Logarithms Exam Questions

1. Solve the following equations: i) $2^x = 5$ ii) $5^{x-1} = 4$ iii) $3^{2x-1} = 4$
   iv) $log_{10}^{(4x-2)} = 1.5$ v) $4^{(x-3)} = 36$ vi) $log_{10}^{(3x-1)} = 0.5$ vii) $5^{(x-2)} = 55$
   viii) $log_{10}^{(2x-3)} = 1.8$ ix) $2^{2x} = 10$ x) $\log_7\left(\frac{1}{7}\right) = x$
   xi) $\log(x-2) + \log(x-4) = \log(3)$ xii) $log_2^{(3-x)} = 4$ xii) $3^x = 2^{(x+1)}$

2. Given that $x = 5 + \sqrt{2}$ and $y = 3 - \sqrt{2}$, calculate a) $x - y$ b) xy c) $\frac{x}{y}$

3. Write $\frac{1+4\sqrt{2}}{3-\sqrt{2}}$ *in the form of* $A + B\sqrt{C}$

4. *Write* $(5 - \sqrt{3})^2$ *in the form* $a + b\sqrt{3}$

5. *If* $m = 3 + \sqrt{5}$ *and* $n = 2 - \sqrt{5}$ *; find* mn *in the form* $p + q\sqrt{5}$.

6. Solve
   (*a*) $5^x = 8$, giving your answers to 3 significant figures, **(3)**
   (*b*) $\log_2 (x + 1) - \log_2 x = \log_2 7$. **(3)**

7. (i) Write down the value of $\log_6 36$. **(1)**
   (ii) Express $2 \log_a 3 + \log_a 11$ as a single logarithm to base a. **(3)**

8. Given that $\log_2 x = a$, find, in terms of a, the simplest form of
   (*a*) $\log_2 (16x)$, **(2)**
   (*b*) $\log_2\left(\frac{x^4}{2}\right)$. **(3)**

9. a) Given that $\log_a x = 2\log_a 6 - \log_a 3$ show that $x = 12$. **(3)**

   b) Given that $\log_a y + \log_a 5 = 7$
   express y in terms of a, giving your answer in a form not involving logarithms. **(3)**

10. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £y, where $y = 1.04^x$.
    (a) Sketch the graph of $y = 1.04^x$, $x \geq 0$. **(2)**
    (*b*) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years. **(2)**
    (*c*) Use logarithms to find the number of years it takes to double the total value of any initial investment. **(3)**

11. Find the value of y such that $\log_2 y = -3$. **(2)**

12. Find the values of x such that $\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$. **(5)**

13. Find, to 3 significant figures, the value of x for which $5^x = 7$. **(2)**

14. Solve the equation i) $5^{2x} - 12(5^x) + 35 = 0$. **(4)**

    ii) $9^x - 10\left(3^x\right) + 9 = 0$

    iii) $4^x - 6\left(2^x\right) + 8 = 0$

    iv) $9^x + 6\left(3^x\right) - 27 = 0$

Logarithms Worksheet 2

1. Write these as a logarithm
   a. $4^2 = 16$
   b. $x^y = z$
2. Write these as an exponential
   a. $\log_3 81 = 4$
   b. $\log_a b = c$
3. Calculate
   a. $\log_{10} 400$
   b. $\log_8 0.5$
4. Simplify these logs and then work out their answers
   a. $\log_3 18 - \log_3 8 + \log_3 4$
   b. $6\log_4 8 - 4\log_4 2$
5. Simplify these logs
   a. $7\log x + 3\log y$
   b. $\frac{1}{2}\log x - 7\log y$
6. Write these as a in terms of $\log_a x, \log_a y$ and $\log_a z$
   a. $\log_a \frac{x}{y^2}$
   b. $\log_a ax^3\sqrt{yz}$
7. Solve for x, give your answers to 3sf
   a. $5^x = 60$
   b. $5^{2x} = 7^{x+1}$
   c. $6^{2x} + 6^{x+1} = 9$
   d. $\log_3 x + 5\log_x 3 = 6$
   e. $2\log_4 x - \log_4(x+24) = 1$

   Hint 7e
   Write the LHS as a single log
8. Using the same axes sketch the graphs of
   a. $y = 3^x$
   b. $y = -2^x$
   c. $y = \left(\frac{1}{2}\right)^x$

Logarithms Questions Worksheet 3

1. Write each of the following in the form $a \log p + b \log q$:

(a) $\log (pq)$

(b) $\log (p^2q^3)$;

(c) $\log \left(\frac{p}{q}\right)$

(d) $\log \sqrt{\frac{p}{q}}$

2. (a) Sketch the graph of $y = 2^x$, stating the coordinates of any points where the graph intersects the coordinate axes.

(b) Write down the value of:

(i) $\log_2 2$

(ii) $\log_2 8$

(c) Find the value of $\log_2 3 - \log_2 24$

3. (a) Given that $\log_a x = 2(\log_a k - \log_a 2)$

where a is a positive constant, show that $k^2 = 4x$.

(b) Given that $\log_3 y = \log_9 27$ find the value of y.

4. (a) Given that $\log_a k = \log_a 2 + 2\log_a 3$,

where a is a positive constant, show that $k = 18$.

(b) Given that $\log_a y = x\log_a 2 + 2x\log_a 3$

(i) express y in the form p^x where p is a constant,

(ii) find the value of x when $y = 40$, giving your answer correct to two decimal places,

(iii) deduce an approximate value of x when $y = 1600$.

5. (a) Given that $\log_a x = \log_a 5 + 2\log_a 3$ where a is a positive constant, show that $x = 45$

(b) (i) Write down the value of $\log_2 2$

(ii) Given that $\log_2 y = \log_4 2$, find the value of y.

Solutions to Logs Questions Worksheet 3

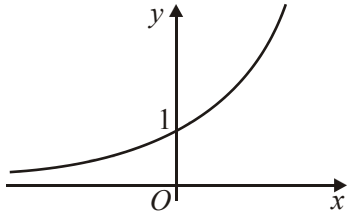
1. (a) $\log(pq) = \log p + \log q$

(b) $\log(p^2q^3) = 2\log p + 3\log q$

(c) $\log\left(\frac{p}{q}\right) = \log p - \log q$

(d) $\log\sqrt{\frac{p}{q}} = \frac{1}{2}\log p - \frac{1}{2}\log q$

2. (a)



(b) (i) 1

(ii) 3

(c) –3

3. (b) $y = 3^{\frac{3}{2}} = \sqrt{27} = 5.20$ to 3sf

4. (b) (i) $y = 18^x$

(ii) 1.28

(iii) 2.55

5. (b) (i) $\log_2 2 = 1$

(ii) $y = 2^{\frac{1}{2}} = \sqrt{2}$

Exponential Equations Worksheet 4

1 (a) (i) Write $\sqrt{2}$ as a power of 2.

(1)

(ii) Hence, express $4\sqrt{2}$ as a power of 2.

(1)

(iii) Hence, solve the equation $2^{3x+4} = 4\sqrt{2}$ (2)

2 (a) Write each of the following as a power of 2:

(i) $\sqrt{2}$;

(1)

(ii) 8^x.

(1)

(b) Hence solve the equation $8^x \times 2^{x+1} = \sqrt{2}$.

(3)

3. Solve the following pair of simultaneous equations to find x and y

$$16^y = \frac{1}{2^x}$$

$$\left(\sqrt{2}\right)^x \times 2^y = 1$$