1. **Statement**: A sentence which is either TRUE or FALSE but not both is known as a statement.
   eg. i) $2 + 2 = 4$ (it is a statement which is true)
   ii) $2 + 3 = 4$ (it is a statement which is false)
   iii) The sum of x and y is greater than zero. (its not a statement since we can't say whether it is true or false without knowing the values of x and y. this is an open sentence.)

2. **Truth Value of a Statement**: The truth or falsity of a statement is called its truth value. If a statement is TRUE it is denoted by "T" and if a statement is FALSE its truth value is denoted by "F".

3. **Quantifiers**: The phrases like "there exists" , " for all" are known as quantifiers.
   i) there exists: ($\exists$) (it means there is atleast one)
   e.g. a) **there exists** a number which is equal to its square.

   ii) for every ($\forall$) (means for all)
   e.g. **for every** prime p, $\sqrt{p}$ is an irrational number.

4. **Simple Statement**: A Statement which cannot be broken down into sub statements is called a simple statement. Simple Statements are represented by small letters usually p, q, r … etc.
   e.g. p : 17 is a prime number.

5. **Compound Statement**: If a Statement is a combination of two or more simple Statements, it is known as a Compound Statement.
   e.g. "All prime numbers are either even or odd" is a compound statement.
   Its component statements are p : All prime numbers are even.
   q : All prime numbers are odd.

6. **Connectives:** The words or phrases used for connecting two or more simple statements like "or", "and", "if …then…" , "… if and only if…." are known as Connectives.

Connectives	Name	Symbol
OR	Disjunction	$\vee$
And	Conjunction	$\wedge$
If …then	Implication/ conditional	$\Rightarrow$
If and only if	Bi-implication / biconditional	$\Leftrightarrow$

7. **Negation:** The denial of a statement is known as negation of the statement.
   e.g. i) *p:* $\sqrt{3}$ *is irrational* the negation of statement p is $\sim p$*:* $\sqrt{3}$ *is not irrational.*

   note that a) $\sim(\sim p) = p$ b) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$ c) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

8. **Conjunction:** A compound statement obtained by combining two or more statements by the connective "and" is known as conjunction.
   Note: A conjunction is true when all its component statements are true.

Truth Table for Conjunction		
p	**q**	**p $\wedge$ q**
T	T	T
T	F	F
F	T	F
F	F	F

Table 14.1

9. **Disjunction:** A compound statement obtained by combining two or more statements by the connective "or" is known as disjunction.
Note: A disjunction is false when all its component statements are false. In other words a disjunction is true if at least one of the component statement is true.

Table 14.2

Truth Table for Disjunction		
p	**q**	**$p \vee q$**
T	T	T
T	F	T
F	T	T
F	F	F

10. **Inclusive OR:** When "or" is used for at least one of the alternatives it is known as "inclusive or".
e.g.i) *The school remains closed on a holiday **or** Friday.*
(this means that the school is closed on a holiday, it also means that the school remains closed on Friday. if a holiday falls on Friday then also the school remains closed.)
ii) *A student who has taken biology **or** biotechnology in Class XII can apply for B.Sc. Biotechnology.*
(this means that the students who have taken both biology and biotechnology can also apply, as well as the students who have taken only one of these subjects.)

11. **Exclusive OR:** When "or" is used for exactly one of the alternatives it is known as "exclusive or".
*e.g. Students can take French **or** Arabic as their third language.*
(this means that a student cannot take both French and Arabic.)

12. **Implication:** If two statements are connected by " if then" it is known as an implication (or conditional).
e.g. " *If* $x = 4$ *then* $x^2 = 16$ "
the component statements are p: $x = 4$ q: $x^2 = 16$

e.g. i) If a natural number is odd, then its square is also odd.
ii) A natural number is odd implies that its square is odd.
iii) A natural number is odd only if its square is odd.
iv) For a natural number to be odd it is necessary that its square is odd.
v) For the square of a natural number to be odd, it is sufficient that the number is odd.
vi) If the square of a natural number is not odd, then the natural number is not odd.

the implication is "if p then q"……….
Here ***p*** is known as antecedent and ***q*** is known as consequent.
Note: a) The conditional $p \Rightarrow q$ is false only when p is true and q is false.(Else true)
i.e. a true statement can't imply a false statement.

Table 14.3

Truth Table for Implication		
p	**q**	**$p \Rightarrow q$**
T	T	T
T	F	F
F	T	T
F	F	T

b) The other equivalent forms (i.e. which means the same) of the implication are :
i) p is sufficient condition for q

ii) q is necessary condition for p
iii) p only if q
iv) if not q then not p.

13. **Converse, Inverse & Contra positive of an Implication:**

Implication:	$p \Rightarrow q$	If p then q
Converse:	$q \Rightarrow p$	If q then p
Inverse:	$\sim p \Rightarrow \sim q$	If not p then not q
Contra positive	$\sim q \Rightarrow \sim p$	If not q then not p

e.g. " If a triangle is equilateral then it is isosceles". (Given Implication) [**True**]

p : the triangle is equilateral ~p : the triangle is not equilateral
q : the triangle is isosceles. ~q : the triangle is not isosceles

converse	($q \Rightarrow p$)	" If a triangle is isosceles then it is equilateral".	[**False**]
Inverse	($\sim p \Rightarrow \sim q$)	" If a triangle is not equilateral then it is not isosceles."	[**False**]
Contra positive	($\sim q \Rightarrow \sim p$)	" If a triangle is not isosceles then it is not equilateral".	[**True**]

14. **Bi-implication:** If two simple statements are connected by "if and only if" it is known as bi-implication or bi-conditional. A bi-implication is true if the component statements have the same truth value(i.e. either both should be **false** or both should be **true**.)
e.g. You will pass in the exam if and only if you work hard.
The component statements are p: you will pass in the exam
q: you will work hard
"If and only if" means the following equivalent forms
a) p if and only if q b) q if and only if p
c) p is necessary and sufficient condition for q
d) q is necessary and sufficient condition for p
The truth table is as given below:

Table 14.4

Truth Table for Implication

p	**q**	**$p \Leftrightarrow q$**
T	T	T
T	F	F
F	T	F
F	F	T

15. ***METHODS OF PROOF***

a. **Validating Statements:**
Rule 1: To prove Statements with "and" to be true
Step 1: Show that the statement p is true.
Step 2: Show that the statement q is true.

Rule 2: To prove statements with " or" to be true
Case 1: By assuming that p is false, show that q must be true.
Case 2: By assuming that q is false, show that p must be true.

Rule 3: To prove statements with "if then' to be true, we need to show that any one of the following case is true.

Case 1: Direct Method

By assuming that p is true, prove that q must be true.

Case 2: Contrapositive Method

By assuming that q is false, prove that p must be false.

Rule 4: To prove statements with "if and only if" to be true

Step 1: if p is true, show that q is true.
Step 2 : if q is true, show that p is true.

b. **Method of Contradiction**

In this method to check whether a statement p is true, we assume that p is not true and arrive at a result which contradicts our assumption.

c. **Method of Counter example**

As the method name suggests we give an example to counter the given statement. Counter examples are used to disprove a statement. (Remember that generating examples in favour of a statement do not provide validity of the statement.)

e.g. p : all odd numbers are prime.
then counter example is 9, which is odd but not prime. thus the statement is false.

PREVIOUS YEAR QUESTIONS

1. Write the component statements of the statement "All prime numbers are either even or odd" and check whether they are true or not.
   Ans: ***p: all prime numbers are even. [False]***
   q: all prime numbers are odd.[False]

2. Write each statement using "if then":
   i) A quadrilateral is a parallelogram if its diagonals bisect each other.
   ii) you get a job implies that your credentials are good.
   Ans: ***i) If the diagonals of a quadrilateral bisect each other then it is a parallelogram.***
   ii) If you get a job then your credentials are good.

3. Combine the two statements using " if and only if":
   p : If a rectangle is a square, then all its four sides are equal.
   q : If all the 4 sides of a rectangle are equal, then the rectangle is a square.
   Ans: A rectangle is a square, if and only if all its four sides are equal.

4. Write the contra positive and converse of the statement "*If a triangle is equilateral, it is isosceles*".
   Ans: Contrapositive: " If a triangle is not isosceles then it is not equilateral".
   Converse: If a triangle is isosceles, then it is equilateral ".

5. Explain "contra positive statements with an example".

6. Write the inverse, converse and contra positive of the statement "If a number is divisible by 9, then it is divisible by 3".
   Ans: ***Inverse: If a number is not divisible by 9, then it is not divisible by 3.***
   Converse: "If a number is divisible by 3, then it is divisible by 9".
   Contra positive: "If a number is not divisible by 3, then it is not divisible by 9".

7. Write the converse of the statement "*If a number 'n' is even, then n^2 is even"*.
   Ans: If n^2 is even, then 'n' is even.

8. Write the component statements of the above compound statements in Q 6 and write whether the statements are true or not.

9. Identify the quantifier in "for every real number x, x + 4 is greater than x".
   Ans: the quantifier is "for every"

10. Write the contra positive of the statement, "If the diagonals of quadrilateral bisect each other, then it is a parallelogram".
    Ans: If the quadrilateral is not a parallelogram, then the diagonals do not bisect each other.

11. Write the contra positive and converse of the following statement. " Something is cold implies that it has low temperature ".
    Ans: ***Converse: Something has low temperature implies that it is cold .***
    Contrapositive: Something does not have low temperature implies that it is not cold.

12. Write the negation of " There exists a number which is equal to its square."
    Ans: There does not exist a number which is equal to its square."

13. Find the truth value of the following compound statements:
    a. " 4 + 2 = 6 or 9 + 7 = 15."
    b. " 4 + 2 = 6 or 9 + 7 = 16."
    c. " 4 + 2 = 5 or 9 + 7 = 16."
    d. " 4 + 2 = 5 or 9 + 7 = 15."
    e. " 4 + 2 = 6 and 9 + 7 = 16."
    f. " 4 + 2 = 6 and 9 + 7 = 15."
    g. " 4 + 2 = 5 and 9 + 7 = 15."
    h. " 4 + 2 = 5 and 9 + 7 = 16."
    i. " If 4 + 2 = 5 then 9 + 7 = 16."
    j. " If 4 + 2 = 5 then 9 + 7 = 15."
    k. " If 4 + 2 = 6 then 9 + 7 = 15."
    l. " If 4 + 2 = 6 then 9 + 7 = 16."
    Ans: a) p: 4 + 2 = 6 [True]
    q : 9 + 7 = 15 [False]
    p or q = $p \vee q$ = True { Refer Table 14.2}

    b) T c) T d) F e) T f) F g) F h) F i) T j) T k) F l) T

14. Prove that $\sqrt{3}$ is irrational by contradiction method. **[4 marks]**

Ans: Assume that $\sqrt{3}$ is a rational number. Then $\sqrt{3} = \frac{p}{q}$, where p & q are coprimes and $q \neq 0$.

Squaring on both sides, we get $3q^2 = p^2$.
This implies that p^2 is divisible by 3 and so, p is divisible by 3.
Let p = 3k which implies $p^2 = 9k^2$. But $p^2 = 3q^2$.
From both the equations we get $9k^2 = 3q^2$ which implies $q^2 = 3k^2$.
This means that q^2 is divisible by 3 and hence, q is divisible by 3.
This implies that p and q have 3 as a common factor. And this is a contradiction to the assumption that p and q are co-primes. Hence $\sqrt{3}$ is not a rational number.

15. Prove that $\sqrt{7}$ is not a rational number by the method of contradiction
Ans: Similar to above proof. (Try Yourself) Refer Textbook pg. 341 e.g. 15

16. Prove that $\sqrt{2}$ is irrational by contradiction method
Ans: Similar to above proof. (Try Yourself)

17. Which are the methods used to check the validity of mathematical statements.
Ans: a) Direct method b) method of contra positive (c) method of contradiction

18. Check the validity of the statement "If x is a real number such that $x^3 + 4x = 0$, then x = 0" is true by
a) Direct method b) method of contra positive (c) method of contradiction
Ans: Let p: "If x is a real number such that $x^3 + 4x = 0$, then x = 0"
It is a compound statement and its component statements are as follows:
q: x is a real number such that $x^3 + 4x = 0$
r : x is 0.

a) Direct method: to show that statement p is true, we assume that q is true and then show that r is true.
Let q be true.
$\therefore\ x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0 \Rightarrow x = 0$ or $x^2 + 4 = 0. \Rightarrow x = 0$ or $x^2 = -4$(this is rejected since x is real), $\therefore\ x = 0$. Thus the statement r is true.

b) method of contra positive: to prove statement p to be true, we assume that r is false and prove that q must be false. Here, r is false means that we should consider the negation of statement r.
i. e. x is not 0. $\Rightarrow x(x^2 + 4) \neq 0$ (since product of two non zero real numbers can't be equal to 0)
$\Rightarrow x^3 + 4x \neq 0$. This shows that the statement q is not true. $\therefore$ the given statement p is true.

c) method of contradiction: Assume that p is not true.
Let x be a real number such that $x^3 + 4x = 0$ and let $x \neq 0$.
$\therefore\ x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0 \Rightarrow x = 0$ or $x^2 + 4 = 0. \Rightarrow x = 0$ or $x^2 = -4$ Since x is real it means that x = 0 which is a contradiction as we assumed that $x \neq 0$.
Thus the given statement p is true.

19. Check the validity of the statement "If x, y are odd, then xy is also odd by **[4 marks]**
a) Direct method b) method of contra positive (c) method of contradiction
Ans: Refer Textbook pg. 340 e.g. 13
let p: x and y are odd.
q: xy is odd

a) Direct Method: we assume that if p is true then q is true.
p is true means that x and y are odd. $\therefore$ ***x = 2m + 1 and y = 2n + 1 for some integer m & n.***
$xy = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$
this shows that xy is odd. $\therefore$ ***given statement is true.***

b) method of contra positive: we assume that q is not true. i.e. xy is not odd $\Rightarrow$ ***xy is even*** $\Rightarrow$ ***either x is even or y is even(since product will be even, only if one of the number is even). This shows that p is not true. Thus we have shown that*** $\sim q \Rightarrow \sim p$

***c) method of contradiction:* (Try Yourself)**