

**TRIGONOMETRY - FORMULAE**

1. Radian Measure =  $\frac{\pi}{180} \times \text{Degree Measure}$       Degree Measure =  $\frac{180}{\pi} \times \text{Radian Measure}$

3.  $\ell = r\theta$  where  $\theta$  is in radians.

4.  $\cos(-x) = \cos x$        $\sin(-x) = -\sin x$        $\tan(-x) = -\tan x$

5.  $\cos(x + y) = \cos x \cos y - \sin x \sin y$        $\cos(x - y) = \cos x \cos y + \sin x \sin y$   
 $\sin(x + y) = \sin x \cos y + \cos x \sin y$        $\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$        $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$        $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

6.  $\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$        $\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$        $\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$        $-2 \sin x \sin y = \cos(x + y) - \cos(x - y)$

$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$        $2 \sin x \sin y = \cos(x - y) - \cos(x + y)$

$2 \cos x \sin y = \sin(x + y) - \sin(x - y)$

8.  $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

from above formula we also get

$1 - \cos x = 2\sin^2(x/2)$  and  $1 + \cos x = 2\cos^2(x/2)$        $\sin x = 2\sin(x/2)\cos(x/2)$

9.  $\sin 2x = 2\sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$       9.1.)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

10.  $\sin 3x = 3\sin x - 4\sin^3 x$       11.  $\cos 3x = 4\cos^3 x - 3\cos x$       12.  $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

13. If  $\sin x = \sin y$  then  $x = n\pi + (-1)^n y$ , where  $n \in \mathbb{Z}$ .       $\sin x = 0$  implies  $x = n\pi$ , where  $n \in \mathbb{Z}$ .

14. If  $\cos x = \cos y$  then  $x = 2n\pi \pm y$ , where  $n \in \mathbb{Z}$ .       $\cos x = 0$  implies  $x = (2n + 1)\pi/2$ , where  $n \in \mathbb{Z}$ .

15. If  $\tan x = \tan y$ , then  $x = n\pi + y$ , where  $n \in \mathbb{Z}$ .       $\tan x = 0$  implies  $x = n\pi$ , where  $n \in \mathbb{Z}$ .

16.  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$       or       $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

17.  $a^2 = b^2 + c^2 - 2bc \cos A$        $b^2 = c^2 + a^2 - 2ac \cos B$        $c^2 = a^2 + b^2 - 2ab \cos C$

18.  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$        $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$        $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

- 1)  $\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
- 2)  $\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
- 3)  $\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

Functions	Domain	Range(Principal Value Branch)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	$\mathbb{R}$	$(0, \pi)$

For suitable values of domain we have,

$$1. \sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$$

$$2. \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$$

$$3. \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$$

$$4. \sin^{-1}(-x) = -\sin^{-1} x$$

$$5. \tan^{-1}(-x) = -\tan^{-1} x$$

$$6. \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$$

$$7. \cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$8. \sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$9. \cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$10. \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$11. \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$12. \operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$$

$$13. \sin^{-1} x + \sin^{-1} y = \sin^{-1} \left[ x\sqrt{1-y^2} + y\sqrt{1-x^2} \right]$$

$$14. \sin^{-1} x - \sin^{-1} y = \sin^{-1} \left[ x\sqrt{1-y^2} - y\sqrt{1-x^2} \right]$$

$$15. \cos^{-1} x + \cos^{-1} y = \cos^{-1} \left[ xy - \sqrt{1-x^2}\sqrt{1-y^2} \right]$$

$$16. \cos^{-1} x - \cos^{-1} y = \cos^{-1} \left[ xy + \sqrt{1-x^2}\sqrt{1-y^2} \right]$$

$$17. \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left[ \frac{x+y}{1-xy} \right], xy < 1$$

$$18. \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left[ \frac{x-y}{1+xy} \right], xy > -1$$

$$19. \tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left[ \frac{x+y}{1-xy} \right], xy > 1$$

$$20. 2 \tan^{-1} x = \tan^{-1} \left( \frac{2x}{1-x^2} \right)$$

$$21. 2 \tan^{-1} x = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$$

$$22. 2 \tan^{-1} x = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$$

LIST OF SUBSTITUTIONS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS:

1. If  $f(x)$  involves  $\sqrt{a^2 - x^2}$  put  $x = a \sin\theta$  ( or  $a \cos\theta$  ) if  $\sqrt{1 - x^2}$  put  $x = \sin\theta$  ( or  $\cos\theta$  )
2. If  $f(x)$  involves  $\sqrt{a^2 + x^2}$  put  $x = a \tan\theta$  ( or  $a \cot\theta$  ) if  $\sqrt{1 + x^2}$  put  $x = \tan\theta$  ( or  $\cot\theta$  )
3. If  $f(x)$  involves  $\sqrt{x^2 - a^2}$  put  $x = a \sec\theta$  ( or  $a \operatorname{cosec}\theta$  ) if  $\sqrt{x^2 - 1}$  put  $x = \sec\theta$  ( or  $\operatorname{cosec}\theta$  )
4. If  $f(x)$  involves both  $\sqrt{a^2 - x^2}$  and  $\sqrt{a^2 + x^2}$  put  $x^2 = a^2 \cos 2\theta$
5. If  $f(x)$  involves both  $\sqrt{a - x}$  and  $\sqrt{a + x}$  put  $x = a \cos 2\theta$

Brain