   
On using theorem  $\frac{d}{dx}(x^n) = nx^{n-1}$ , we obtain  
 $f'(x) = 2x - (a+b) + 0 = 2x - a - b$   
(ii) Let  $f(x) = (ax^2 + b)^2$   
 $\Rightarrow f(x) = a^2x^4 + 2abx^2 + b^2$   
 $\therefore f'(x) = \frac{d}{dx}(a^2x^4 + 2abx^2 + b^2) = a^2\frac{d}{dx}(x^4) + 2ab\frac{d}{dx}(x^2) + \frac{d}{dx}(b^2)$   
On using theorem  $\frac{d}{dx}x^n = nx^{n-1}$ , we obtain  
 $f'(x) = a^2(4x^3) + 2ab(2x) + b^2(0)$   
 $= 4a^2x^3 + 4abx$   
 $= 4ax(ax^2 + b)$   
Let  $f(x) = \frac{(x-a)}{(x-b)}$   
(iii)

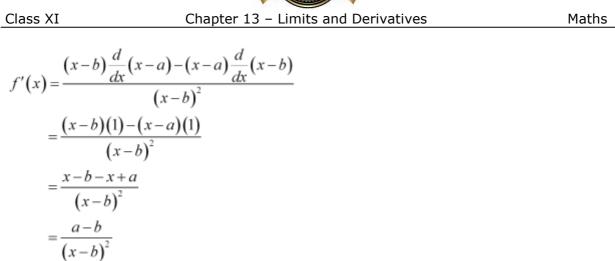
By quotient rule,

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**Question 8:** 

$$\frac{x^n-a^n}{a}$$

Find the derivative of x-a for some constant *a*. Answer

Let 
$$f(x) = \frac{x^n - a^n}{x - a}$$
  
 $\Rightarrow f'(x) = \frac{d}{dx} \left( \frac{x^n - a^n}{x - a} \right)$ 

By quotient rule,

$$f'(x) = \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$
$$= \frac{(x-a)(nx^{n-1} - 0) - (x^n - a^n)}{(x-a)^2}$$
$$= \frac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$$

**Question 9:** 

Find the derivative of

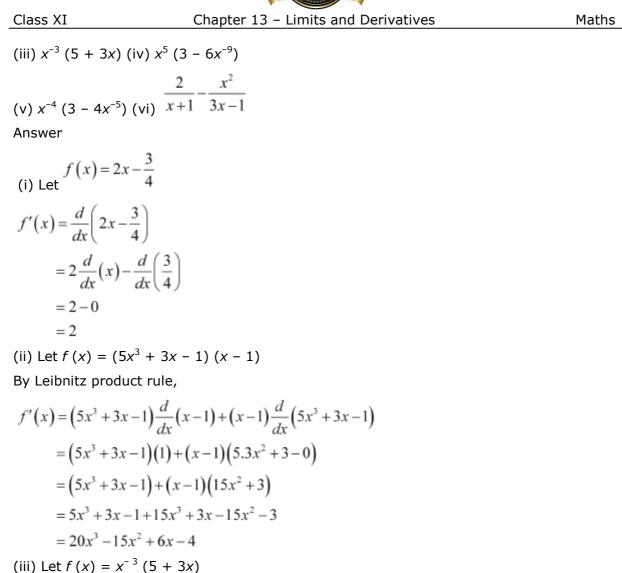
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(i) 
$$2x - \frac{3}{4}$$
 (ii)  $(5x^3 + 3x - 1)(x - 1)$ 

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By Leibnitz product rule,

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Chapter 13 – Limits and Derivatives  $f'(x) = x^{-3} \frac{d}{dx} (5+3x) + (5+3x) \frac{d}{dx} (x^{-3})$  $= x^{-3} (0+3) + (5+3x) (-3x^{-3-1})$  $= x^{-3} (3) + (5+3x) (-3x^{-4})$ 

$$= x^{-3} (3) + (5+3x)(-3x)$$
  
=  $3x^{-3} - 15x^{-4} - 9x^{-3}$   
=  $-6x^{-3} - 15x^{-4}$   
=  $-3x^{-3} \left(2 + \frac{5}{x}\right)$   
=  $\frac{-3x^{-3}}{x} (2x+5)$   
=  $\frac{-3}{x^4} (5+2x)$ 

(iv) Let  $f(x) = x^5 (3 - 6x^{-9})$ 

By Leibnitz product rule,

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$$f'(x) = x^{5} \frac{d}{dx} (3 - 6x^{-9}) + (3 - 6x^{-9}) \frac{d}{dx} (x^{5})$$
  
=  $x^{5} \{0 - 6(-9)x^{-9-1}\} + (3 - 6x^{-9})(5x^{4})$   
=  $x^{5} (54x^{-10}) + 15x^{4} - 30x^{-5}$   
=  $54x^{-5} + 15x^{4} - 30x^{-5}$   
=  $24x^{-5} + 15x^{4}$   
=  $15x^{4} + \frac{24}{x^{5}}$ 

(v) Let  $f(x) = x^{-4} (3 - 4x^{-5})$ 

By Leibnitz product rule,

$$f'(x) = x^{-4} \frac{d}{dx} (3 - 4x^{-5}) + (3 - 4x^{-5}) \frac{d}{dx} (x^{-4})$$
  
=  $x^{-4} \{0 - 4(-5)x^{-5-1}\} + (3 - 4x^{-5})(-4)x^{-4-1}$   
=  $x^{-4} (20x^{-6}) + (3 - 4x^{-5})(-4x^{-5})$   
=  $20x^{-10} - 12x^{-5} + 16x^{-10}$   
=  $36x^{-10} - 12x^{-5}$   
=  $-\frac{12}{x^5} + \frac{36}{x^{10}}$ 

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(vi) Let 
$$f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$$
  
$$f'(x) = \frac{d}{dx} \left(\frac{2}{x+1}\right) - \frac{d}{dx} \left(\frac{x^2}{3x-1}\right)$$

By quotient rule,

$$f'(x) = \left[\frac{(x+1)\frac{d}{dx}(2) - 2\frac{d}{dx}(x+1)}{(x+1)^2}\right] - \left[\frac{(3x-1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}\right]$$
$$= \left[\frac{(x+1)(0) - 2(1)}{(x+1)^2}\right] - \left[\frac{(3x-1)(2x) - (x^2)(3)}{(3x-1)^2}\right]$$
$$= \frac{-2}{(x+1)^2} - \left[\frac{6x^2 - 2x - 3x^2}{(3x-1)^2}\right]$$
$$= \frac{-2}{(x+1)^2} - \left[\frac{3x^2 - 2x^2}{(3x-1)^2}\right]$$
$$= \frac{-2}{(x+1)^2} - \left[\frac{3x^2 - 2x^2}{(3x-1)^2}\right]$$

**Question 10:** 

Find the derivative of  $\cos x$  from first principle.

## Answer

Let  $f(x) = \cos x$ . Accordingly, from the first principle,

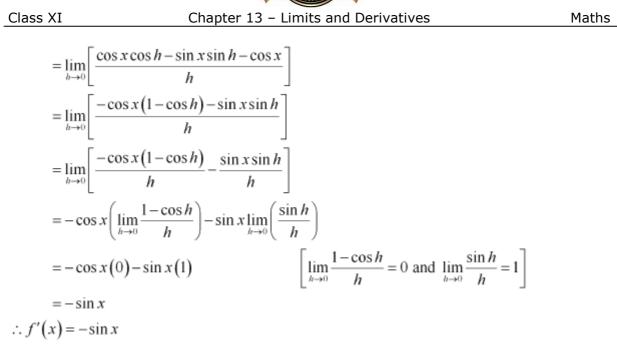
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

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**Question 11:** 

Find the derivative of the following functions:

(i)  $\sin x \cos x$  (ii)  $\sec x$  (iii)  $5 \sec x + 4 \cos x$ 

(iv)  $\operatorname{cosec} x$  (v)  $\operatorname{3cot} x$  +  $\operatorname{5cosec} x$ 

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(vi) 5sin x - 6cos x + 7 (vii) 2tan x - 7sec x

#### Answer

(i) Let  $f(x) = \sin x \cos x$ . Accordingly, from the first principle,

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
  
=  $\lim_{h \to 0} \frac{\sin(x+h)\cos(x+h) - \sin x \cos x}{h}$   
=  $\lim_{h \to 0} \frac{1}{2h} \Big[ 2\sin(x+h)\cos(x+h) - 2\sin x \cos x \Big]$   
=  $\lim_{h \to 0} \frac{1}{2h} \Big[ \sin 2(x+h) - \sin 2x \Big]$   
=  $\lim_{h \to 0} \frac{1}{2h} \Big[ 2\cos\frac{2x+2h+2x}{2} \cdot \sin\frac{2x+2h-2x}{2} \Big]$   
=  $\lim_{h \to 0} \frac{1}{h} \Big[ \cos\frac{4x+2h}{2}\sin\frac{2h}{2} \Big]$   
=  $\lim_{h \to 0} \frac{1}{h} \Big[ \cos(2x+h)\sin h \Big]$   
=  $\lim_{h \to 0} \cos(2x+h) \cdot \lim_{h \to 0} \frac{\sin h}{h}$   
=  $\cos(2x+0) \cdot 1$   
=  $\cos 2x$ 

(ii) Let  $f(x) = \sec x$ . Accordingly, from the first principle,

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{1}{\cos(x+h)} - \frac{1}{\cos x} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\cos x - \cos(x+h)}{\cos x \cos(x+h)} \right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h \to 0} \frac{1}{h} \left[ \frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos(x+h)} \right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h \to 0} \frac{1}{h} \left[ \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos(x+h)} \right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h \to 0} \frac{\left[ \sin\left(\frac{2x+h}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right]}{\cos(x+h)}$$

$$= \frac{1}{\cos x} \cdot \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \cdot \lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)}$$

$$= \frac{1}{\cos x} \cdot 1 \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \tan x$$

(iii) Let  $f(x) = 5 \sec x + 4 \cos x$ . Accordingly, from the first principle,

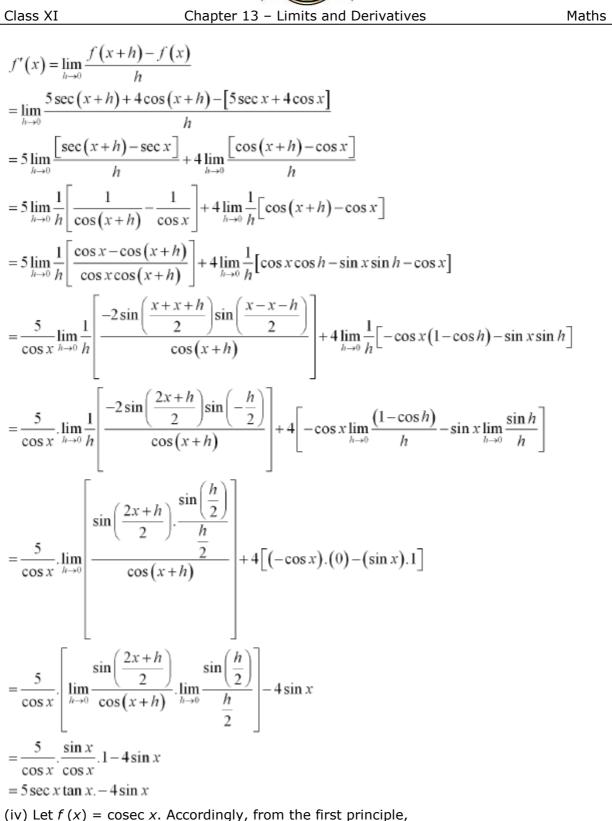
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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{1}{h} \Big[ \operatorname{cosec}(x+h) - \operatorname{cosec} x \Big]$$

$$= \lim_{h \to 0} \frac{1}{h} \Big[ \frac{1}{\sin(x+h)} - \frac{1}{\sin x} \Big]$$

$$= \lim_{h \to 0} \frac{1}{h} \Big[ \frac{\sin x - \sin(x+h)}{\sin(x+h)\sin x} \Big]$$

$$= \lim_{h \to 0} \frac{1}{h} \Big[ \frac{2 \cos\left(\frac{x+x+h}{2}\right) \cdot \sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)\sin x} \Big]$$

$$= \lim_{h \to 0} \frac{1}{h} \Big[ \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(-\frac{h}{2}\right)}{\sin(x+h)\sin x} \Big]$$

$$= \lim_{h \to 0} \frac{-\cos\left(\frac{2x+h}{2}\right) \cdot \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}}{\sin(x+h)\sin x}$$

$$= \lim_{h \to 0} \left( \frac{-\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x} \right) \cdot \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= \left( \frac{-\cos x}{\sin x \sin x} \right) \cdot 1$$

$$= -\operatorname{cosecx \ cot \ x}$$

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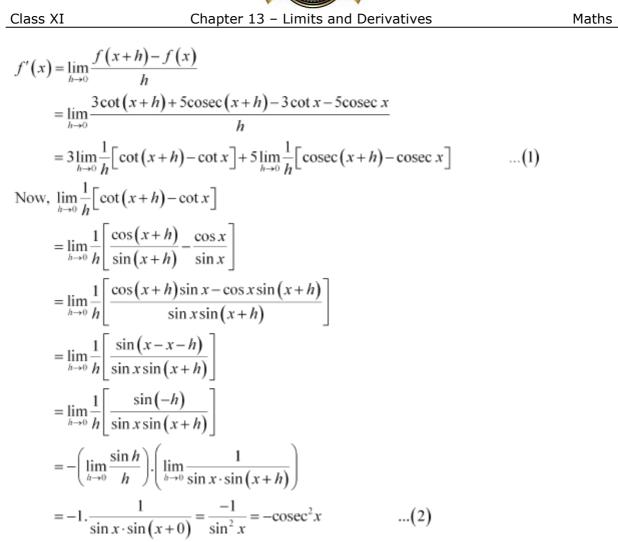
(v) Let  $f(x) = 3\cot x + 5\operatorname{cosec} x$ . Accordingly, from the first principle,

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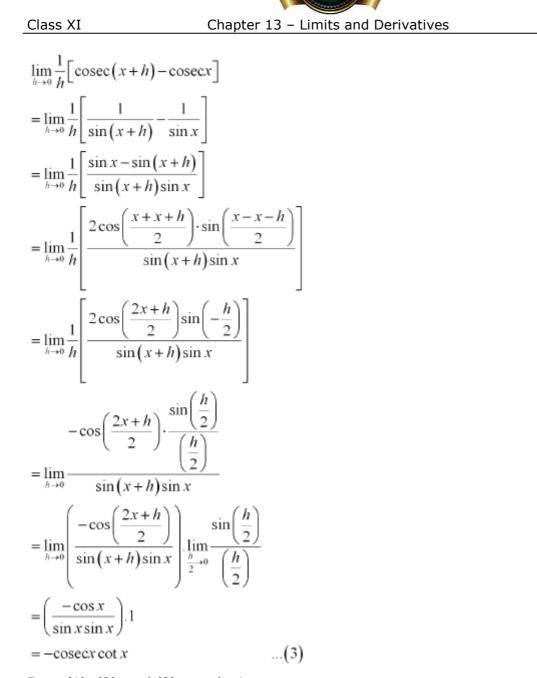


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From (1), (2), and (3), we obtain

 $f'(x) = -3\csc^2 x - 5\csc x \cot x$ 

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(vi) Let  $f(x) = 5\sin x - 6\cos x + 7$ . Accordingly, from the first principle,

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$$\begin{aligned} \int f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{h}{h} \Big[ 5\sin(x+h) - 6\cos(x+h) + 7 - 5\sin x + 6\cos x - 7 \Big] \\ &= \lim_{h \to 0} \frac{h}{h} \Big[ 5\sin(x+h) - 6\cos(x+h) + 7 - 5\sin x + 6\cos x - 7 \Big] \\ &= \lim_{h \to 0} \frac{h}{h} \Big[ 5\left\{ \sin(x+h) - \sin x \right\} - 6\left\{ \cos(x+h) - \cos x \right\} \Big] \\ &= 5\lim_{h \to 0} \frac{h}{h} \Big[ 2\cos\left(\frac{x+h+x}{2}\right) \sin\left(\frac{x+h-x}{2}\right) \Big] - 6\lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\ &= 5\lim_{h \to 0} \frac{h}{h} \Big[ 2\cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{x}{2}\right) - 6\lim_{h \to 0} \Big[ \frac{-\cos x(1-\cos h) - \sin x \sin h}{h} \Big] \\ &= 5\lim_{h \to 0} \left( \cos\left(\frac{2x+h}{2}\right) \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right) - 6\lim_{h \to 0} \Big[ \frac{-\cos x(1-\cos h) - \sin x \sin h}{h} \Big] \\ &= 5 \Big[ \lim_{h \to 0} \cos\left(\frac{2x+h}{2}\right) \Big] \Bigg[ \lim_{h \to 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \Bigg] - 6 \Big[ (-\cos x) \Big( \lim_{h \to 0} \frac{1-\cos h}{h} \Big) - \sin x \lim_{h \to 0} \left( \frac{\sin h}{h} \right) \Big] \\ &= 5 \cos x \cdot 1 - 6 \Big[ (-\cos x) \cdot (0) - \sin x \cdot 1 \Big] \\ &= 5 \cos x + 6 \sin x \end{aligned}$$

(vii) Let  $f(x) = 2 \tan x - 7 \sec x$ . Accordingly, from the first principle,

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Chapter 13 – Limits and Derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
  
=  $\lim_{h \to 0} \frac{1}{h} \Big[ 2 \tan(x+h) - 7 \sec(x+h) - 2 \tan x + 7 \sec x \Big]$   
=  $\lim_{h \to 0} \frac{1}{h} \Big[ 2 \{ \tan(x+h) - \tan x \} - 7 \{ \sec(x+h) - \sec x \} \Big]$   
=  $2 \lim_{h \to 0} \frac{1}{h} \Big[ \tan(x+h) - \tan x \Big] - 7 [\lim_{h \to 0} \frac{1}{h} \Big[ \sec(x+h) - \sec x \Big]$   
=  $2 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \Big] - 7 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{1}{\cos(x+h)} - \frac{1}{\cos x} \Big]$   
=  $2 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos x\cos(x+h)} \Big] - 7 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{\cos x - \cos(x+h)}{\cos x\cos(x+h)} \Big]$   
=  $2 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{\sin(x+h-x)}{\cos x\cos(x+h)} \Big] - 7 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos x\cos(x+h)} \Big]$   
=  $2 \lim_{h \to 0} \Big[ \Big( \frac{\sin h}{h} \Big) \frac{1}{\cos x\cos(x+h)} \Big] - 7 \lim_{h \to 0} \frac{1}{h} \Big[ \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos x\cos(x+h)} \Big]$   
=  $2 \Big[ \lim_{h \to 0} \frac{\sin h}{h} \Big] \Big( \lim_{h \to 0} \frac{1}{\cos x\cos(x+h)} \Big] - 7 \Big[ \lim_{h \to 0} \frac{1}{h} \Big[ \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos x\cos(x+h)} \Big]$   
=  $2 \Big[ (\lim_{h \to 0} \frac{\sin h}{h} \Big] \Big( \lim_{h \to 0} \frac{1}{\cos x\cos(x+h)} \Big) - 7 \Big[ \lim_{h \to 0} \frac{\sin h}{2} \Big] \Big[ \lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos x\cos(x+h)} \Big]$   
=  $2 (\lim_{h \to 0} \frac{1}{h} \Big[ \lim_{h \to 0} \frac{1}{\cos x\cos(x+h)} \Big] - 7 \Big[ \lim_{h \to 0} \frac{\sin h}{2} \Big] \Big[ \lim_{h \to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos x\cos(x+h)} \Big]$ 

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## Chapter 13 – Limits and Derivatives

## **NCERT Miscellaneous Solutions**

**Question 1:** 

Find the derivative of the following functions from first principle:

(i) -x (ii)  $(-x)^{-1}$  (iii) sin (x + 1) $(\pi)$ 

(iv) 
$$\cos\left(x-\frac{\pi}{8}\right)$$

Answer

(i) Let 
$$f(x) = -x$$
. Accordingly,  $f(x+h) = -(x+h)$ 

By first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
  
=  $\lim_{h \to 0} \frac{-(x+h) - (-x)}{h}$   
=  $\lim_{h \to 0} \frac{-x - h + x}{h}$   
=  $\lim_{h \to 0} \frac{-h}{h}$   
=  $\lim_{h \to 0} (-1) = -1$   
(ii) Let  $f(x) = (-x)^{-1} = \frac{1}{-x} = \frac{-1}{x}$ . Accordingly,  $f(x+h) = \frac{-1}{(x+h)}$ 

(ii) Let  $(-x) = (-x)^{-1}$ 

By first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{-1}{x+h} - \left( \frac{-1}{x} \right) \right]$$
$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{-1}{x+h} + \frac{1}{x} \right]$$
$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{-x + (x+h)}{x(x+h)} \right]$$
$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{-x + x + h}{x(x+h)} \right]$$

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$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{h}{x(x+h)} \right]$$
$$= \lim_{h \to 0} \frac{1}{x(x+h)}$$
$$= \frac{1}{x \cdot x} = \frac{1}{x^{2}}$$

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(iii) Let  $f(x) = \sin (x + 1)$ . Accordingly,  $f(x+h) = \sin(x+h+1)$ By first principle,

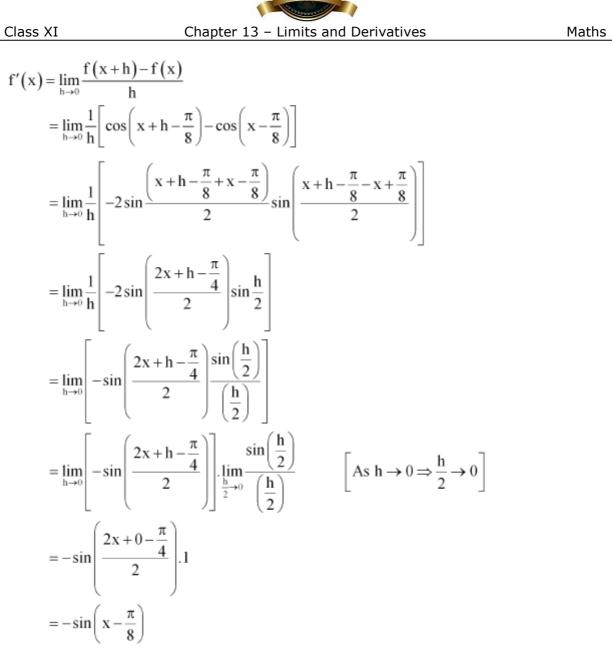
$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{1}{h} \Big[ \sin(x+h+1) - \sin(x+1) \Big] \\ &= \lim_{h \to 0} \frac{1}{h} \Big[ 2\cos\left(\frac{x+h+1+x+1}{2}\right) \sin\left(\frac{x+h+1-x-1}{2}\right) \Big] \\ &= \lim_{h \to 0} \frac{1}{h} \Big[ 2\cos\left(\frac{2x+h+2}{2}\right) \sin\left(\frac{h}{2}\right) \Big] \\ &= \lim_{h \to 0} \left[ \cos\left(\frac{2x+h+2}{2}\right) \cdot \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right] \\ &= \lim_{h \to 0} \cos\left(\frac{2x+h+2}{2}\right) \cdot \frac{1}{h} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \quad \left[ As \ h \to 0 \Rightarrow \frac{h}{2} \to 0 \right] \\ &= \cos\left(\frac{2x+0+2}{2}\right) \cdot 1 \qquad \left[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \right] \\ &= \cos(x+1) \\ (iv) \ Let \qquad f(x) = \cos\left(x - \frac{\pi}{8}\right) \text{. Accordingly, } f(x+h) = \cos\left(x+h - \frac{\pi}{8}\right) \\ By \ first principle, \end{aligned}$$

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**Question 2:** 

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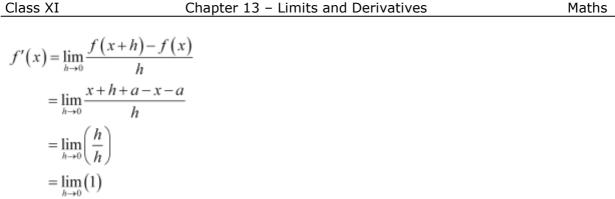
Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, rand s are fixed non-zero constants and m and n are integers): (x + a) Answer

Let 
$$f(x) = x + a$$
. Accordingly,  $f(x+h) = x+h+a$   
By first principle,

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#### **Question 3:**

= 1

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

 $(px+q)\left(\frac{r}{x}+s\right)$ 

and s are fixed non-zero constants and m and n are integers): Answer

Let 
$$f(x) = (px+q)\left(\frac{r}{x}+s\right)$$

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By Leibnitz product rule,

$$f'(x) = (px+q)\left(\frac{r}{x}+s\right)' + \left(\frac{r}{x}+s\right)(px+q)'$$
$$= (px+q)(rx^{-1}+s)' + \left(\frac{r}{x}+s\right)(p)$$
$$= (px+q)(-rx^{-2}) + \left(\frac{r}{x}+s\right)p$$
$$= (px+q)\left(\frac{-r}{x^2}\right) + \left(\frac{r}{x}+s\right)p$$
$$= \frac{-pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + ps$$
$$= ps - \frac{qr}{x^2}$$

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# **Question 4:**

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Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(ax + b) (cx + d)^2$ Answer

Let 
$$f(x) = (ax+b)(cx+d)^2$$

By Leibnitz product rule,

$$f'(x) = (ax+b)\frac{d}{dx}(cx+d)^{2} + (cx+d)^{2}\frac{d}{dx}(ax+b)$$
  
=  $(ax+b)\frac{d}{dx}(c^{2}x^{2} + 2cdx + d^{2}) + (cx+d)^{2}\frac{d}{dx}(ax+b)$   
=  $(ax+b)\left[\frac{d}{dx}(c^{2}x^{2}) + \frac{d}{dx}(2cdx) + \frac{d}{dx}d^{2}\right] + (cx+d)^{2}\left[\frac{d}{dx}ax + \frac{d}{dx}b\right]$   
=  $(ax+b)(2c^{2}x+2cd) + (cx+d^{2})a$   
=  $2c(ax+b)(cx+d) + a(cx+d)^{2}$ 

**Question 5:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

ax+b

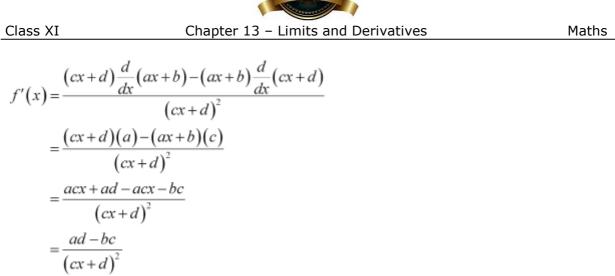
and *s* are fixed non-zero constants and *m* and *n* are integers): cx + dAnswer

$$f(x) = \frac{ax+b}{cx+d}$$

By quotient rule,

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**Question 6:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Answer

Let 
$$f(x) = \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$$
, where  $x \neq 0$ 

By quotient rule,

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$$f'(x) = \frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}, x \neq 0, 1$$
$$= \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}, x \neq 0, 1$$
$$= \frac{x-1-x-1}{(x-1)^2}, x \neq 0, 1$$
$$= \frac{-2}{(x-1)^2}, x \neq 0, 1$$

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Maths

1

### **Question 7:**

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\overline{ax^2 + bx + c}$ Answer

$$f(x) = \frac{1}{ax^2 + bx + c}$$

By quotient rule,

$$f'(x) = \frac{\left(ax^2 + bx + c\right)\frac{d}{dx}(1) - \frac{d}{dx}\left(ax^2 + bx + c\right)}{\left(ax^2 + bx + c\right)^2}$$
$$= \frac{\left(ax^2 + bx + c\right)(0) - \left(2ax + b\right)}{\left(ax^2 + bx + c\right)^2}$$
$$= \frac{-\left(2ax + b\right)}{\left(ax^2 + bx + c\right)^2}$$

**Question 8:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\frac{ax+b}{px^2+qx+r}$ 

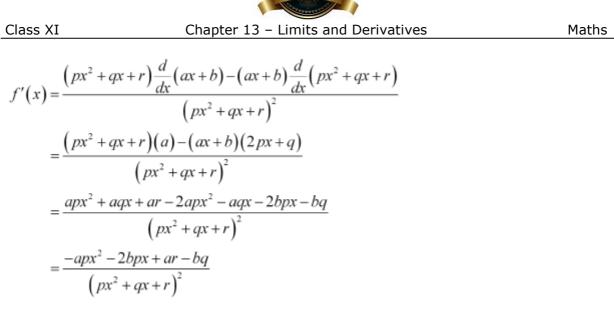
Answer

Let 
$$f(x) = \frac{ax+b}{px^2+qx+r}$$

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By quotient rule,





**Question 9:** 

Answer

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

 $\frac{px^2 + qx + r}{ax + b}$ and *s* are fixed non-zero constants and *m* and *n* are integers):

$$\operatorname{Let} f(x) = \frac{px^2 + qx + r}{ax + b}$$

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By quotient rule,

$$f'(x) = \frac{(ax+b)\frac{d}{dx}(px^2+qx+r) - (px^2+qx+r)\frac{d}{dx}(ax+b)}{(ax+b)^2}$$
$$= \frac{(ax+b)(2px+q) - (px^2+qx+r)(a)}{(ax+b)^2}$$
$$= \frac{2apx^2 + aqx + 2bpx + bq - apx^2 - aqx - ar}{(ax+b)^2}$$
$$= \frac{apx^2 + 2bpx + bq - ar}{(ax+b)^2}$$

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Maths

**Question 10:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$ Answer

Let 
$$f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$$
  
 $f'(x) = \frac{d}{dx} \left(\frac{a}{x^4}\right) - \frac{d}{dx} \left(\frac{b}{x^2}\right) + \frac{d}{dx} (\cos x)$   
 $= a \frac{d}{dx} (x^{-4}) - b \frac{d}{dx} (x^{-2}) + \frac{d}{dx} (\cos x)$   
 $= a (-4x^{-5}) - b (-2x^{-3}) + (-\sin x) \qquad \left[\frac{d}{dx} (x^n) = nx^{n-1} \text{and } \frac{d}{dx} (\cos x) = -\sin x\right]$   
 $= \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$ 

**Question 11:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

and *s* are fixed non-zero constants and *m* and *n* are integers):  $4\sqrt{x}-2$ 

Answer

Let 
$$f(x) = 4\sqrt{x} - 2$$
  
 $f'(x) = \frac{d}{dx} (4\sqrt{x} - 2) = \frac{d}{dx} (4\sqrt{x}) - \frac{d}{dx} (2)$   
 $= 4\frac{d}{dx} (x^{\frac{1}{2}}) - 0 = 4 (\frac{1}{2}x^{\frac{1}{2}-1})$   
 $= (2x^{-\frac{1}{2}}) = \frac{2}{\sqrt{x}}$ 

**Question 12:** 

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Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(ax + b)^n$ Answer

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Let 
$$f(x) = (ax+b)^n$$
. Accordingly,  $f(x+h) = \{a(x+h)+b\}^n = (ax+ah+b)^n$ 

By first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
  
=  $\lim_{h \to 0} \frac{(ax+ah+b)^n - (ax+b)^n}{h}$   
=  $\lim_{h \to 0} \frac{(ax+b)^n \left(1 + \frac{ah}{ax+b}\right)^n - (ax+b)^n}{h}$   
=  $(ax+b)^n \lim_{h \to 0} \frac{\left(1 + \frac{ah}{ax+b}\right)^n - 1}{h}$   
=  $(ax+b)^n \lim_{h \to 0} \frac{1}{n} \left[ \left\{ 1 + n \left(\frac{ah}{ax+b}\right) + \frac{n(n-1)}{\underline{12}} \left(\frac{ah}{ax+b}\right)^2 + \ldots \right\} - 1 \right]$   
(Using binomial theorem)

$$= (ax+b)^{n} \lim_{b \to 0} \frac{1}{h} \left[ n \left( \frac{ah}{ax+b} \right) + \frac{n(n-1)a^{2}h^{2}}{\left[ \frac{1}{2}(ax+b)^{2} + \dots \right]} + \dots \right]$$

$$= (ax+b)^{n} \lim_{b \to 0} \left[ \frac{na}{(ax+b)} + \frac{n(n-1)a^{2}h}{\left[ \frac{1}{2}(ax+b)^{2} + \dots \right]} \right]$$

$$= (ax+b)^{n} \left[ \frac{na}{(ax+b)} + 0 \right]$$

$$= na \frac{(ax+b)^{n}}{(ax+b)}$$

$$= na(ax+b)^{n-1}$$

**Question 13:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(ax + b)^n (cx + d)^m$ Answer

Let 
$$f(x) = (ax+b)^n (cx+d)^m$$

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By Leibnitz product rule,

$$f'(x) = (ax + b)^a \frac{d}{dx} (cx + d)^m + (cx + d)^m \frac{d}{dx} (ax + b)^a \qquad \dots(1)$$
Now, let  $f_1(x) = (cx + d)^m$ 

$$f_1(x + h) = (cx + ch + d)^m$$

$$f_1'(x) = \lim_{b \to 0} \frac{f_1(x + h) - f_1(x)}{h}$$

$$= \lim_{b \to 0} \frac{(cx + ch + d)^m - (cx + d)^m}{h}$$

$$= (cx + d)^m \lim_{b \to 0} \frac{1}{h} \left[ \left( 1 + \frac{ch}{cx + d} \right)^m - 1 \right]$$

$$= (cx + d)^m \lim_{b \to 0} \frac{1}{h} \left[ \left( 1 + \frac{mch}{(cx + d)} + \frac{m(m - 1)}{2} \frac{(c^2 h^2)}{(cx + d)^2} + \dots \right) - 1 \right]$$

$$= (cx + d)^m \lim_{b \to 0} \frac{1}{h} \left[ \frac{mch}{(cx + d)} + \frac{m(m - 1)c^2 h^2}{2(cx + d)^2} + \dots \right]$$

$$= (cx + d)^m \lim_{b \to 0} \left[ \frac{mc}{(cx + d)} + \frac{m(m - 1)c^2 h}{2(cx + d)^2} + \dots \right]$$

$$= (cx + d)^m \left[ \frac{mc}{(cx + d)} + \frac{m(m - 1)c^2 h}{2(cx + d)^2} + \dots \right]$$

$$= (cx + d)^m \left[ \frac{mc}{(cx + d)} + \frac{m(m - 1)c^2 h}{2(cx + d)^2} + \dots \right]$$

$$= (cx + d)^m \left[ \frac{mc}{(cx + d)} + \frac{m(m - 1)c^2 h}{2(cx + d)^2} + \dots \right]$$

$$= mc(cx + d)^{m-1}$$

$$= mc(cx + d)^{m-1}$$

$$= mc(cx + d)^{m-1}$$

$$= mc(cx + d)^{m-1}$$

$$(2)$$
Similarly,  $\frac{d}{dx} (ax + b)^a = na(ax + b)^{n-1}$ 

$$(3)$$

Therefore, from (1), (2), and (3), we obtain

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$$f'(x) = (ax+b)^{n} \{ mc(cx+d)^{m-1} \} + (cx+d)^{m} \{ na(ax+b)^{n-1} \}$$
$$= (ax+b)^{n-1} (cx+d)^{m-1} [mc(ax+b) + na(cx+d)]$$

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### **Question 14:**

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): sin (x + a)Answer

Let 
$$f(x) = \sin(x+a)$$
  
 $f(x+h) = \sin(x+h+a)$ 

By first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h+a) - \sin(x+a)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ 2\cos\left(\frac{x+h+a+x+a}{2}\right) \sin\left(\frac{x+h+a-x-a}{2}\right) \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ 2\cos\left(\frac{2x+2a+h}{2}\right) \sin\left(\frac{h}{2}\right) \right]$$

$$= \lim_{h \to 0} \left[ \cos\left(\frac{2x+2a+h}{2}\right) \left\{ \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right\} \right]$$

$$= \lim_{h \to 0} \cos\left(\frac{2x+2a+h}{2}\right) \lim_{\frac{h}{2} \to 0} \left\{ \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right\}$$

$$= \cos\left(\frac{2x+2a}{2}\right) \times 1$$

$$= \cos\left(x+a\right)$$

$$\left[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

**Question 15:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, rand s are fixed non-zero constants and m and n are integers): cosec x cot xAnswer

Let  $f(x) = \csc x \cot x$ 

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By Leibnitz product rule,

$$f'(x) = \operatorname{cosec} x (\operatorname{cot} x)' + \operatorname{cot} x (\operatorname{cosec} x)' \qquad \dots (1)$$

Let 
$$f_1(x) = \cot x$$
. Accordingly,  $f_1(x+h) = \cot(x+h)$ 

By first principle,

$$f_{1}'(x) = \lim_{h \to 0} \frac{f_{1}(x+h) - f_{1}(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cot(x+h) - \cot x}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left( \frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sin x \cos(x+h) - \cos x \sin(x+h)}{\sin x \sin(x+h)} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sin(x-x-h)}{\sin x \sin(x+h)} \right]$$

$$= \frac{1}{\sin x} \cdot \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sin(-h)}{\sin(x+h)} \right]$$

$$= \frac{-1}{\sin x} \cdot \left( \lim_{h \to 0} \frac{\sin h}{h} \right) \left( \lim_{h \to 0} \frac{1}{\sin(x+h)} \right)$$

$$= \frac{-1}{\sin x} \cdot 1 \cdot \left( \frac{1}{\sin(x+0)} \right)$$

$$= \frac{-1}{\sin^{2} x}$$

$$= -\csc^{2} x$$

$$\cdots (\cot x)' = -\csc^{2} x \qquad \cdots (2)$$

Now, let  $f_2(x) = \operatorname{cosec} x$ . Accordingly,  $f_2(x+h) = \operatorname{cosec}(x+h)$ By first principle,

$$f_{2}'(x) = \lim_{h \to 0} \frac{f_{2}(x+h) - f_{2}(x)}{h}$$
$$= \lim_{h \to 0} \frac{1}{h} \left[ \operatorname{cosec}(x+h) - \operatorname{cosec} x \right]$$

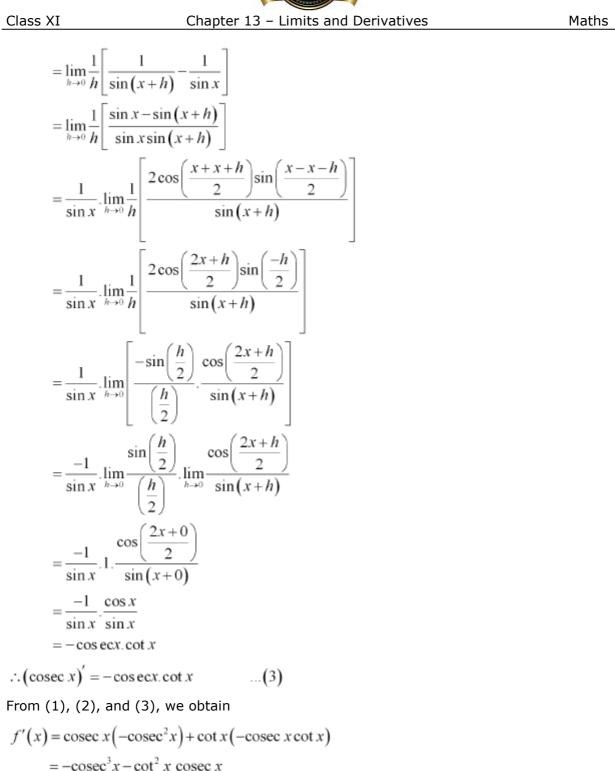
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 $\cos x$ 

**Question 16:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $1 + \sin x$ Answer

$$f(x) = \frac{\cos x}{1 + \sin x}$$

By quotient rule,

$$f'(x) = \frac{(1+\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$
$$= \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2}$$
$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$
$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$$
$$= \frac{-\sin x - 1}{(1+\sin x)^2}$$
$$= \frac{-(1+\sin x)}{(1+\sin x)^2}$$
$$= \frac{-1}{(1+\sin x)}$$

**Question 17:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\sin x - \cos x$ Answer

 $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$ By quotient rule,

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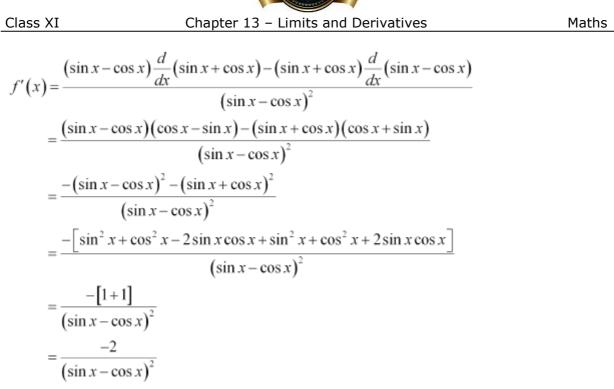
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 $\sin x + \cos x$ 





#### **Question 18:**

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

 $\sec x - 1$ 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\sec x + 1$ Answer

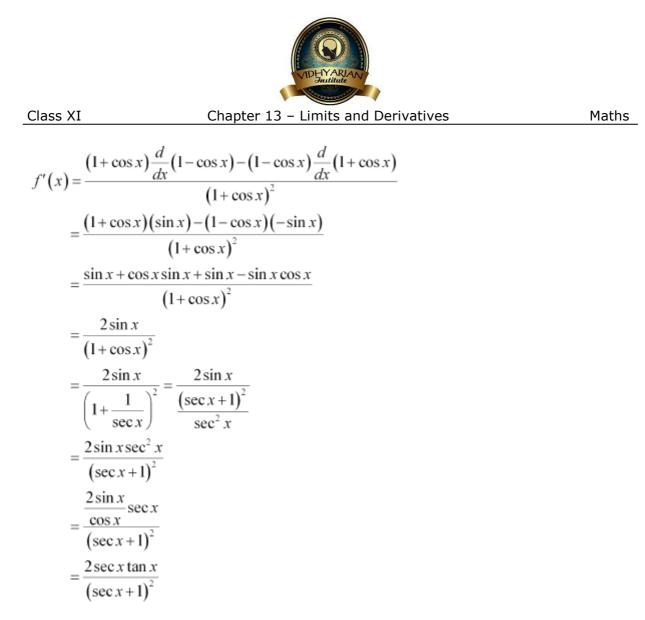
Let  $f(x) = \frac{\sec x - 1}{\sec x + 1}$  $f(x) = \frac{\frac{1}{\cos x} - 1}{\frac{1}{1 + \cos x} + 1} = \frac{1 - \cos x}{1 + \cos x}$ 

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By quotient rule,

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**Question 19:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\sin^n x$ 

Answer

Let  $y = \sin^n x$ . Accordingly, for n = 1,  $y = \sin x$ .

$$\therefore \frac{dy}{dx} = \cos x, \text{ i.e., } \frac{d}{dx} \sin x = \cos x$$
  
For  $n = 2$ ,  $y = \sin^2 x$ .

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Class XI  

$$\begin{array}{l} (-\frac{dy}{dx} = \frac{d}{dx}(\sin x \sin x)) \\ = (\sin x)' \sin x + \sin x(\sin x)' \qquad [By Leibnitz product rule] \\ = \cos x \sin x + \sin x \cos x \\ = 2 \sin x \cos x \qquad ...(1) \\ \text{For } n = 3, y = \sin^3 x. \\ (-\frac{dy}{dx} = \frac{d}{dx}(\sin x \sin^2 x)) \\ = (\sin x)' \sin^2 x + \sin x(\sin^2 x)' \qquad [By Leibnitz product rule] \\ = \cos x \sin^2 x + \sin x(2 \sin x \cos x) \qquad [Using (1)] \\ = \cos x \sin^2 x + 2 \sin^2 x \cos x \\ = 3 \sin^2 x \cos x \\ \text{We assert that} \quad \frac{d}{dx}(\sin^a x) = n \sin^{(a-1)} x \cos x \\ \text{Let our assertion be true for } n = k. \\ (-\frac{d}{dx}(\sin^a x) = k \sin^{(1-1)} x \cos x \qquad ...(2) \\ \text{Consider} \\ \frac{d}{dx}(\sin^{a+1} x) = \frac{d}{dx}(\sin x \sin^{a} x) \\ = (\sin x)' \sin^{a} x + \sin x(\sin^{a} x)' \qquad [By Leibnitz product rule] \\ = \cos x \sin^{a} x + \sin x(\sin^{a} x) \qquad [Using (2)] \\ = \cos x \sin^{a} x + \sin x(x \sin^{a} x \cos x) \\ = (k+1) \sin^{a} x \cos x
\end{array}$$

Thus, our assertion is true for n = k + 1.

al induction, 
$$\frac{d}{dx}(\sin^n x) = n\sin^{(n-1)}x\cos x$$

Hence, by mathematical induction

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Question 20:

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# Chapter 13 – Limits and Derivatives

Maths

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\overline{c+d\cos x}$ Answer

$$\operatorname{Let} f(x) = \frac{a + b \sin x}{c + d \cos x}$$

By quotient rule,

$$f'(x) = \frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x) - (a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$$
$$= \frac{(c+d\cos x)(b\cos x) - (a+b\sin x)(-d\sin x)}{(c+d\cos x)^2}$$
$$= \frac{cb\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c+d\cos x)^2}$$
$$= \frac{bc\cos x + ad\sin x + bd(\cos^2 x + \sin^2 x)}{(c+d\cos x)^2}$$
$$= \frac{bc\cos x + ad\sin x + bd}{(c+d\cos x)^2}$$

**Question 21:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

 $\sin(x+a)$ 

 $a + b \sin x$ 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\cos x$ Answer

$$f(x) = \frac{\sin(x+a)}{\cos x}$$

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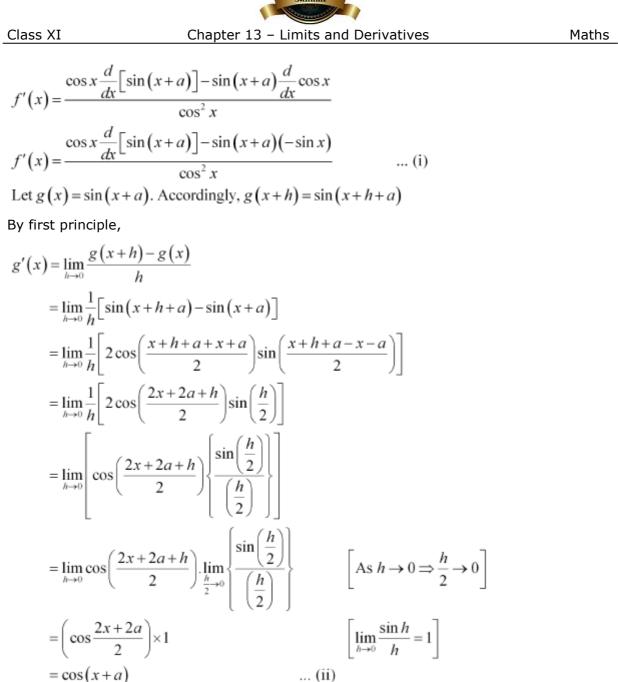
By quotient rule,

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Mobile: 9999 249717





From (i) and (ii), we obtain

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$$f'(x) = \frac{\cos x \cdot \cos(x+a) + \sin x \sin(x+a)}{\cos^2 x}$$
$$= \frac{\cos(x+a-x)}{\cos^2 x}$$
$$= \frac{\cos a}{\cos^2 x}$$

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Chapter 13 – Limits and Derivatives

**Question 22:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $x^4$  (5 sin x – 3 cos x) Answer

Let 
$$f(x) = x^4 (5\sin x - 3\cos x)$$

By product rule,

$$f'(x) = x^{4} \frac{d}{dx} (5\sin x - 3\cos x) + (5\sin x - 3\cos x) \frac{d}{dx} (x^{4})$$
  
$$= x^{4} \left[ 5 \frac{d}{dx} (\sin x) - 3 \frac{d}{dx} (\cos x) \right] + (5\sin x - 3\cos x) \frac{d}{dx} (x^{4})$$
  
$$= x^{4} \left[ 5\cos x - 3(-\sin x) \right] + (5\sin x - 3\cos x) (4x^{3})$$
  
$$= x^{3} \left[ 5x\cos x + 3x\sin x + 20\sin x - 12\cos x \right]$$

Question 23:

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(x^2 + 1) \cos x$ Answer

Let 
$$f(x) = (x^2 + 1)\cos x$$

By product rule,

$$f'(x) = (x^{2} + 1)\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x^{2} + 1)$$
$$= (x^{2} + 1)(-\sin x) + \cos x(2x)$$
$$= -x^{2}\sin x - \sin x + 2x\cos x$$

**Question 24:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(ax^2 + \sin x) (p + q \cos x)$ 

Answer

Let  $f(x) = (ax^2 + \sin x)(p + q\cos x)$ 

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By product rule,

$$f'(x) = (ax^{2} + \sin x)\frac{d}{dx}(p + q\cos x) + (p + q\cos x)\frac{d}{dx}(ax^{2} + \sin x)$$
$$= (ax^{2} + \sin x)(-q\sin x) + (p + q\cos x)(2ax + \cos x)$$
$$= -q\sin x(ax^{2} + \sin x) + (p + q\cos x)(2ax + \cos x)$$

**Question 25:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(x + \cos x)(x - \tan x)$ Answer

Let 
$$f(x) = (x + \cos x)(x - \tan x)$$

By product rule,

$$f'(x) = (x + \cos x) \frac{d}{dx} (x - \tan x) + (x - \tan x) \frac{d}{dx} (x + \cos x)$$
  
=  $(x + \cos x) \left[ \frac{d}{dx} (x) - \frac{d}{dx} (\tan x) \right] + (x - \tan x) (1 - \sin x)$   
=  $(x + \cos x) \left[ 1 - \frac{d}{dx} \tan x \right] + (x - \tan x) (1 - \sin x)$  ... (i)

Let  $g(x) = \tan x$ . Accordingly,  $g(x+h) = \tan(x+h)$ 

By first principle,

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Chapter 13 – Limits and Derivatives

Maths

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left( \frac{\tan(x+h) - \tan x}{h} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[ \frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos(x+h)\cos x} \right]$$

$$= \frac{1}{h} \frac{1}{h} \left[ \frac{\sin(x+h)\cos x}{\cos(x+h)\cos x} \right]$$

$$= \frac{1}{1} \frac{1}{1} \frac{1}{1} \left[ \frac{\sin(x+h-x)}{\cos(x+h)} \right]$$

$$= \frac{1}{1} \frac{1}{1} \frac{1}{1} \frac{1}{1} \left[ \frac{\sin h}{\cos(x+h)} \right]$$

$$= \frac{1}{1} \frac{1}{1$$

Therefore, from (i) and (ii), we obtain

$$f'(x) = (x + \cos x)(1 - \sec^2 x) + (x - \tan x)(1 - \sin x)$$
  
=  $(x + \cos x)(-\tan^2 x) + (x - \tan x)(1 - \sin x)$   
=  $-\tan^2 x(x + \cos x) + (x - \tan x)(1 - \sin x)$ 

**Question 26:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

 $4x + 5\sin x$ 

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\overline{3x + 7\cos x}$ Answer

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 Website:
 www.vidhyarjan.com
 Email:
 contact@vidhyarjan.com
 Mobile:
 9999
 249717

 Head Office:
 1/3-H-A-2,
 Street # 6,
 East Azad Nagar,
 Delhi-110051

 (One
 Km from 'Welcome' Metro Station)



Chapter 13 – Limits and Derivatives

 $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$ 

By quotient rule,

$$f'(x) = \frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$
$$= \frac{(3x+7\cos x)\left[4\frac{d}{dx}(x)+5\frac{d}{dx}(\sin x)\right]-(4x+5\sin x)\left[3\frac{d}{dx}x+7\frac{d}{dx}\cos x\right]}{(3x+7\cos x)^2}$$
$$= \frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$
$$= \frac{12x+15x\cos x+28\cos x+35\cos^2 x-12x+28x\sin x-15\sin x+35\sin^2 x}{(3x+7\cos x)^2}$$
$$= \frac{15x\cos x+28\cos x+28x\sin x-15\sin x+35(\cos^2 x+\sin^2 x)}{(3x+7\cos x)^2}$$
$$= \frac{35+15x\cos x+28\cos x+28x\sin x-15\sin x}{(3x+7\cos x)^2}$$

**Question 27:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):

$$\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$$

Answer

$$f(x) = \frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$$

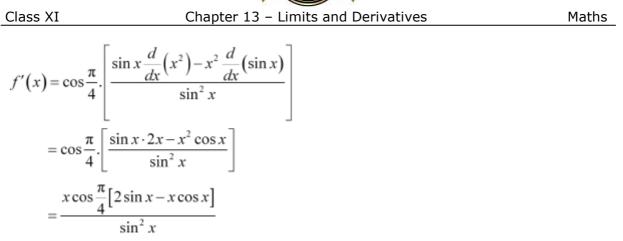
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By quotient rule,

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**Question 28:** 

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

х

and *s* are fixed non-zero constants and *m* and *n* are integers):  $1 + \tan x$ Answer

Let  $f(x) = \frac{x}{1 + \tan x}$ 

$$f'(x) = \frac{(1+\tan x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$
$$f'(x) = \frac{(1+\tan x) - x \cdot \frac{d}{dx}(1+\tan x)}{(1+\tan x)^2} \qquad \dots (i)$$

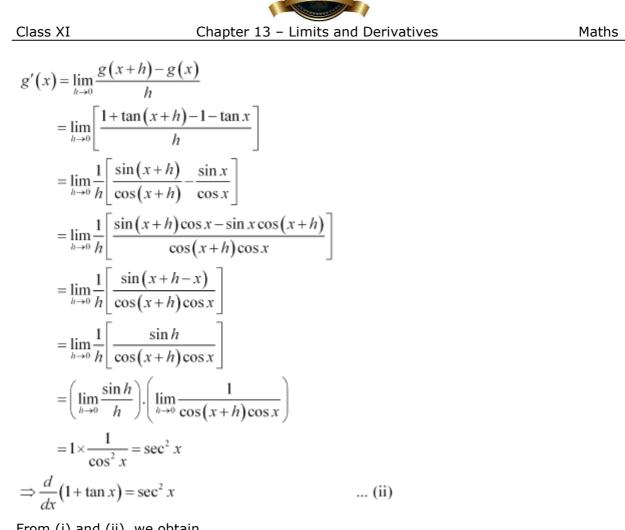
Let  $g(x) = 1 + \tan x$ . Accordingly,  $g(x+h) = 1 + \tan(x+h)$ . By first principle,

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From (i) and (ii), we obtain

$$f'(x) = \frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2}$$

**Question 29:** 

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers):  $(x + \sec x) (x - \tan x)$ Answer

Let 
$$f(x) = (x + \sec x)(x - \tan x)$$

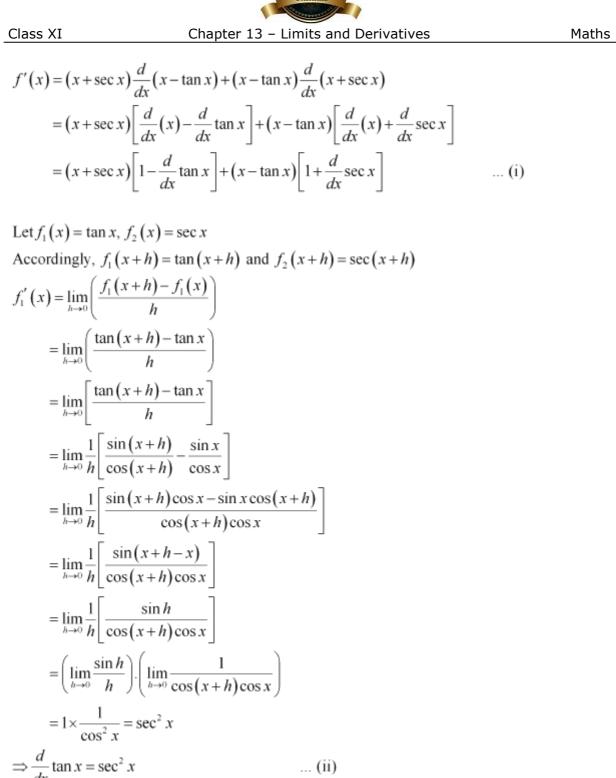
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By product rule,

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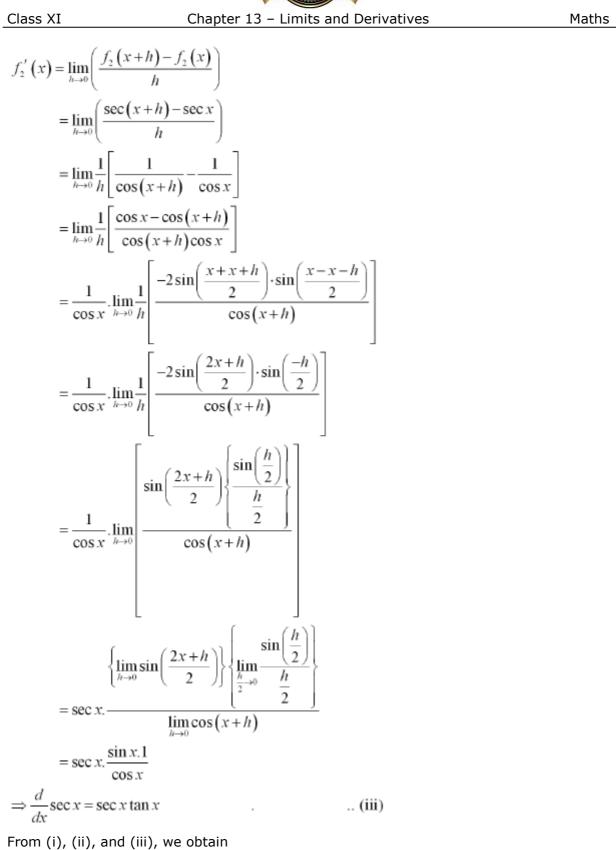


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Class XI Chapter 13 – Limits and Derivatives Maths

$$f'(x) = (x + \sec x)(1 - \sec^2 x) + (x - \tan x)(1 + \sec x \tan x)$$

Question 30:

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* 

х

and *s* are fixed non-zero constants and *m* and *n* are integers):  $\overline{\sin^n x}$ Answer

$$f(x) = \frac{x}{\sin^n x}$$

By quotient rule,

$$f'(x) = \frac{\sin^n x \frac{d}{dx} x - x \frac{d}{dx} \sin^n x}{\sin^{2n} x}$$

$$\frac{d}{dx}\sin^n x = n\sin^{n-1}x\cos x$$

It can be easily shown that dxTherefore,

$$f'(x) = \frac{\sin^n x \frac{d}{dx} x - x \frac{d}{dx} \sin^n x}{\sin^{2n} x}$$
$$= \frac{\sin^n x \cdot 1 - x \left(n \sin^{n-1} x \cos x\right)}{\sin^{2n} x}$$
$$= \frac{\sin^{n-1} x \left(\sin x - nx \cos x\right)}{\sin^{2n} x}$$
$$= \frac{\sin x - nx \cos x}{\sin^{n+1} x}$$

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