Assignment (Complex Numbers)

1. Prove that i) $1 + i^2 + i^4 + i^6 = 0$ ii) $6i^{54} + 5i^{37} - 2i^{11} + 6i^{68} = 7i$ iii) $i^n . i^{n+1} . i^{n+2} . i^{n+3} = -1$, $n \in N$

   iv) $(1+i)^4 \left(1 + \frac{1}{i}\right)^4 = 16$ v) $(1+i)^8\left(1+\frac{1}{i}\right)^8 = 256$

2. Solve for x and y. i) $2x + 3iy = 5 + 6i$ ii) $(3 + i)x + (1 - 2i)y + 7i = 0$

   iii) $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

3. Find the real values of x and y if i) $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$ ii) $(x - iy)(2 + 3i) = \frac{x+2i}{1-i}$

4. Express the following numbers in the form x + iy : i) $\frac{(2+3i)^2}{1+i}$ ii) $\frac{(1+i)^3}{1-i^3}$ iii) $\frac{3}{1+i} - \frac{2}{2-i} + \frac{2}{1-i}$

   iv) $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$ v) $\frac{(1-i)^3}{1-i^3}$ vi) $\frac{3+2i}{-2+i}$ vii) $\frac{1+\sqrt{-36}}{-2+\sqrt{-4}}$

   viii) $\frac{(3+2i)(2+3i)}{(1+2i)(2-i)}$ ix) $\frac{1}{1-\cos x+2i\sin x}$ x) $\frac{(\cos x+i\sin x)(\cos y+i\sin y)}{(\cos u+i)(1+i\tan v)}$

5. Prove that the following complex numbers are purely real: i) $\left(\frac{3+4i}{2-3i}\right)\left(\frac{3-4i}{2+3i}\right)$ ii) $\left(\frac{3+2i}{2-3i}\right)+\left(\frac{3-2i}{2+3i}\right)$

6. Find the multiplicative inverse of the following i) $(2 - i)(3 + i)$ ii) $\frac{2+3i}{3-2i}$ iii) $\frac{(i+1)(i+2)}{(i-1)(i-2)}$

   iv) $\frac{(1-2i)^2-3}{7i+2(2-3i)}$ v) $2 + \frac{1}{1+i}$ vi) $2+\sqrt{3}i$

7. Find the modulus and amplitude of the following complex numbers and express in the polar form.

   i) $1 + \sqrt{3}\,i$ ii) $\frac{1+i}{1-i}$ iii) $-1 + \sqrt{3}\,i$ iv) $\frac{1-i\sqrt{3}}{2}$ v) $-i\sqrt{2} - \sqrt{2}$

   vi) $-1 - \sqrt{-1}$ vii) $\frac{1+3i}{1-2i}$ viii) $4\sqrt{3}+4i$ ix) $\frac{1+i}{1-i}$ x) $3\sqrt{2}\,i - 3\sqrt{2}$

   xi) $-2i$ xii) $\frac{1+2i}{1-3i}$ xiii) $\frac{1-3i}{1+2i}$ xiv) -4

8. Find the real values of x and y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each.

9. Find the conjugate of i) $\frac{1}{1+i}$ ii) $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ iii) $\frac{4}{i^5}$

10. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

11. Find the modulus and argument of $\frac{(1+i)^{13}}{(1-i)^7}$

12. Find the conjugate and modulus of $\frac{(1+i)^{2n+1}}{(1-i)^{2n-1}}$

13. Express in the standard form: i) $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{3})}$ ii) $\left(\frac{3+4i}{2-4i}\right)\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)$ iii) $\frac{(1+i)^2}{3-i}$

    iv) $\frac{(3-i)^2}{2+i}$ v) $\frac{(1+i)(2+i)}{3+i}$ vi) i^{-39}

14. Find the conjugate of $\frac{Z_1Z_2 - Z_3}{Z_1 + Z_3}$ when $z_1 = 1 - 2i$, $z_2 = -1 + i$ and $z_3 = 1 + i$

15. Convert $\dfrac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ into polar form.

16. Find real θ such that $\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is i) purely real ii) purely imaginary.

17. Find the modulus of $\dfrac{(3-4i)^{10}}{(2+i)^8}$

18. Find the square root of i) – 15 – 8i ii) 3 + 4i iii) – 15 + 8i iv) 5 – 12i
v) 8 + 6i vi) 7 – 24i vii) 1 + i viii) 1 – i

19. Solve : i) $2x^2+\sqrt{15}\ ix-i=0$ ii) $x^2-x+(1+i)=0$ iii) $x^2-(3\sqrt{2}-2i)x-\sqrt{2}i=0$

20. If z = 2 – 3i, prove that $z^2 - 4z + 13 = 0$

21. If $x - iy = \sqrt{\dfrac{(x+i)^2}{2x^2+1}}$ Prove that $a^2 + b^2 = \dfrac{(x^2+1)^2}{(2x^2+1)^2}$

22. If $(x+iy)^{\frac{1}{3}} = u + iv$,then Prove that $\dfrac{x}{u}+\dfrac{y}{v}=4(u^2-v^2)$

23. If $a+ ib = \dfrac{c+i}{c-i}, showthat\ a^2+b^2=1\ and\ \dfrac{b}{a}=\dfrac{2c}{c^2-1}$

24. If a and b are two complex numbers such that $|b|=3$ find the value of $\left|\dfrac{b-a}{9-\bar{a}b}\right|$

25. If $x + iy = \sqrt{\dfrac{a+ib}{c+id}}$, then prove that $(x^2+y^2)^2 = \dfrac{a^2+b^2}{c^2+d^2}$

26. Find the smallest possible integer n such that $\left(\dfrac{1+i}{1-i}\right)^n = 1$

27. If (1 + 2i)(2 + 3i)(3 + 4i) = a + ib, prove that $a^2 + b^2 = 625$

28. If $|a+ib|=1 then\ showthat\ \dfrac{1+b+ai}{1+b-ai}=b+ai$

29. If $|z_1|=|z_2|=1, Provethat\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}\right|=|z_1+z_2|$

30. Solve the equation for z : i) $2z = |z| + 2i$ ii) $|z| + z = 2 + I$ iii) $z + 2 = \dfrac{1}{4-3i}$

31. If $iz^3 + z^2 - z + i = 0$, show that $|z| = 1$.

32. If z = 1 + i, evaluate: $z^3 - 2z^2 + 3z - 4$

33. If $z = -1 + \sqrt{-2}$, evaluate $z^4 + 4z^3 + 6z^2 + 4z + 9$