TRIGONOMETRY - FORMULAE

1. Radian Measure = $\frac{\pi}{180} \times$ Degree Measure   Degree Measure = $\frac{180}{\pi} \times$ Radian Measure

3. $\ell = r\theta$ where θ is in radians.

4. cos(-x) = cos x   sin(-x) = -sinx   tan(-x) = - tanx

5. cos(x + y) = cosxcosy – sinxsiny   cos(x – y) = cosxcosy + sinxsiny

sin(x +y) = sinxcosy + cosxsiny   sin(x – y) = sinxcosy – cosxsiny

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x} \qquad \cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

6. $$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \qquad \sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \qquad \cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$2\cos x \cos y = \cos(x+y) + \cos(x-y) \qquad -2\sin x \sin y = \cos(x+y) - \cos(x-y)$$

$$2\sin x \sin y = \cos(x-y) - \cos(x+y)$$

$$2\sin x \cos y = \sin(x+y) + \sin(x-y) \qquad 2\cos x \sin y = \sin(x+y) - \sin(x-y)$$

8. cos2x = $2\cos^2 x - 1$ = $1 - 2\sin^2 x$ = $\cos^2 x - \sin^2 x$ = $\frac{1 - \tan^2 x}{1 + \tan^2 x}$

from above formula we also get

$1 - \cos x = 2\sin^2(x/2)$ and $1 + \cos x = 2\cos^2(x/2)$ sinx = 2sin(x/2)cos(x/2)

9. sin2x = 2sinxcosx = $\frac{2\tan x}{1 + \tan^2 x}$ 9.1. Tan 2x = $\frac{2\tan x}{1 - \tan^2 x}$

10. $\sin 3x = 3\sin x - 4\sin^3 x$ 11. $\cos 3x = 4\cos^3 x - 3\cos x$ 12. Tan3x = $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

13. If sin x = sin y then $x = n\pi + (-1)^n y$, where n ε Z.   sin x = 0 implies x = nπ , where n ε Z.

14. If cos x = cos y then x = 2nπ ± y, where n ε Z.   cos x =0 implies x = (2n + 1)π/2, where n ε Z.

15. If tan x = tan y, then x = nπ + y, where n ε Z.   tan x = 0 implies x = nπ , where n ε Z.

16. $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

17. $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = c^2 + a^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$

18. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

1) $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
2) $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
3) $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

Functions	**Domain**	**Range(Principal Value Branch)**
y = $\sin^{-1} x$	[-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
y = $\cos^{-1} x$	[-1, 1]	$[0, \pi]$
y = $\text{cosec}^{-1} x$	R – (–1, 1)	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ – {0}
y = $\sec^{-1} x$	R – (–1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
y = $\tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
y = $\cot^{-1} x$	R	$(0, \pi)$

For suitable values of domain we have,

1. $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1} x$
2. $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$
3. $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$
4. $\sin^{-1}(-x) = -\sin^{-1} x$
5. $\tan^{-1}(-x) = -\tan^{-1} x$
6. $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$
7. $\cos^{-1}(-x) = \pi - \cos^{-1} x$
8. $\sec^{-1}(-x) = \pi - \sec^{-1} x$
9. $\cot^{-1}(-x) = \pi - \cot^{-1} x$
10. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
11. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
12. $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
13. $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$
14. $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$
15. $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$
16. $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right]$
17. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$, xy < 1
18. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$, xy > −1
19. $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left[\frac{x+y}{1-xy}\right]$, xy > 1
20. $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$
21. $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$
22. $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

LIST OF SUBSTITUTIONS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS:

1, If f(x) involves $\sqrt{a^2 - x^2}$ put x = a sinθ (or a cosθ) if $\sqrt{1-x^2}$ put x = sinθ (or cosθ)

2. If f(x) involves $\sqrt{a^2 + x^2}$ put x = a tanθ (or a cotθ) if $\sqrt{1+x^2}$ put x = tanθ (or cot θ)

3. If f(x) involves $\sqrt{x^2 - a^2}$ put x = a secθ (or a cosecθ) if $\sqrt{x^2 - 1}$ put x = secθ (or cosecθ)

4. If f(x) involves both $\sqrt{a^2 - x^2}$ and $\sqrt{a^2 + x^2}$ put $x^2 = a^2 \cos 2\theta$

5. If f(x) involves both $\sqrt{a - x}$ and $\sqrt{a + x}$ put x = a cos2θ