# DEVELOPMENT OF SUPPORT MATERIAL IN MATHEMATICS FOR CLASS XI

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# CHAPTER - 1

# SETS

## KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set :–
  - (a) Roster or Tabular form.
  - (b) Set-builder form or Rule method.
- Types of sets :–
  - (i) Empty set or Null set or void set
  - (ii) Finite set
  - (iii) Infinite set
  - (iv) Singleton set
- Subset :– A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B$, $\forall \; a \in A$
- Equal sets :– Two sets A and B are equal if they have exactly the same elements i.e $A = B$ if $A \subset B$ and $B \subset A$
- Power set : The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. $P(A) = \{ B : B \subset A \}$
- If A is a set with $n(A) = m$ then $n[P(A)] = 2^m$.

**Types of Intervals**

Open Interval $(a, b) = \{ x \in R : a < x < b \}$

Closed Interval $[a, b] = \{ x \in R : a \leq x \leq b \}$

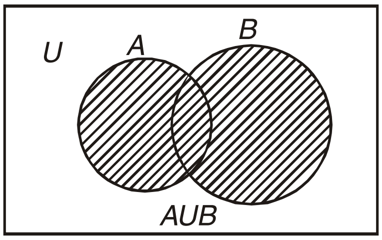
Semi open or Semi closed Interval,

$(a,b] = \{ x \in R : a < x \leq b\}$

$[a,b) = \{ x \in R : a \leq x < b\}$

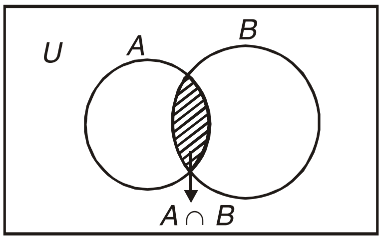
- Union of two sets A and B is,

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

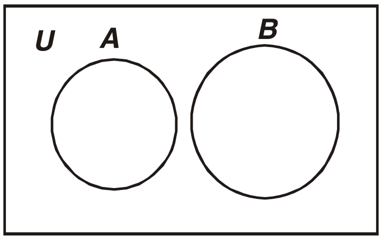



- Intersection of two sets A and B is,

  $A \cap B = \{ x : x \in A \text{ and } x \in B\}$

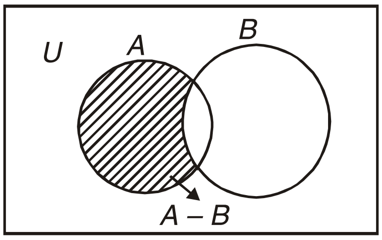



- Disjoint sets : Two sets A and B are said to be disjoint if $A \cap B = \phi$



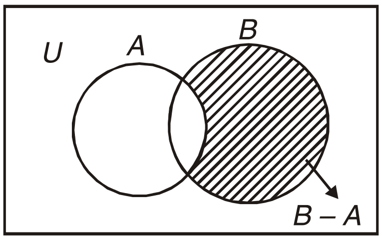
- Difference of sets A and B is,

  A – B = { x : x $\in$ A and x $\notin$ B}



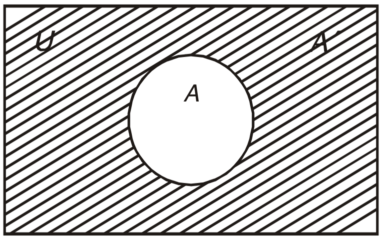

- Difference of sets B and A is,

  B – A = { x : x $\in$ B and x $\notin$ A }




- Complement of a set A, denoted by A' is

  A' = { x : x $\in$ U and x $\notin$ A}




- Properties of complement sets :

  1. Complement laws

     (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

3. $\phi' = \bigcup$ and $\bigcup' = \phi$

- $A - B = A \cap B'$
- Commutative Laws :–

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

- Associative Laws :–

(i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive Laws :–

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M
2. The collection of difficult topics in Mathematics.

Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces :– (Question- 3,4)

3. 2 —— A
4. 5 —— A
5. Write the set A = { x : x is an integer, $-1 \le x < 4$} in roster form
6. List all the elements of the set,

$$A = \left\{x : x \in z, -\frac{1}{2} < x < \frac{11}{2}\right\}$$

7. Write the set B = {3,9,27,81} in set-builder form.

Which of the following are empty sets? Justify. (Question- 8,9)

8. A = { x : x $\in$ N and 3 <x <4}

9. B = { x : x $\in$ N and $x^2 = x$}

Which of the following sets are finite or Infinite? Justify. (Question-10,11)

10. The set of all the points on the circumference of a circle.

11. B = { x : x $\in$ N and x is an even prime number}

12. Are sets A = { –2,2}, B = { x : x $\in$ z, $x^2 - 4 = 0$} equal? Why?

13. Write (–5,9] in set-builder form

14. Write { x : –3 $\leq$ x <7} as interval.

15. If A = { 1,3,5}, how many elements has P(A)?

16. Write all the possible subsets of A = {5,6}.

If A = {2,3,4,5}, B = { 3,5,6,7} find (Question- 17,18)

17. A $\cup$ B

18. A $\cap$ B

19. If A = {1,2,3,6} , B = {1, 2, 4, 8} find B – A

20. If A = {p, q}, B = {p, q, r}, is B superset of A? Why?

21. Are sets A = {1,2,3,4}, B = { x : x $\in$ N and 5 $\leq$ x $\leq$ 7} disjoint? Why?

22. If X and Y are two sets such that n(X) = 19, n(Y) = 37 and n(X $\cap$ Y) = 12, find n(X $\cup$ Y).

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

23. If $\bigcup$ = {1,2,3,4,5,6,7,8,9}, A = {2,3,5,7,9}, B = { 1,2,4,6}, verify

    (i) $(A \cup B)' = A' \cap B'$

    (ii) $B - A = B \cap A' = B - (A \cap B)$

24. Let A, B be any two sets. Using properties of sets prove that,

(i) $(A - B) \cup B = A \cup B$

(ii) $(A \cup B) - A = B - A$

[**Hint :** $A - B = A \cap B'$ and use distributive law.]

25. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find

(i) How many can speak both Hindi and English?

(ii) How many can speak Hindi only?

26. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?

27. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

28. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

29. In a survey it is found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B, 15 people like product B and C, 12 people like product C and A, and 8 people like all the three products. Find

(i) How many people are surveyed in all?

(ii) How many like product C only?

30. A college awarded 38 medals in football, 15 in basket ball and 20 in cricket. If these medals went to a total of 50 men and only five men got medals in all the three sports, how many received medals in exactly two of the three sports?

## ANSWERS

1. Set
2. Not a set
3. $\notin$
4. $\in$
5. A = {–1, 0, 1, 2, 3}
6. A = { 0,1,2,3,4,5}
7. B = { x : x = $3^n$, n $\in$ N and 1 $\leq$ n $\leq$ 4}
8. Empty set
9. Non-empty set
10. Infinite set
11. Finite set
12. Yes
13. { x : x $\in$ R, –5 < x $\leq$ 9}
14. [ –3,7)
15. $2^3 = 8$
16. $\phi$, { 5}, {6}, {5,6}
17. A $\cup$ B = {2,3,4,5,6,7}
18. A $\cap$ B = {3, 5}
19. B – A = {4,8}
20. Yes, because A is a subset of B
21. Yes, because A $\cap$ B = $\phi$
22. n(X $\cup$ Y) = 44
25. (i) 20 people can speak both Hindi and English

    (ii) 480 people can speak Hindi only
26. 29% of the Indians like both grapes and pineapple.
27. **Hint :** $\cup$ – set of people surveyed

    A – set of people who play cricket

    B – set of people who play tennis

    Number of people who play neither cricket nor tennis

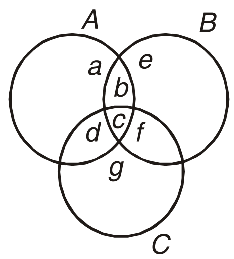
    = n[(A $\cup$ B)'] = n(U) – n(A $\cup$ B)

    = 450 – 200

    = 250
28. There are 280 students in the group.

29. **Hint :** Let A, B, C denote respectively the set of people who like product A, B, C.

a, b, c, d, e, f, g – Number of elements in bounded region




(i) Total number of Survyed people = a + b + c + d + e + f + g = 43

(ii) Number of people who like product C only = g = 10

30. 13 people got medals in exactly two of the three sports.

## CHAPTER - 2

# RELATIONS AND FUNCTIONS

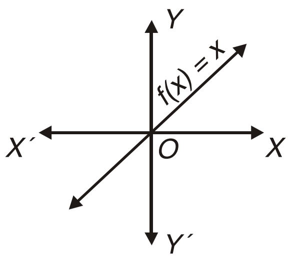
## KEY POINTS

- Cartesian Product of two non-empty sets A and B is given by,

  $A \times B = \{ (a,b) : a \in A, b \in B\}$

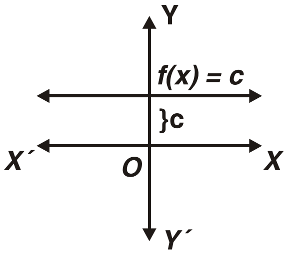
- If $(a,b) = (x, y)$, then $a = x$ and $b = y$
- Relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.
- Domain of $R = \{a : (a,b) \in R\}$
- Range of $R = \{ b : (a,b) \in R\}$
- Co-domain of R = Set B
- Range $\subseteq$ Co-domain
- If $n(A) = p$, $n(B) = q$ then $n(A\times B) = pq$ and number of relations $= 2^{pq}$
- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.
- Identity function, $f : R \rightarrow R$; $f(x) = x \;\; \forall x \in R$ where R is the set of real numbers.

  $D_f = R$ $\qquad$ $R_f = R$



- Constant function, f : R → R; f(x) = c $\forall x \in R$ where c is a constant

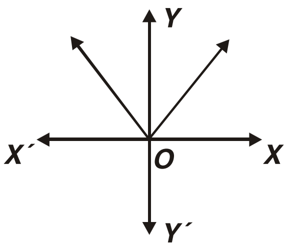
  $D_f$ = R $\qquad$ $R_f$ = {c}




- Modulus function, f : R → R; f(x) = |x| $\forall x \in R$

  $D_f$ = R

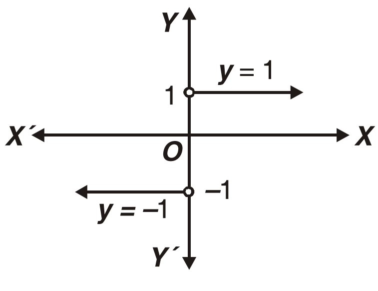
  $R_f = R^+ = \{ x \in R: x \geq 0\}$




- Signum function, f : R → R ; $f(x) = \begin{cases} 1, \text{If } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$

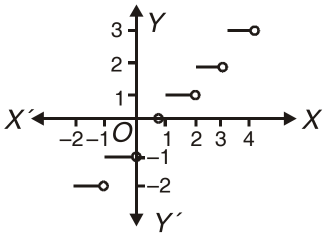
  $D_f$ = R

  $R_f$ = {–1,0,1}



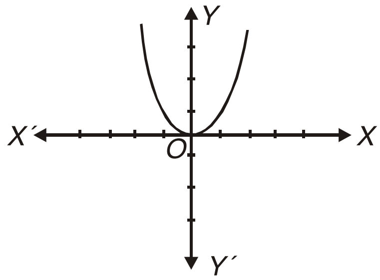

- Greatest Integer function, f : R → R; f(x) = [x], x ∈ R assumes the value of the greatest integer, less than or equal to x

$D_f = R$ $\qquad$ $R_f = Z$



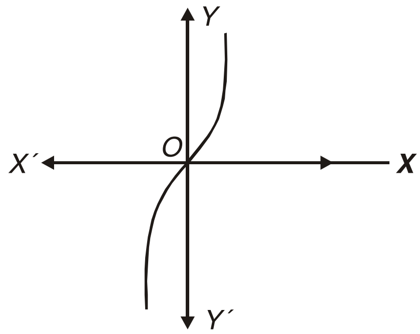

- $f : R \to R,\ f(x) = x^2$

  $D_f = R$ $\qquad$ $R_f = [0, \infty)$




- $f : R \to R,\ f(x) = x^3$

  $D_f = R$ $\qquad$ $R_f = R$




- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

  $(f \pm g)(x) = f(x) \pm g(x) \quad \forall x \in X$

  $(fg)(x) = f(x)\, g(x) \quad \forall x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \forall\ x \in X \text{ provided } g(x) \neq 0$$

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if (a – 1, b + 5) = (2, 3)

   If A = {1,3,5}, B = {2,3} find : (Question-2, 3)

2. $A \times B$

3. $B \times A$

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. $A \times (B \cap C)$

5. $A \times (B \cup C)$

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B

7. If A = {1,2,3,5} and B = {4,6,9},

   $R = \{(x, y) : |x - y| \text{ is odd}, x \in A, y \in B\}$

   Write R in roster form

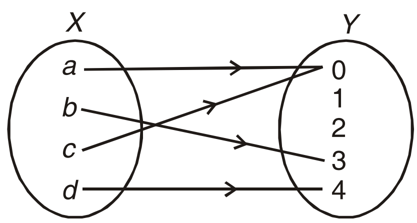
   Which of the following relations are functions. Give reason. (Questions 8 to 10)

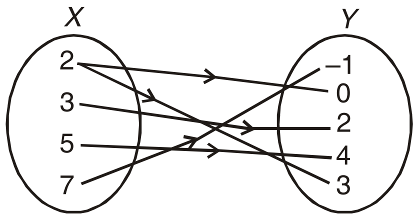
8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

    Which of the following arrow diagrams represent a function? Why? (Question- 11,12)

11. 

12. 


Let f and g be two real valued functions, defined by, $f(x) = x^2$, $g(x) = 3x + 2$, find : (Question 13 to 16)

13. $(f + g)(-2)$

14. $(f - g)(1)$

15. $(fg)(-1)$

16. $\left(\frac{f}{g}\right)(0)$

17. If $f(x) = x^3$, find the value of,

$$\frac{f(5) - f(1)}{5 - 1}$$

18. Find the domain of the real function,

$$f(x) = \sqrt{x^2 - 4}$$

19. Find the domain of the function, $f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$

Find the range of the following functions, (Question- 20,21)

20. $f(x) = \frac{1}{1 - x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,

$R = \{ (x, y) : x, y \in z, xy = 4\}$

Find the range of the following relations : (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R = \left\{\left(x, \frac{1}{x}\right) : x \in z, 0 < x < 6\right\}$

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

26. Let $R = \{ (x, y) : x, y \in N \text{ and } y = 2x\}$ be a relation on N. Find :

(i) Domain

(ii) Codomain

(iii) Range

Is this relation a function from N to N?

27. Let $f(x) = \begin{cases} x^2, \text{when } 0 \le x \le 2. \\ 2x, \text{ when } 2 \le x \le 5 \end{cases}$

$g(x) = \begin{cases} x^2, \text{when } 0 \le x \le 3. \\ 2x, \text{ when } 3 \le x \le 5 \end{cases}$

Show that f is a function while g is not a function.

28. Find the domain and range of,

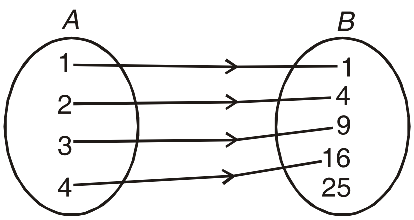
$f(x) = |2x - 3| - 3$

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant function, $f : R \to R; f(x) = 2 \ \forall x \in R$. Also find its domain and range.

## ANSWERS

1. $a = 3, b = -2$
2. $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$
3. $B \times A = \{ (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$
4. $\{(1,4), (2,4)\}$
5. $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$
6. $2^6 = 64$
7. $R = \{ (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$
8. Not a function
9. Not a function
10. Function
11. Function
12. Not a function
13. 0
14. –4
15. –1
16. 0
17. 31
18. $(-\infty, -2] \cup [2, \infty)$
19. $R - \{2,3\}$
20. $(-\infty, 0) \cup [1, \infty)$
21. $[2,\infty)$
22. $\{-4, -2, -1,1,2,4\}$
23. $\{2,4,6,8\}$
24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$
25. (a)




(b) $\{1,2,3,4\}$

(c) {1,4,9,16}

(d) {1,4,9,16,25}

26. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

28. Domain is R

Range is $[-3, \infty)$

# CHAPTER - 3

# TRIGONOMETRIC FUNCTIONS

## KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by $1^c$.
- $\pi$ radian = 180 degree

  1 radian = $\frac{180}{\pi}$ degree

  1 degree = $\frac{\pi}{180}$ radian
- If an arc of length $l$ makes an angle $\theta$ radian at the centre of a circle of radius r, we have

  $$\theta = \frac{l}{r}$$

- 

| Quadrant → | I | II | III | IV |
|---|---|---|---|---|
| t- functions which are positive | All | sin x<br>cosec x | tan x<br>cot x | cos x<br>sec x |

- 

| Function | –x | $\frac{\pi}{2} - x$ | $\frac{\pi}{2} + x$ | $\pi - x$ | $\pi + x$ | $2\pi - x$ | $2\pi + x$ |
|---|---|---|---|---|---|---|---|
| sin | –sin x | cos x | cos x | sin x | –sin x | –sin x | sin x |
| cos | cos x | sin x | –sin x | –cos x | –cos x | cos x | cos x |
| tan | –tan x | cot x | –cot x | –tan x | tan x | –tan x | tan x |
| cosec | –cosec x | sec x | sec x | cosec x | –cosec x | –cosec x | cosec x |
| sec | sec x | cosec x | –cosec x | –sec x | –sec x | sec x | sec x |
| cot | –cot x | tan x | –tan x | –cot x | cot x | –cot x | cot x |

| Function | Domain | Range |
|---|---|---|
| sin x | R | [–1,1] |
| cos x | R | [–1,1] |
| tan x | $R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$ | R |
| Cosec x | R – {nπ; n ∈ z} | R – (–1,1) |
| Sec x | $R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$ | R – (–1,1) |
| cot x | R – {nπ, n ∈ z} | R |

**Some Standard Results**

- sin (x + y) = sinx cosy + cosx siny

  cos (x + y) = cosx cosy – sinx siny

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x.\tan y}$$

  $$\cot(x+y) = \frac{\cot x.\cot y - 1}{\cot y + \cot x}$$

- sin (x – y) = sinx cosy – cosx siny

  cos (x – y) = cosx cosy + sinx siny

  $$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x.\tan y}$$

  $$\cot(x-y) = \frac{\cot x.\cot y + 1}{\cot y - \cot x}$$

- $$\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y.\tan z - \tan z \tan x}$$

- 2sinx cosy = sin(x + y) + sin(x – y)

  2cosx siny = sin(x + y) – sin(x – y)

  2cosx cosy = cos(x + y) + cos(x – y)

  2sinx siny = cos(x – y) – cos(x + y)

- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

  $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

  $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

  $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

- $\text{Sin } 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

- $\sin 3x = 3\sin x - 4\sin^3 x$

- $\cos 3x = 4\cos^3 x - 3\cos x$

- $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

- $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

  $= \cos^2 y - \cos^2 x$

- $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

  $= \cos^2 y - \sin^2 x$

- Principal solutions – The solutions of a trigonometric equation for which $0 \le x < 2\pi$ are called its principal solutions.

- General solution – A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.

General solutions of trigonometric equations :

$\sin\theta = 0 \Rightarrow \theta = n\pi,\ n \in z$

$\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in z$

$\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in z$

$\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha,\ n \in z$

$\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha,\ n \in z$

$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha,\ n \in z$

- Law of sines or sine formula

  The lengths of sides of a triangle are proportional to the sines of the angles opposite to them i.e..

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Law of cosines or cosine formula

  In any $\Delta$ ABC

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the radian measure corresponding to 5° 37' 30"

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

4. Find the value of $\tan\frac{19\pi}{3}$

5. Find the value of sin(–1125°)

6. Find the value of tan 15°

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find cos A

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of A + B.

9. Express sin 12θ + sin 4θ as the product of sines and cosines.

10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

11. Write the range of cosθ

12. What is domain of secθ ?

13. Find the principal solution of $\cot x = -\sqrt{3}$

14. Write the general solution of cos θ = 0

15. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$ find the value of cos 2x

16. If $\cos x = \frac{-1}{3}$ and x lies in quadrant III, find the value of $\sin\frac{x}{2}$

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

18. It the angles of a triangle are in the ratio 3:4:5, find the smallest angle in degrees and the greatest angle in radians.

19. If $\sin x = \frac{12}{13}$ and x lies in the second quadrant, show that secx + tanx = –5

20. If $\cot \alpha = \frac{1}{2}$, $\sec \beta = \frac{-5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, find the value of $\tan(\alpha + \beta)$

**Prove the following Identities**

21. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta$

22. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$

23. $\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x$

24. $\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\frac{\theta}{2}$

25. $\tan \alpha . \tan(60° - \alpha) . \tan(60° + \alpha) = \tan 3\alpha$

26. Show that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$

27. Show that $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

28. Pr ove that $\frac{\cos x}{1 - \sin x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

29. Draw the graph of cosx in $[0, 2\pi]$

**Find the general solution of the following equations (Q.No. 30 to Q. No. 33)**

30. $\cos\left(x + \frac{\pi}{10}\right) = 0$

31. $\sin 7x = \sin 3x$

32. $\sqrt{3}\cos x - \sin x = 1$

33. $3 \tan x + \cot x = 5 \operatorname{cosec} x$

34. In any triangle ABC, prove that

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$$

35. In any triangle ABC, prove that

$$a = b \cos C + c \cos B$$

36. In any triangle ABC, prove that

$$\frac{a+b}{c} = \frac{\cos \frac{A-B}{2}}{\sin \frac{C}{2}}$$

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

37. Prove that

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}$$

38. Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

39. Find the general solution of

$$\sin 2x + \sin 4x + \sin 6x = 0$$

40. Find the general solution of

$$\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$$

41. Draw the graph of tanx in $\left(\frac{-3\pi}{2}, \frac{3\pi}{2}\right)$

42. In any triangle ABC, prove that

$$\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{\left(a^2 - b^2\right)}{c^2} \sin 2C = 0$$

## ANSWERS

1. $\left(\frac{\pi}{32}\right)^c$

2. 39°22'30"

3. $\frac{5\pi}{12}$ cm

4. $\sqrt{3}$

5. $\frac{-1}{\sqrt{2}}$

6. $2 - \sqrt{3}$

7. $\frac{-4}{5}$

8. 45°

9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x - \sin 2x$

11. [–1,1]

12. $R - \left\{(2n+1)\frac{\pi}{2};\ n \in z\right\}$

13. $\frac{5\pi}{6}$

14. $(2n+1)\frac{\pi}{2}, n \in z$

15. $-\frac{1}{9}$

16. $\frac{\sqrt{6}}{3}$

17. 70 m

18. 45°, $\frac{5\pi}{12}$ radians

20 $\frac{2}{11}$

30. $\left(n\pi + \frac{2\pi}{5}\right), n \in z$

31. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, n \in z$

32. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, n \in z$

33. $2n\pi \pm \frac{\pi}{3}, n \in z$

39. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in z$

40. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$

# CHAPTER - 4

# PRINCIPLE OF MATHEMATICAL INDUCTION

## KEY POINTS

- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Principle of Mathematical Induction :

  Let P(n) be any statement involving natural number n such that

  (i) P(1) is true, and

  (ii) If P(k) is true implies that P(k +1) is also true for some natural number k

  then P(n) is true $\forall n \in N$

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Using the principle of mathematical induction prove the following for all $n \in N$ :

1. 3.6 + 6.9 + 9.12 + ......... + 3n (3n + 3) = 3n(n + 1)(n + 2)
2. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)---\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$
3. $n^2 + n$ is an even natural number.
4. $2^{3n} - 1$ is divisible by 7
5. $3^{2n}$ when divided by 8 leaves the remainder 1.

6. $4^n + 15n - 1$ is divisible by 9

7. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9.

8. $x^{2n-1} - 1$ is divisible by $x - 1$, $x \neq 1$

9. $3^n > n$

10. If x and y are any two distinct integers then $x^n - y^n$ is divisible by $(x - y)$

11. $n < 2^n$

12. $a + (a + d) + (a + 2d) + \dots\dots\dots + [a + (n - 1)d] = \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$

13. $3x + 6x + 9x + \dots\dots\dots$ to n terms $= \frac{3}{2}n\left(n + 1\right)x$

14. $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a positive integer

15. $11^{n+2} + 12^{2n+1}$ is divisible by 133.

## CHAPTER - 5

# COMPLEX NUMBERS AND QUADRATIC EQUATIONS

## KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form z = a + ib, where a, b $\in$ R is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id if a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$

  but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- $-z = -a + i(-b)$ is called the Additive Inverse or negative of z = a + ib
- $\bar{z} = a - ib$ is called the conjugate of z = a + ib

  $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of z = a + ib

  $(a \neq 0, b \neq 0)$

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane
- Polar form of $z = a + ib$ is,

  $z = r(\cos\theta + i\sin\theta)$ where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z,

  $\theta$ is called the argument or amplitude of z.
- The value of $\theta$ such that, $-\pi < \theta \leq \pi$ is called the principle argument of z.
- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 z_2| = |z_1| . |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $\left|z^n\right| = |z|^n$, $|z| = |\bar{z}| = |-z| = |-\bar{z}|$, $z\bar{z} = |z|^2$
- $|z_1 - z_2| \leq |z_1| + |z_2|$
- $|z_1 - z_2| \geq \left||z_1| - |z_2|\right|$
- For the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$, $a \neq 0$,

  if $b^2 - 4ac < 0$ then it will have complex roots given by,

$$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Evaluate, $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

2. Evaluate, $i^{29} + \frac{1}{i^{29}}$

3. Find values of x and y if,

   $(3x - 7) + 2iy = -5y + (5 + x)i$

4. Express $\frac{i}{1+i}$ in the form a + ib

5. If $z = \frac{1}{3+4i}$, find the conjugate of z

6. Find the modulus of z = 3 – 2i

7. If z is a purely imaginary number and lies on the positive direction of y-axis then what is the argument of z?

8. Find the multiplicative inverse of 5 + 3i

9. If |z| = 4 and argument of z = $\frac{5\pi}{6}$ then write z in the form x + iy; x, y $\in$R

10. If z = 1 – i, find $\text{Im}\left(\frac{1}{z\,\bar{z}}\right)$

11. Simplify (–i)(3 i) $\left(\frac{-1\,i}{6}\right)^3$

12. Find the solution of the equation $x^2 + 5 = 0$ in complex numbers.

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

13. For Complex numbers $z_1 = -1 + i$, $z_2 = 3 - 2i$

show that,

$$\text{Im}(z_1z_2) = \text{Re}(z_1)\,\text{Im}(z_2) + \text{Im}(z_1)\,\text{Re}(z_2)$$

14. Convert the complex number $-3\sqrt{2} + 3\sqrt{2}\,i$ in polar form

15. If $x + iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

16. Find real value of $\theta$ such that,

$$\frac{1 + i\cos\theta}{1 - 2i\cos\theta} \text{ is a real number}$$

17. If $\left|\frac{z-5i}{z+5i}\right| = 1$, show that z is a real number.

18. If $(x + iy)^{\frac{1}{3}} = a + ib$, prove that, $\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)$

19. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$

20. If $\left(\frac{2 + 2i}{2 - 2i}\right)^n = 1$, find the least positive integral value of n.

21. Find the modulus and argument of $z = 2 - 2i$

22. Solve the equation, $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

23. If $z_1$, $z_2$ are complex numbers such that, $\left|\frac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1$ then find $|z_1|$

24. Find the square root of $-3 + 4i$ and verify your answer.

25. If $x = -1 + i$ then find the value of $x^4 + 4x^3 + 4x^2 + 2$

## ANSWERS

| | |
|---|---|
| 1. 0 | 2. 0 |
| 3. $x = -1,\ y = 2$ | 4. $\frac{1}{2} + \frac{1}{2}i$ |
| 5. $\bar{z} = \frac{3}{25} + \frac{4i}{25}$ | 6. $\sqrt{13}$ |
| 7. $\frac{\pi}{2}$ | 8. $\frac{5}{34} - \frac{3i}{34}$ |
| 9. $z = -2\sqrt{3} + 2i$ | 10. 0 |
| 11. $\frac{i}{72}$ | 12. $x = \pm i\sqrt{5}$ |

14. $z = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

16. $\theta = (2n+1)\frac{\pi}{2}, n \in z$

17. **Hint :** use property $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

19. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

20. n = 4

21. modulus = $2\sqrt{2}$, argument $= \frac{-\pi}{4}$

22. $x = \frac{\sqrt{2} \pm i\sqrt{34}}{2\sqrt{3}}$

23. **Hint :** use $|z|^2 = z.\bar{z}$, $|z_1| = 3$

24. ± (1 + 2i)

25. 6

## CHAPTER - 6

# LINEAR INEQUALITIES

## KEY POINTS

- Two real numbers or two algebraic expressions related by the symbol '$<$', '$>$', '$\leq$' or '$\geq$' form an inequality.
- The inequalities of the form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$, $ax + b \leq 0$ ; $a \neq 0$ are called linear inequalities one variable x
- The inequalities of the form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$, $ax + by \leq 0$, $a \neq 0$, $b \neq 0$ are called linear inequalities in two variables x and y
- Rules for solving inequalities :

  (i) $a \geq b$ then $a \pm k \geq b \pm k$

  where k is any real number.

  (ii) but if $a \geq b$ then ka is not always $\geq$ kb.

  If $k > 0$ (i.e. positive) then $a \geq b \Rightarrow ka \geq kb$

  If $k < 0$ (i.e. negative) then $a \geq b \Rightarrow ka \leq kb$

- **Solution Set :** A solution of an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set of the inequality.
- The graph of the inequality $ax + by > c$ is one of the half planes and is called the solution region
- When the inequality involves the sign $\leq$ or $\geq$ then the points on the line are included in the solution region but if it has the sign $<$ or $>$ then the points on the line are not included in the solution region and it has to be drawn as a dotted line.

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Solve $5x < 24$ when $x \in N$
2. Solve $3x < 11$ when $x \in Z$
3. Solve $3 - 2x < 9$ when $x \in R$
4. Show the graph of the solution of $2x - 3 > x - 5$ on number line.
5. Solve $5x - 8 \geq 8$ graphically
6. Solve $\frac{1}{x-2} \leq 0$
7. Solve $0 < \frac{-x}{3} < 1$

**Write the solution in the form of intervals for $x \in R$. for Questions 8 to 10**

8. $\frac{2}{x-3} < 0$
9. $-3 \leq -3x + 2 < 4$
10. $3 + 2x > -4 - 3x$
11. Draw the graph of the solution set of $x + y \geq 4$.
12. Draw the graph of the solution set of $x \leq y$

## SHORT ANSWER TYPE QUSTIONS (4 MARKS)

**Solve the inequalities for real x**

13. $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$
14. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$

15. $-5 \leq \frac{2-3x}{4} \leq 9$

16. $|x - 2| \geq 5$

17. $|4 - x| + 1 < 3$

18. $\frac{3}{x-2} < 1$

19. $\frac{x}{x-5} > \frac{1}{2}$

20. $\frac{x+3}{x-2} > 0$

21. $x + 2 \leq 5$, $3x - 4 > -2 + x$

22. $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

23. The water acidity in a pool is considered normal when the average PH reading of three daily measurements is between 7.2 and 7.8. If the first two PH readings are 7.48 and 7.85, find the range of PH value for the third reading that will result in the acidity level being normal.

24. While drilling a hole in the earth, it was found that the temperature (T °C) at x km below the surface of the earth was given by

    $T = 30 + 25(x - 3)$, when $3 \leq x \leq 15$.

    Between which depths will the temperature be between 200°C and 300°C?

    Solve the following systems of inequalities graphically : (Questions 25, 26)

25. $x + y > 6$, $2x - y > 0$

26. $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

**Solve the system of inequalities for real x**

27. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ and

$$\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$$

**Solve the following system of inequalities graphically (Questions 28 to 30)**

28. $3x + 2y \le 24$, $x + 2y \le 16$, $x + y \le 10$, $x \ge 0$, $y \ge 0$

29. $2x + y \ge 4$, $x + y \le 3$, $2x - 3y \le 6$

30. $x + 2y \le 2000$, $x + y \le 1500$, $y \le 600$, $x \ge 0$, $y \ge 0$

## ANSWERS

1. {1,2,3,4}
2. {......, –2, –1, 0,1, 2, 3}
3. $x > -3$
6. $x < 2$
7. $-3 < x < 0$
8. $(-\infty, 3)$
9. $\left(\frac{-2}{3}, \frac{5}{3}\right]$
10. $\left(\frac{-7}{5}, \infty\right)$
13. $\left(-\infty, \frac{63}{10}\right]$
14. $\left(-\infty, \frac{-13}{2}\right)$
15. $\left[\frac{-34}{3}, \frac{22}{3}\right]$
16. $(-\infty, -3] \cup [7, \infty)$
17. $(2, 6)$
18. $(-\infty, 2) \cup (5, \infty)$
19. $(-\infty, -5) \cup (5, \infty)$
20. $(-\infty, -3) \cup (2, \infty)$
21. $(1, 3]$
22. $(5, \infty)$
23. Between 6.27 and 8.07
24. Between 9.8 m and 13.8 m
27. $(3, \infty)$

## CHAPTER - 7

# PERMUTATIONS AND COMBINATIONS

## KEY POINTS

- When a job (task) is performed in different ways then each way is called the permutation.
- **Fundamental Principle of Counting :** If a job can be performed in m different ways and for each such way, second job can be done in n different ways, then the two jobs (in order) can be completed in $m \times n$ ways.
- **Fundamental Principle of Addition :** If there are two events such that they can be performed independently in m and n ways respectively, then either of the two events can be performed in $(m + n)$ ways.
- The number of arrangements (permutations) of n different things taken r at a time is $^nP_r$ or $P(n, r)$
- The number of selections (Combinations) of n different things taken r at a time is $^nC_r$.
- $^nP_r = \dfrac{n!}{(n-r)!}, \quad {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$
- No. of permutations of n things, taken all at a time, of which p are alike of one kind, q are alike of $2^{nd}$ kind such that $p + q = n$, is $\dfrac{n!}{p!\,q!}$
- $0! = 1$, $^nC_o = {}^nC_n = 1$
- $^nP_r = r!\,{}^nC_r$

## (1 MARK QUESTIONS)

1. Using the digits 1, 2, 3, 4, 5 how many 3 digit numbers (without repeating the digits) can be made?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If $\frac{1}{6!} + \frac{1}{8!} = \frac{x}{9!}$ find x

7. If ${}^nP_4 : {}^nP_2 = 12$, find n.

8. How many different words (with or without meaning) can be made using all the vowels at a time?

9. Using 1, 2, 3, 4, 5 how many numbers greater than 10000 can be made? (Repetition not allowed)

10. If ${}^nC_{12} = {}^nC_{13}$ then find the value of ${}^{25}C_n$.

11. In how many ways 4 boys can be choosen from 7 boys to make a committee?

12. How many different words can be formed by using all the letters of word SCHOOL?

13. In how many ways can the letters of the word PENCIL be arranged so that I is always next to L.

## (4 MARKS QUESTIONS)

14. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seat?

15. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

16. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

17. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

18. In how many ways 11 players can be choosen from 16 players so that 2 particular players are always excluded?

19. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

20. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be $182^{nd}$ word.

21. From a class of 15 students, 10 are to choosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

22. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

23. A polygon has 35 diagnals. Find the number of its sides.

    [**Hint :** Number of diagnals of n sided polygon is given by ${}^{n}C_{2} - n$]

24. How many different products can be obtained by multiplying two or more of the numbers 2, 3, 6, 7, 9?

25. Determine the number of 5 cards combinations out of a pack of 52 cards if atleast 3 out of 5 cards are ace cards?

26. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

## LONG ANSWER TYPE QUESTION (6 MARKS)

27. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

28. There are 15 points in a plane out of which 6 are in a straight line, then

   (a) How many different straight lines can be made?

   (b) How many triangles can be made?

   (c) How many quadrilaterals can be made?

29. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

   (i) No two girls sit together?

   (ii) All the girls never sit together?

30. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

31. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

   (a) 3 are red and 1 is black.

   (b) All 4 cards are from different suits.

   (c) Atleast 3 are face cards.

   (d) All 4 cards are of the same colour.

32. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

33. How many 5 letter words containing 3 vowels and 2 consonants can be formed using the letters of the word EQUATION so that 3 vowels always occur together?

## ANSWERS

1. 60
2. $\frac{9!}{2!}$
3. 100
4. 45
5. 120
6. 513

7. $n = 6$
8. 120
9. 120
10. 1
11. 35
12. 360
13. 120
14. $90 \times {}^{10}P_8$
15. 56
16. 350
17. 2772
18. 364
19. 36
20. PAANVR
21. ${}^{13}C_{10} + {}^{13}C_8$
22. 5040
23. 10
24. 26
25. 4560
26. 576
27. 420
28. (a) 91

    (b) 435

    (c) Hint : For quadrilaterals selection of 4 points can be done like this, ${}^{9}C_4 + {}^{9}C_2 \times {}^{6}C_2 + {}^{9}C_3 \times {}^{6}C_1$
29. (i) $7! \times {}^{8}P_5$ (ii) $12! - 8! \times 5!$
30. 24480
31. ${}^{52}C_4$

    (a) ${}^{26}C_1 \times {}^{26}C_3$

    (b) $(13)^4$

    (c) 9295 (Hint : Face cards : 4J + 4K + 4Q)

    (d) $2 \times {}^{26}C_4$

32. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different

(ii) 2 are identical 1 different

(iii) All are identical, then form the words.)

33. 1080

## CHAPTER - 8

# BINOMIAL THEOREM

## KEY POINTS

- $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + --- + {}^nC_n b^n$

  $= \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r, n \in N$

- $T_{r+1}$ = General term

  $= {}^nC_r a^{n-r} b^r \quad 0 \leq r \leq n$

- Total number of terms in $(a + b)^n$ is $(n + 1)$

- If n is even, then in the expansion of $(a + b)^n$, middle term is $\left(\frac{n}{2}+1\right)^{th}$ term i.e. $\left(\frac{n+2}{2}\right)^{th}$ term.

- If n is odd, then in the expansion of $(a + b)^n$, middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms

- In $(a + b)^n$, $r^{th}$ term from the end is same as $(n - r + 2)^{th}$ term from the beginning.

- $r^{th}$ term from the end in $(a + b)^n$

  = $r^{th}$ term from the beginning in $(b + a)^n$

- In $(1 + x)^n$, coefficient of $x^r$ is ${}^nC_r$

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Compute $(98)^2$, using binomial theorem.

2. Expand $\left(x - \frac{1}{x}\right)^3$ using binomial theorem.

3. Write number of terms in the expansion of $(1 + 2x + x^2)^{10}$.

4. Write number of terms in $(2a - b)^{15}$

5. Simplify :

$$\frac{{}^nC_r}{{}^nC_{r-1}}$$

6. Write value of

$${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$$

[**Hint :** Use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

7. In the expansion, $(1 + x)^{14}$, write the coefficient of $x^{12}$

8. Find the sum of the coefficients in $(x + y)^8$

[**Hint :** Put x = 1, y = 1]

9. If ${}^nC_{n-3} = 120$, find n.

[**Hint :** Express 720 as the product of 3 consecutive positive integers]

10. In $\left(\frac{x}{2} - \frac{2}{x}\right)^8$, write $5^{th}$ term.

## SHORT ANSWER TYPE QUESTION (4 MARKS)

11. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

12. In $\left(3x^2 - \frac{1}{x}\right)^{18}$, which term contains $x^{12}$?

13. In $\left(2x - \frac{1}{x^2}\right)^{15}$, find the term independent of x.

14. Evaluate : $\left(\sqrt{2} + 1\right)^5 - \left(\sqrt{2} - 1\right)^5$ using binomial theorem.

15. Evaluate $(0.9)^4$ using binomial theorem.

16. Prove that if n is odd, then $a^n + b^n$ is divisible by a + b.

    [**Hint :** $a^n = (a + b - b)^n$. Now use binomial theorem]

17. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of $x^6$ and $x^4$.

18. In $\left(2x - \frac{3}{x}\right)^8$, find $7^{th}$ term from end.

19. In $\left(2x^3 - \frac{1}{x^2}\right)^{12}$, find the coefficient of $x^{11}$.

20. Find the coefficient of $x^4$ in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

21. Using binomial theorem, show that

    $3^{2n+2} - 8n - 9$ is divisible by 8.

    [**Hint :** $3^{2n+2} = 9\left(3^2\right)^n = 9(1 + 8)^n$, Now use binomial theorem.]

22. Prove that,

$$\sum_{r=0}^{20} {}^{20}C_{20-r} (2 - t)^{20-r} (t - 1)^r = 1$$

23. Find the middle term(s) in $\left(x - \frac{1}{x}\right)^8$

24. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

25. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

26. Show that the coefficient of $x^5$ in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

27. If the $3^{rd}$, $4^{th}$ and $5^{th}$ terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n

28. In the expansion of $(1 - x)^{2n-1}$, find the sum of coefficients of $x^{r-1}$ and $x^{2n-r}$

29. If the coefficients of $x^7$ in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and $x^{-7}$ in $\left(ax - \frac{1}{bx^2}\right)^{11}$ are equal, then show that ab = 1

## ANSWERS

| | | | |
|---|---|---|---|
| 1. | 9604 | 2. | $x^3 - \frac{1}{x^3} - 3x + \frac{3}{x}$ |
| 3. | 21 | 4. | 16 |
| 5. | $\frac{n - r + 1}{r}$ | 6. | ${}^{2n+1}C_7$ |
| 7. | 91 | 8. | 256 |
| 9. | n = 10 | 10. | 70 |
| 11. | a = 3, b = 2, n = 3 | 12. | $9^{th}$ term |
| 13. | $-2^{10} \times {}^{15}C_5$ | 14. | 82 |
| 15. | 0.6561 | 17. | 28 |
| 18. | $16128\ x^4$ | 19. | –101376 |
| 20. | 10 | 23. | 70 |
| 27. | a = 2, x = 1, n = 7 | 28. | 0 |

## CHAPTER - 9

# SEQUENCES AND SERIES

## KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers.
- A sequence whose range is a subset of R is called a real sequence.
- General A.P. is,

  a, a + d, a + 2d, ...........
- $a_n = a + (n - 1)d = n^{th}$ term
- $S_n$ = Sum of first n terms of A.P.

  $= \frac{n}{2}\left[a + l\right]$ where $l$ = last term.

  $= \frac{n}{2}\left[2a + \left(n - 1\right)d\right]$
- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P., ak, bk, ck are in A.P., k ≠ 0
- Three numbers in A.P.

  a – d, a, a + d
- Arithmetic mean between a and b is $\frac{a + b}{2}$.
- If $A_1$, $A_2$, $A_3$, ........$A_n$ are inserted between a and b, such that the resulting sequence is A.P. then,

  $A_n = a + n\left(\frac{b - a}{n + 1}\right)$

- $a_m = n, a_n = m \Rightarrow a_r = m + n - r$
- $S_m = S_n \Rightarrow S_{m+n} = 0$
- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
- In an A.P., the sum of the terms equidistant from the beginning and from the end is always equal, and equal to the sum of first and last term
- G.P. (Geometrical Progression)

  $a, ar, ar^2, \ldots\ldots\ldots$ (General G.P.)

  $a_n = ar^{n-1}$

  $$S_n = \frac{a(r^n - 1)}{r - 1}, \qquad r \neq 1$$

- Geometric mean between a and b is $\sqrt{ab}$
- If $G_1, G_2, G_3, \ldots\ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}, \ 1 \leq k \leq n$$

- In a G.P., the product of the terms equidistant from beginning and from end is always a constant and equal to the product of first and last term.
- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$
- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$
- $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

## VERY SHORT ANSWER TYPE QUESTION (1 MARK)

1. If $n^{th}$ term of an A.P. is $6n - 7$ then write its $50^{th}$ term.

2. If $S_n = 3n^2 + 2n$, then write $a_2$

3. Which term of the sequence,

   3, 10, 17, ........... is 136?

4. If in an A.P. $7^{th}$ term is 9 and $9^{th}$ term is 7, then find $16^{th}$ term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its $n^{th}$ term.

6. Which term of the G.P.,

   $2, 1, \frac{1}{2}, \frac{1}{4}$, .................. is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.

8. In G.P. $2, 2\sqrt{2}, 4$, ........., $128\sqrt{2}$, find the $4^{th}$ term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}$, ......

11. Find the value of $5^{1/2}, 5^{1/4}, 5^{1/8}$ ....... upto infinity.

12. Write the value of $0.\overline{3}$

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. Write the $n^{th}$ term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \ldots\ldots$

15. Find $S_n$ of the series whose $n^{th}$ term is $2^n + 3$.

16. In an infinite G.P., every term is equal to the sum of all terms that follow it. Find r

17. In an A.P.,

    8, 11, 14, .......... find $S_n - S_{n-1}$

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

18. Write the first negative term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, .......

19. Determine the number of terms in A.P. 3, 7, 11, ........ 407. Also, find its $11^{th}$ term from the end.

20. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

21. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

22. Find the sum of the sequence,

$$-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ----, \frac{10}{3}$$

23. If in an A.P. $\frac{a_7}{a_{10}} = \frac{5}{7}$ find $\frac{a_4}{a_7}$

24. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

25. Solve : 1 + 6 + 11 + 16 + .......... + x = 148

26. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their $10^{th}$ terms.

27. If the $I^{st}$, $2^{nd}$ and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

28. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

29. Prove that the sum of n numbers inserted between a and b such that the series becomes A.P. is $\frac{n(a+b)}{2}$

30. Insert 5 numbers between 7 and 55 , so that resulting series is A.P.

31. Find the sum of first n terms of the series, 0.7 + 0.77 + 0.777 + ......

32. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

33. Prove that, $0.03\bar{1} = \frac{7}{225}$

[**Hint :** $0.03\bar{1}$ = 0.03 + 0.001 + 0.0001 + ........ Now use infinite G.P.]

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

34. If $A = 1 + r^a + r^{2a} + ...............$ up to infinity then express r in terms of a and A.

35. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.

36. If a, b, c are in G.P., then prove that

$$\frac{1}{a^2 - b^2} = \frac{1}{b^2 - c^2} - \frac{1}{b^2}$$

37. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

38. If a is A.M. of b and c and c, $G_1$, $G_2$, b are in G.P. then prove that

$$G_1^3 + G_2^3 = 2abc$$

39. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + .......... upto n terms.

40. Evaluate $\sum_{r=1}^{10} (2r - 1)^2$

## ANSWERS

1. 293
2. 11
3. $20^{th}$
4. 0

5. $4n + 5$

6. $12^{th}$

7. $\frac{9}{2}$

8. 64

9. 3

10. $20\left(1 - \frac{1}{2^8}\right)$

11. 5

12. $\frac{1}{3}$

13. $\frac{2}{3}$

14. $\frac{2n + 1}{(n + 6)(n + 10)^2}$

15. $2^{n+1} + 3n - 2$

16. $r = \frac{1}{2}$

17. $3n + 5$

18. $-\frac{1}{4}$

19. 102, 367

20. 33

21. 7999

22. $\frac{63}{2}$

23. $\frac{3}{5}$

24. 11

25. 36

26. 33 : 17

27. $\frac{(b + c - 2a)(a + c)}{2(b - a)}$

30. 15, 23, 31, 39, 47

31. $\frac{7}{81}\left(9n - 1 + 10^{-n}\right)$

32. $\frac{15}{7}\left(2^n - 1\right)$

34. $\left(\frac{A - 1}{A}\right)^{1/a}$

35. 450 $cm^2$

37. 16, 4

39. $\frac{n(n + 1)}{3}\left(48n^2 - 16n - 14\right)$

40. 1330

## CHAPTER - 10

# STRAIGHT LINES

- Slope or gradient of a line is defined as $m = \tan \theta$, $(\theta \neq 90°)$, where $\theta$ is angle which the line makes with positive direction of x-axis measured in anticlockwise direction, $0 \leq \theta < 180°$
- Slope of x-axis is zero and slope of y-axis is not defined.
- Slope of a line through given points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $\frac{y_2 - y_1}{x_2 - x_1}$
- Two lines are parallel to each other if and only if their slopes are equal.
- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.
- Acute angle $\alpha$ between two lines, whose slopes are $m_1$ and $m_2$ is given by $\tan \alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$
- $x = a$ is a line parallel to y-axis at a distance of a units from y-axis. $x = a$ lies on right or left of y-axis according as a is positive or negative.
- $y = b$ is a line parallel to x-axis at a distance of 'b' units from x-axis. $y=b$ lies above or below x-axis, according as b is positive or negative.
- Equation of a line passing through given point $(x_1, y_1)$ and having slope m is given by

  $$y - y_1 = m(x - x_1)$$

- Equation of a line passing through given points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Equation of a line having slope m and y-intercept c is given by

  $$y = mx + c$$

- Equation of line having intercepts a and b on x and y-axis respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Equation of line in normal form is given by x cos$\alpha$ + y sin$\alpha$ = p,

  p = Length of perpendicular segment from origin to the line

  $\alpha$ = Angle which the perpendicular segment makes with positive direction of x-axis

- Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non zero.

- Distance of a point ($x_1$, $y_1$) from line Ax + By + C = 0 is given by

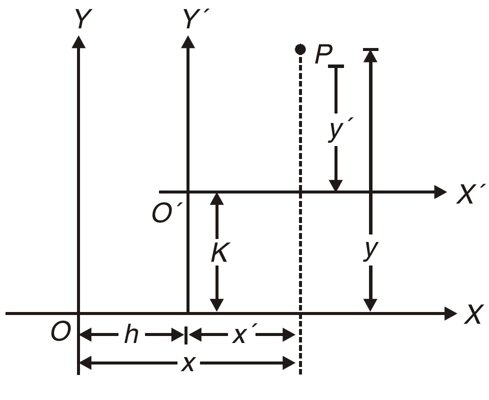
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines Ax + By + $C_1$ = 0 and Ax + By + $C_2$ = 0 is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

- Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

  Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



Let O'X' and O'Y' be drawn parallel to and in same direction as OX and OY respectively. Let coordinates of P referred to new axes O'X' and O'Y' be (x', y') then $x = x' + h$, $y = y' + k$

or $$x' = x - h, \quad y' = y - k$$

Thus

(i) The point whose coordinates were (x, y) has now coordinates (x − h, y − k) when origin is shifted to (h, k).

(ii) Coordinates of old origin referred to new axes are (–h, –k).

- Equation of family of lines parallel to $Ax + By + C = 0$ is given by $Ax + By + k = 0$, for different real values of k
- Equation of family of lines perpendicular to $Ax + By + C = 0$ is given by $Bx - Ay + k = 0$, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

## VERY SHORT ANSWER TYPE QUESTIONS

1. Three consecutive vertices of a parallegram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.
2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
3. The mid point of the segment joining (a, b) and (–3, 4b) is (2, 3a + 4). Find a and b.
4. Coordinates of centroid of $\Delta ABC$ are (1, –1). Vertices of $\Delta ABC$ are A(–5, 3), B(p, –1) and C(6, q). Find p and q.
5. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1) ?
6. What are the possible slopes of a line which makes equal angle with both axes?
7. Determine x so that slope of line through points (2, 7) and (x, 5) is 2.
8. Show that the points (a, 0), (0, b) and (3a – 2b) are collinear.

9. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

10. Find k so that the line $2x + ky - 9 = 0$ may be perpendicular to $2x + 3y - 1 = 0$

11. Find the acute angle between lines $x + y = 0$ and $y = 0$

12. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

13. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

14. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

## SHORT ANSWER TYPE QUESTIONS

15. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is the point (–1, –4), then find the value of p.

16. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$

17. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

18. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

19. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

20. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

21. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

22. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0).

23. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

24. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

## LONG ANSWER TYPE QUESTIONS

25. Find points on the line $x + y + 3 = 0$ that are at a distance of $\sqrt{5}$ units from the line $x + 2y + 2 = 0$

26. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

27. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

28. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

29. Find the equations of the straight lines which cut off intercepts on x–axis twice that on y–axis and are at a unit distance from origin.

30. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

## ANSWERS

| | |
|---|---|
| 1. (1, 2) | 2. $k = 3$ |
| 3. $a = 7, b = 10$ | 4. $p = 2, q = -5$ |
| 5. 3 : 2 (internally) | 6. $\pm 1$ |
| 7. 1 | 9. $x + y = 7$ |
| 10. $\frac{-4}{3}$ | 11. $\frac{\pi}{4}$ |

12. $\frac{2\pi}{3}$

13. (–3, –1)

14. p = –3, q = –3

15. 1

16. $\frac{10}{\sqrt{26}}$

17. (1, –5)

18. c = –4, (2,0), (4, 4)

19. 49 square units

20. $x + y + 5\sqrt{2} = 0$, $x + y - 5\sqrt{2} = 0$

21. 4 square units

22. (–4, –3)

23. $x + 2y = 1$

24. $x + y - 5 = 0$

25. (1, –4), (–9 , 6)

26. $3x - 4y + 15 = 0$

27. $4x + 7y - 11 = 0$, $7x - 4y + 25 = 0$

    $7x - 4y - 3 = 0$

28. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,

    $x - 2y + 13 = 0$, $2x + y - 4 = 0$

    $3x - y - 1 = 0$, $x + 3y - 17 = 0$

29. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$

30. $x = y$

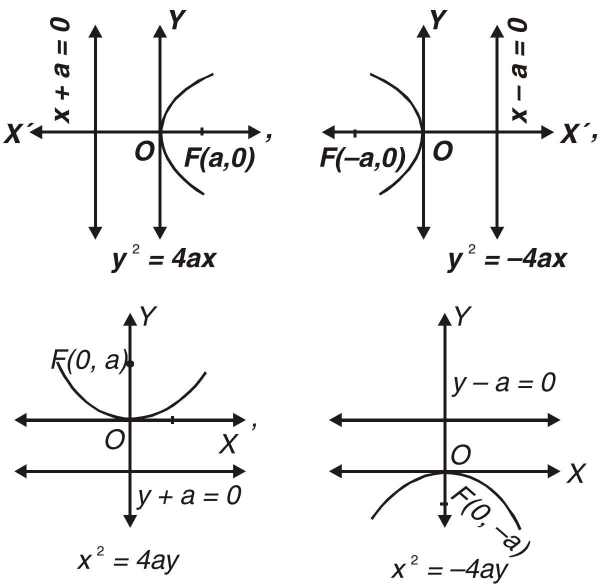
# CHAPTER - 11

# CONIC SECTIONS

## KEY POINTS

- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions
- **Circle :** It is the set of all points in a plane that are equidistant from a fixed point in that plane
- Equation of circle : $(x - h)^2 + (y - k)^2 = r^2$

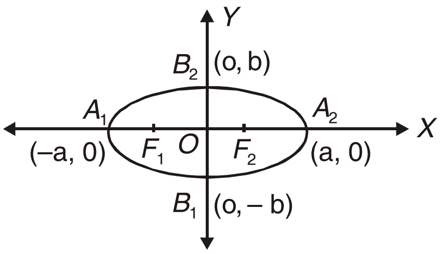
  Centre (h, k), radius = r
- **Parabola :** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line.

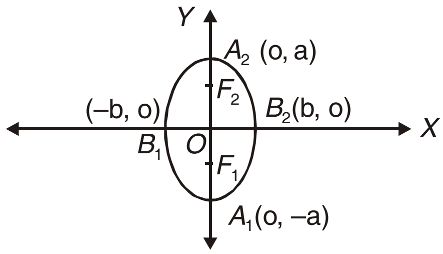


**Main facts about the parabola**

| Equation | $y^2 = 4ax$ (a > 0) Right hand | $y^2 = -4ax$ a > 0 Left hand | $x^2 = 4ay$ a > 0 Upwards | $x^2 = -4ay$ a > 0 Downwards |
|---|---|---|---|---|
| Axis | y = 0 | y = 0 | x = 0 | x = 0 |
| Directrix | x + a = 0 | x – a = 0 | y + a = 0 | y – a = 0 |
| Focus | (a, 0) | (–a, 0) | (0, a) | (0, –a) |
| Length of latus–rectum | 4a | 4a | 4a | 4a |
| Equation of latus–rectum | x – a = 0 | x + a = 0 | y – a = 0 | y + a = 0 |

- **Latus Rectum :** A chord through focus perpendicular to axis of parabola is called its latus rectum.
- **Ellipse :** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points




$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

a > b > 0, a> b > 0

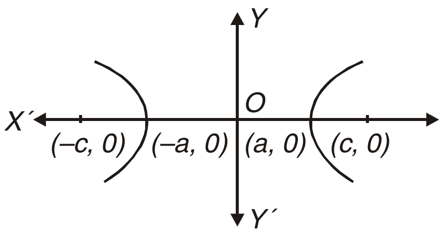
$C = \sqrt{a^2 - b^2}$

**Main facts about the ellipse**

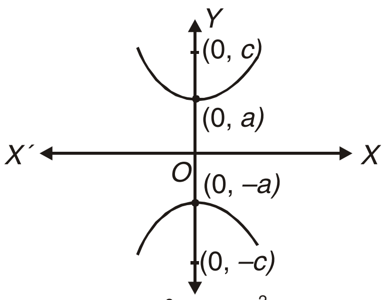
| Equation | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > 0, b > 0 | $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ a > 0, b > 0 |
|---|---|---|
| Centre | (0,0) | (0,0) |
| Major axis lies along | x–axis | y–axis |
| Length of major axis | 2a | 2a |

| | | |
|---|---|---|
| Length of minor axis | 2b | 2b |
| Foci | (–c, 0), (c, 0) | (0, –c),(0, c) |
| Vertices | (–a, 0), (a, 0) | (0, –a), (0, a) |
| Eccentricity e | $\frac{c}{a}$ | $\frac{c}{a}$ |
| Length of latus–rectum | $\frac{2b^2}{a}$ | $\frac{2b^2}{a}$ |

- **Latus rectum :** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola :** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.




$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$




$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c = \sqrt{a^2 + b^2}$$

**Main facts about the hyperbola**

| Equation | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, <br> $a > 0, b > 0$ | $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ <br> $a > 0, b > 0$ |
|---|---|---|
| Centre | (0,0) | (0,0) |
| Transverse axis lies along | x–axis | y–axis |
| Length of transverse axis | 2a | 2a |
| Length of conjugate axis | 2b | 2b |
| Foci | (–c, 0), (c, 0) | (0, –c),(0, c) |

| | | |
|---|---|---|
| Vertices | (–a, 0), (a, 0) | (0, –a), (0, a) |
| Eecentricity e | $\frac{c}{a}$ | $\frac{c}{a}$ |
| Length of latus–rectum | $\frac{2b^2}{a}$ | $\frac{2b^2}{a}$ |

- **Latus Rectum :** Chord through foci perpendicular to transverse axis is called latus rectum.

## VERY SHORT ANSWER TYPE QUESTIONS

1. Find the centre and radius of the circle

$$3x^2 + 3y^2 + 6x - 4y - 1 = 0$$

2. Does $2x^2 + 2y^2 + 3y + 10 = 0$ represent the equation of a circle? Justify.
3. Find equation of circle whose end points of one of its diameter are (–2, 3) and (0, –1).
4. Find the value(s) of p so that the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ may represent a circle of radius 5 units.
5. If parabola $y^2 = px$ passes through point (2, –3), find the length of latus rectum.
6. Find the coordinates of focus, and length of latus rectum of parabola $3y^2 = 8x$.
7. Find the eccentricity of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

## SHORT ANSWER TYPE QUESTIONS

8. One end of diameter of a circle $x^2 + y^2 - 6x + 5y - 7 = 0$ is (7, –8). Find the coordinates of other end.
9. Find the equation of the ellipse coordinates of whose foci are (±2, 0) and length of latus rectum is $\frac{10}{3}$.

10. Find the equation of ellipse with eccentricity $\frac{3}{4}$, centre at origin, foci on y-axis and passing through point (6, 4).

11. Find the equation of hyperbola with centre at origin, transverse axis along x-axis, eccentricity $\sqrt{5}$ and sum of lengths of whose axes is 18.

12. Two diameters of a circle are along the lines $x - y - 9 = 0$ and $x - 2y - 7 = 0$ and area of circle is 154 square units, find its equation.

13. Find equation(s) of circle passing through points (1,1), (2,2) and whose radius is 1 unit.

14. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

15. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

## LONG ANSWER TYPE QUESTIONS

16. Show that the four points (7,5), (6, –2) (–1,–1) and (0,6) are concyclic.

## ANSWERS

1. $\left(-1, \frac{2}{3}\right)$, $\frac{4}{3}$

2. No

3. $x^2 + y^2 + 2x - 2y - 3 = 0$ or $(x + 1)^2 + (y - 1)^2 = 5$

4. –3, +3

5. $\frac{9}{2}$

6. $\left(\frac{2}{3}, 0\right)$, $\frac{8}{3}$

7. $\frac{4}{5}$

8. (–1, 3)

9. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

10. $16x^2 + 7y^2 = 688$

11. $4x^2 - y^2 = 36$

12. $x^2 + y^2 - 22x - 4y + 76 = 0$

13. $x^2 + y^2 - 2x - 4y + 4 = 0$, $x^2 + y^2 - 4x - 2y + 4 = 0$

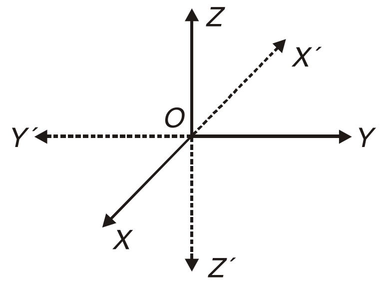
14. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

15. $x^2 + y^2 - 8x + 4y - 5 = 0$

## CHAPTER - 12

# INTRODUCTION TO THREE DIMENSIONAL COORDINATE GEOMETRY

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.




| | |
|---|---|
| Coordinate axes : | XOX', YOY', ZOZ' |
| Coordinate planes : | XOY, YOZ, ZOX or |
| | XY, YX, ZX planes |
| Octants : | OXYZ, OX'YZ, OXY'Z, OXYZ' |
| | OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z' |

- Coordinates of a point P are the perpendicular distances of P from three coordinate planes YZ, ZX and XY respectively.

- The distance between the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio $m_1 : m_2$

(i) internally, then the coordinates of R are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

(ii) externally, then coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

- Coordinates of centroid of a triangle whose vertices are $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ are

$$\left(\frac{x_1 + y_1 + z_1}{3}, \frac{x_2 + y_2 + z_2}{3}, \frac{x_3 + y_3 + z_3}{3}\right)$$

## VERY SHORT ANSWER TYPE QUESTIONS

1. Find image of (–2, 3, 5) in YZ plane.
2. Name the octant in which (–5, 4, –3) lies.
3. Find the distance of the point P(4, –3, 5) from XY plane.
4. Find the distance of point P(3, –2,1) from z–axis.
5. Write coordinates of foot of perpendicular from (3, 7, 9) on x axis.
6. Find the distance between points (2, 3, 4) and (–1, 3, –2).

## SHORT ANSWER TYPE QUESTIONS

7. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.
8. Find the point on y–axis which is equidistant from the point (3, 1, 2) and (5, 5, 2).
9. Find the coordinates of a points equidistant from four points (0,0,0), (2,0,0), (0,3,0) and (0,0,8).
10. The centroid of $\Delta$ABC is at (1,1,1). If coordiantes of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of points C.

11. If the extremities of diagonal of a square are (1,–2,3) and (2,–3,5) then find the length of the side of square.

12. Determine the point in XY plane which is equidistant from the points A (1, –1, 0) B(2, 1, 2) and C(3, 2, –1).

13. If the points A(1, 0, –6), B(–5, 9, 6) and C(–3, p, q) are collinear, find the value of p and q.

14. Show that the points A(3,3,3), B(0,6,3), C(1,7,7) and D(4,4,7) are the vertices of a square.

15. The coordinates of mid point of sides of $\Delta$ ABC are (–2, 3, 5), (4, –1, 7) and (6, 5, 3). Find the coordinates of vertices of $\Delta$ABC.

16. Find the coordinates of the point P which is five-sixth of the way from A(2, 3, –4) to B(8, –3, 2).

## ANSWERS

| | | | |
|---|---|---|---|
| 1. | (2,3,5) | 2. | OX' YZ' |
| 3. | 5 units | 4. | $\sqrt{13}$ units |
| 5. | (3,0,0) | 6. | $\sqrt{45}$ units |
| 8. | (0, 5, 0) | 9. | $\left(1, \frac{3}{2}, 4\right)$ |
| 10. | (1,1,2) | 11. | $\sqrt{3}$ units |
| 12. | $\left(\frac{3}{2}, 1, 0\right)$ | 13. | p = 6, q = 2 |
| 15. | $\left[\begin{array}{l}(0, 9, 1), \\ (-4, -3, 9), \\ (12, 1, 5)\end{array}\right.$ | 16. | (7,–2,1) |

## CHAPTER - 13

# LIMITS AND DERIVATIVES

## KEY POINTS

- $\lim_{x \to c} f(x) = l$ if and only if

  $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

- $\lim_{x \to c} \alpha = \alpha$, where $\alpha$ is a fixed real number.
- $\lim_{x \to c} x^n = c^n$, for all $n \in N$
- $\lim_{x \to c} f(x) = f(c)$, where $f(x)$ is a real polynomial in x.

**Algebra of limits**

Let f, g be two functions such that $\lim_{x \to c} f(x) = l$ and $\lim_{x \to c} g(x) = m$, then

- $\lim_{x \to c} [\alpha\, f(x)] = \alpha \lim_{x \to c} f(x)$

  $= \alpha\, l$ for all $\alpha \in R$

- $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$
- $\lim_{x \to c} [f(x).g(x)] = \lim_{x \to c} f(x). \lim_{x \to c} g(x) = lm$
- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{l}{m}$, $m \neq 0$ $g(x) \neq 0$

- $\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l}$ provided $l \neq 0$ $f(x) \neq 0$

- $\lim_{x \to c} [(f(x)]^n = \left[\left(\lim_{x \to c} f(x)\right)\right]^n = l^n$, for all $n \in N$

**Some important theorems on limits**

- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$ where x is measured in radians.

- $\lim_{x \to 0} \frac{\tan x}{x} = 1$

- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e^a$

- $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

**Derivative of a function at any point**

- A function f is said to have a derivative at any point x if it is defined in some neighbourhood of the point x and $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ exists.

  The value of this limit is called the derivative of f at any point x and is denoted by f'(x) i.e.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Algebra of derivatives :**

- $\frac{d}{dx}\left(cf(x)\right) = c.\frac{d}{dx}\left(f(x)\right)$ where c is a constant
- $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$
- $\frac{d}{dx}\left(f(x).g(x)\right) = f(x).\frac{d}{dx}\left(g(x)\right) + g(x)\frac{d}{dx}\left(f(x)\right)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}\left(f(x)\right) - f(x).\frac{d}{dx}\left(g(x)\right)}{\left(g(x)\right)^2}$

## 1 MARK QUESTIONS

**Evaluate the following Limits :**

1. $\lim_{x \to 3} \frac{\sqrt{2x+3}}{x+3}$

2. $\lim_{x \to 0} \frac{\sin 3x}{x}$

3. $\lim_{x \to 0} \frac{\tan^2 3x}{x^2}$

4. $\lim_{x \to 2} \left(x^2 - 5x + 1\right)$

**Differentiate the following functions with respect to x :**

5. $\frac{x}{2} + \frac{2}{x}$

6. $x^2 \tan x$

7. $\dfrac{x}{\sin x}$

8. $\log_x x$

9. $2^x$

10. If $f(x) = x^2 - 5x + 7$, find $f'(3)$

11. If $y = \sin x + \tan x$, find $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{3}$

## 4 MARKS QUESTIONS

12. If $f(x) = \begin{cases} 5x - 4, & 0 < x \le 1, \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$ show that $\lim\limits_{x \to 1} f(x)$ exists.

13. If $f(x) = \begin{cases} \dfrac{x - |x|}{x}, & x \ne 0, \\ 2, & x = 0 \end{cases}$ show that $\lim\limits_{x \to 0} f(x)$ does not exist.

14. Let f(x) be a function defined by

$$f(x) = \begin{cases} 4x - 5, & \text{If } x \le 2, \\ x - \lambda, & \text{If } x > 2, \end{cases}$$ Find $\lambda$, if $\lim\limits_{x \to 2} f(x)$ exists

**Evaluate the following Limits :**

15. $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3}$

16. $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

17. $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 + x} - \sqrt{1 - x}}$

18. $\lim\limits_{x \to a} \dfrac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$

19. $\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x - a}$

20. $\lim_{x \to 0} \frac{1 - \cos 2mx}{1 - \cos 2nx}$

21. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$

22. $\lim_{x \to 0} \frac{x \tan x}{1 - \cos x}$

23. $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \sin a}{x - a}$

24. $\lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a}$

25. $\lim_{x \to \pi} \frac{1 + \sec^3 x}{\tan^2 x}$

26. $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$

27. $\lim_{x \to 1} \frac{x - 1}{\log_e x}$

28. $\lim_{x \to e} \frac{\log x - 1}{x - e}$

29. $\lim_{x \to 4} \frac{3 - \sqrt{5 + x}}{1 - \sqrt{5 - x}}$

30. $\lim_{x \to a} \frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$

31. $\lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x}$

**Differentiate the following functions with respect to x from first-principles:**

32. $\sqrt{2x + 3}$

33. $\frac{x^2 + 1}{x}$

34. $e^x$

35. $\log x$

36. cosec x

37. cot x

38. $a^x$

**Differentiate the following functions with respect to x :**

39. $\frac{(3x + 1)\,(2\sqrt{x} - 1)}{\sqrt{x}}$

40. $\left(x - \frac{1}{\sqrt{x}}\right)^3$

41. $\left(x - \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)$

42. $\frac{\sin x - x\cos x}{x\sin x + \cos x}$

43. $x^3\, e^x \sin x$

44. $x^n \log_a x\; e^x$

45. $\frac{e^x + \log x}{\sin x}$

46. $\frac{1 + \log x}{1 - \log x}$

47. $e^x \sin x + x^n \cos x$

48. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that $2x\frac{dy}{dx} + y = 2\sqrt{x}$

49. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ find $\frac{dy}{dx}$

50. If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that

$$(2xy)\frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$$

## 6 MARKS QUESTIONS

**Differentiate the following functions with respect to x from first-principles:**

51. $\frac{\cos x}{x}$

52. $x^2 \sin x$

**Evaluate the following limits :**

53. $\lim_{x \to \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

54. $\lim_{x \to 0} \frac{\cos 2x - \cos 3x}{\cos 4x - 1}$

## ANSWERS

1. $\frac{1}{2}$

2. 3

3. 9

4. –5

5. $\frac{1}{2} - \frac{2}{x^2}$

6. $2x \tan x + x^2 \sec^2 x$

7. cosecx – x cotx cosecx

8. 0

9. $2^x \log_e 2$

10. 1

11. $\frac{9}{2}$

14. $\lambda = -1$

15. $\frac{1}{2}$

16. $\frac{1}{2\sqrt{2}}$

17. 1

18. $\frac{5}{2}a^{\frac{3}{7}}$

19. $\frac{5}{2}(a+2)^{\frac{3}{2}}$

20. $\frac{m^2}{n^2}$

21. $\frac{1}{2}$

22. 2

23. cosa

24. $\sin^3 a$

25. $-\frac{3}{2}$

26. 2

27. 1

28. $\frac{1}{e}$

29. $-\frac{1}{3}$

30. $\frac{2}{3\sqrt{3}}$

31. 2 cos2

32. $\frac{1}{\sqrt{2x+3}}$

33. $\frac{x^2-1}{x^2}$

34. $e^x$

35. $\frac{1}{x}$

36. – cosecx. cotx

37. $-\operatorname{cosec}^2 x$

38. $a^x \log_e a$

39. $6-\frac{3}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$

40. $3x^2+\frac{3}{2x^{5/2}}-\frac{9}{2}\sqrt{x}$

41. $3x^2+\frac{1}{x^2}-1-\frac{3}{x^4}$

42. $\frac{x^2}{(x\sin x+\cos x)^2}$

43. $x^2e^x(3\sin x+x\sin x+x\cos x)$

44. $e^x x^{n-1}\{n\log_a x+\log a+x\log_a x\}$

45. $\dfrac{\left(e^{x}+\dfrac{1}{x}\right)\sin x-\left(e^{x}+\log x\right)\cos x}{\sin^{2} x}$

46. $\dfrac{2}{x\,(1-\log x)^{2}}$

47. $e^{x}\left(1+\dfrac{1}{x}+x+\log x\right)$

49. $\sec^{2}x$

51. $\dfrac{-(x\sin x+\cos x)}{x^{2}}$

52. $2x\sin x + x^{2}\cos x$

53. –3

54. $-\dfrac{5}{16}$

## CHAPTER – 14

# MATHEMATICAL REASONING

## KEY POINTS

- A sentence is called a statement if it is either true or false but not both.
- The denial of a statement p is called its negative and is written as ~p and read as not p.
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If–then', 'only if' 'If and only if' etc are connecting words, which are used to form a compound statement.
- Compound statement with **'And'** is
  - (a) true if all its component statements are true
  - (b) false if any of its component statement is false
- Compound statement with **'Or'** is
  - (a) true when at least one component statement is true
  - (b) false when any of its component statement is false.
- A statement with **"If p then q"** can be rewritten as
  - (a) p implies q
  - (b) p is sufficient condition for q
  - (c) q is necessary condition for p
  - (d) p only if q
  - (e) (~q) implies (~p)

- Contrapositive of the statement p $\Rightarrow$ q is the statement ~ q $\Rightarrow$ ~ p
- Converse of the statement p $\Rightarrow$ q is the statement q $\Rightarrow$ p
- "For all", "For every" are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

**Identify which of the following are statements (Q. No 1 to 7)**

1. Prime factors of 6 are 2 and 3.
2. $x^2 + 6x + 3 = 0$
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.
6. Tomorrow is a holiday.
7. Answer this question.

**Write negation of the following statements (Q. No 8 to 12)**

8. All men are mortal.
9. $\pi$ is not a rational number.
10. Every one in Spain speaks Spanish.
11. Zero is a positive number.

**Write the component statements of the following compound statements**

12. 7 is both odd and prime number.
13. All integers are positive or negative.
14. 36 is a multiple of 4, 6 and 12.
15. Jack and Jill went up the hill.

**Identify the type 'Or' (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)**

16. Students can take French or Spainish as their third language.
17. To enter in a country you need a visa or citizenship card.
18. $\sqrt{2}$ is a rational number or an irrational number.
19. 125 is a multiple of 5 or 8.

**Which of the following statements are true or false. Give Reason. (Question No. 20 to 23)**

20. 48 is a multiple of 6, 7 and 8
21. $\pi > 2$ and $\pi < 3$.
22. Earth is flat or it revolves around the moon.
23. $\sqrt{2}$ is a rational number or an irrational number.

**Identify the quantifiers in the following statements (Q. No. 24 to 26)**

24. For every integer p, $\sqrt{p}$ is a real number.
25. There exists a capital for every country in the world.
26. There exists a number which is equal to its square.

**Write the converse of the following statements (Q. No. 27 to 30)**

27. If a number x is even then $x^2$ is also even.
28. If $3 \times 7 = 21$ then $3 + 7 = 10$
29. If n is a prime number then n is odd.
30. Some thing is cold implies that it has low temperature.

**Write contrapositive of the following statements (Q. No. 31 and 32)**

31. If $5 > 7$ then $6 > 7$.
32. x is even number implies that $x^2$ is divisible by 4.

33. Check the validity of the statement 'An integer x is even if and only if $x^2$ is even.

## ANSWERS

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. Not a Statement
7. Not a statement
8. All men are not mortal
9. $\pi$ is a rational number.
10. Everyone in Spain doesn't speak Spainish.
11. Zero is not a positive number.
12. 7 is an odd number. 7 is a prime number.
13. All integer are positive. All integers are negative.
14. 36 is a multiple of 4.

    36 is a multiple of 6.

    36 is a multiple of 12.
15. Jack went up the hill.

    Jill went up the hill.
16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. False
21. False
22. False
23. True
24. For every
25. For every, there exists
26. There exists
27. If $x^2$ is even then x is even
28. If $3 + 7 = 10$ then $3 \times 7 = 21$

29. If n is odd then n a prime number.

30. If some thing has low temperature then it is cold.

31. If $6 < 7$ then $5 < 7$

32. If $x^2$ is not divisible by 4 then x is not even.

33. Valid

# CHAPTER - 15

# STATISTICS

- Mean deviation for ungrouped data

$$\text{M.D.}(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

$$\text{M.D.}(M) = \frac{\sum |x_i - M|}{n}, \qquad M = \text{Median}$$

- Mean deviation for grouped data

$$\text{M.D.}(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$\text{M.D.}(M) = \frac{\sum f_i |x_i - M|}{N}$$

where $N = \Sigma f_i$

- Standard deviation is positive square root of variance.
- Variance and standard deviation for ungrouped data

$$\sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

- Variance and standard deviation of a discrete frequency distribution

$$\sigma^2 = \frac{1}{N}\sum f_i (x_i - \bar{x})^2, \ \sigma = \frac{1}{N}\sqrt{N \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

- Short cut method to find variance and standard deviation

$$\sigma^2 = \frac{h^2}{N^2}\left[N \sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$\sigma = \frac{h}{N}\sqrt{N \sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

where $y_i = \frac{x_i - A}{h}$

- Coefficient of variation $(C.V) = \frac{\sigma}{\bar{x}} \times 100, \ \bar{x} \neq 0$
- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes $k^2$ times of the original value and standard deviation becomes k times of the original value.
- If each observation is increased by k, where k is positive or negative, the variance and standard deviation remains same.
- Standard deviation is independent of choice of origin but depends on the scale of measurement.
- The series having higher coefficient of variation is called more variable than the other while the series having lesser C.V. is called more consistent or more stable.

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What is the mean deviation about the mean of the following data

   1, 3, 7, 9,10,12

2. What is the mean deviation about the median of the following data

   3, 6,11,12,18

3. The mean of 2, 7, p, 4, 6 and 8 is 7. What is the median of these observations.

4. What is the variance for the following data

   4, 8, 5, 2, 17, 6

5. What is the standard deviation for the following data 1, 3, 7, 9, 10

6. Coefficient of variation of two distributions are 70 and 75, and their standard deviations are 28 and 27 respectively. What are their arithmetic means?

7. Find mean of first ten multiples of 3.

8. Two plants X and Y of a factory perform as

| | X | Y |
|---|---|---|
| Number of workers | 5000 | 6000 |
| Average monthly wages | Rs. 2500 | Rs. 2550 |
| Variance of distribution of wages | 81 | 100 |

Which plant shows the greater variability in individual wages?

9. Write the variance of first n natural numbers.

10. If the sum of the squares of deviations of 10 observations taken from their mean is 2.5 then what is their standard derivation?

11. If each observation of raw data whose standard deviation is $\sigma$ is multiplied by a then what is the standard deviation of the new set of observations?

12. If a variable X takes values 0, 1, 2, ......... n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$ ......... ${}^nC_n$ then find variance of X.

13. If n = 10, $\overline{X}$ = 12 and $\sum x_i^2 = 1530$ then find coefficient of variation.

14.

| | *Factory A* | *Factory B* |
|---|---|---|
| Number of workers | 1000 | 1200 |
| Average monthly wages | Rs. 2500 | Rs. 2500 |
| Variance of distribution of wages | 100 | 159 |

In which factory A or B is there greater variability in individual wages?

15. What formula is used to compute the variance of a grouped or continuous frequency distribution?

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. Find mean deviation from the mean for the following data

| $x_i$ | 10 | 30 | 50 | 70 | 90 |
|---|---|---|---|---|---|
| $f_i$ | 4 | 24 | 28 | 16 | 8 |

17. Calculate the mean deviation about the meadian for the following data.

| $x_i$ | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
|---|---|---|---|---|---|---|---|---|---|
| $f_i$ | 15 | 20 | 32 | 35 | 35 | 22 | 20 | 10 | 8 |

18. Find standard deviation for the given data

| Age in years | 25 | 35 | 45 | 55 | 65 | 75 | 85 |
|---|---|---|---|---|---|---|---|
| Number of persons | 3 | 61 | 132 | 153 | 140 | 51 | 2 |

19. The mean and the standard deviation of 25 observations are 60 and 3. Later on it was decided to omit an observation which was wrongly recorded as 50. Find the mean and standard deviation of remaining 24 observations.

20. From the data given below, find which group is more consistent A or B?

| Marks | 15 | 25 | 35 | 45 | 55 | 65 |
|---|---|---|---|---|---|---|
| Group A | 10 | 16 | 30 | 40 | 26 | 18 |
| Group B | 22 | 18 | 32 | 34 | 18 | 16 |

21. Find the mean deviation from the mean for the following data

| $x_i$ | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| $f_i$ | 3 | 12 | 18 | 12 | 3 |

22. Find coefficient of variation for the following data

| $x_i$ | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 7 | 11 | 15 | 10 | 4 | 1 |

23. The mean of 5 observations is 6 and the standard derivation is 2. If the three observations are 5, 7 and 9 then find other two observations.

24. Calculate the possible values of x if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

## LONG ANSWER TYPE QUESTIONS (6 MARKS)

25. Find mean deviation about the median of the following data

| Marks | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 |
|---|---|---|---|---|---|---|
| Number of students | 8 | 10 | 10 | 16 | 4 | 2 |

26. Calculate the standard deviation for the following distribution giving the age distribution of persons.

| Age in years | 20–30 | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 | 80–90 |
|---|---|---|---|---|---|---|---|
| Number of persons | 3 | 61 | 132 | 153 | 140 | 51 | 2 |

27. Mean and standard deviation of data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find correct mean and standard deviation.

28. Find the coefficient of variation for the following data :

| Marks | 20–29 | 30–39 | 40–49 | 50–59 | 60–69 | 70–79 | 80–89 | 90–99 |
|---|---|---|---|---|---|---|---|---|
| Number of students | 5 | 12 | 15 | 20 | 18 | 10 | 6 | 4 |

29. Calculate mean deviation about median for the following frequency distribution of population of males in different age groups given below :

| Age group in years | 5–14 | 15–24 | 25–34 | 35–45 | 45–54 | 55–64 | 65–74 |
|---|---|---|---|---|---|---|---|
| Number of males (in lakhs) | 447 | 307 | 279 | 220 | 157 | 91 | 39 |

30. Find mean and variance of the following data by using step deviation method

| Class | 0–5 | 5–10 | 10–15 | 15–20 | 20–25 | 25–30 | 30–35 | 35–40 | 40–45 |
|---|---|---|---|---|---|---|---|---|---|
| Frequency | 20 | 24 | 32 | 28 | 20 | 11 | 26 | 15 | 24 |

## ANSWERS

1. $\frac{10}{3}$

2. 4.2

3. 6.5

4. $\frac{70}{3}$

5. $\sqrt{12}$

6. 40, 36

7. 16.5

8. Plant y

9. $\frac{n^2 - 1}{12}$

10. 0.5

11. $|a| \sigma$

12. $\frac{n}{4}$

13. 25

14. Factory B

15. $\text{Var}(x) = h^2 \left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]$

16. 16

17. 1.493

18. 11.87

19. Mean 60.42, S.D = 2.24

20. A is more consistent as C.V is 32.73 compared to 39.38 for B

21. 1.48

22. 10.47

23. 3 and 6

24. 3, $\frac{7}{3}$, use $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$, we get $3x^2 - 16x + 21 = 0$

25. 11.44

26. 11.87

27. 6.5 and 2.5

28. 31.24

29. 14.14

30. 21.5 and 164.75

## CHAPTER - 16

# PROBABILITY

- **Random Experiment :** If an experiment has more than one possible out come and it is not possible to predict the outcome in advance then experiment is called random experiment.
- **Sample Space :** The collection of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.
- **Event :** A subset of the sample space associated with a random experiment is called an event.
- **Simple Event :** Simple event is a single possible outcome of an experiment.
- **Compound Event :** Compound event is the joint occurence of two or more simple events.
- **Sure Event :** If event is same as the sample space of the experiment, then event is called sure event.
- **Impossible Event :** Let S be the sample space of the experiment, $\phi \subset S$, $\phi$ is an event called impossible event.
- **Exhaustive and Mutually Exclusive Events :** Events $E_1$, $E_2$, $E_3$ ------ $E_n$ are mutually exclusive and exhaustive if

  $E_1 \cup E_2 \cup E_3$ ------- $\cup E_n = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$
- **Probability of an Event :** For a finite sample space S with equally likely outcomes, probability of an event A is $P(A) = \frac{n(A)}{n(S)}$, where n(A) is number of elements in A and n(S) is number of elements in set S.
- (a) If A and B are any two events then

  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b) If A and B are mutually exclusive events then

$P(A \cup B) = P(A) + P(B)$

(c) $P(A) + P(\overline{A}) = 1$

or $P(A) + P(\text{not } A) = 1$

- $P(A - B) = P(A) - P(A \cap B)$
- If $S = \{w_1, w_2, \ldots, w_n\}$ then

  (i) $0 \leq P(w_i) \leq 1$ for each $w_i \in S$

  (ii) $P(w_1) + p(w_2) + \ldots + p(w_n) = 1$

  (iii) $P(A) = \Sigma P(w_i)$ for any event A containing elementary events $w_i$.

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

**Describe the Sample Space for the following experiments (Q. No. 1 to 4)**

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up for the first time.
4. A coin is tossed. If it shows head we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Write an example of an impossible event.
6. Write an example of a sure event.
7. Three coins are tossed. Write three events which are mutually exclusive and exhaustive.
8. A coin is tossed n times. What is the number of elements in its sample space?

   If E, F and G are the subsets representing the events of a sample space S. What are the sets representing the following events? (Q No 9 to 12).

9. Out of three events atleast two events occur.

10. Out of three events only one occurs.

11. Out of three events only E occurs.

12. Out of three events exactly two events occur.

13. If probability of event A is 1 then what is the type of event 'not A'?

14. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?

15. What is the probability that a given two digit number is divisible by 15?

16. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?

17. If A and B are mutually exclusive events then what is the probability of $A \cap B$ ?

18. If A and B are mutually exclusive and exhaustive events then what is the probability of $A \cup B$?

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

19. The letters of the word EQUATION are arranged in a row. Find the probability that

    (i) all vowels are together

    (ii) the arrangement starts with a vowel and ends with a consonant.

20. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

21. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

22. Find the probability of atmost two tails or atleast two heads in a toss of three coins.

23. A, B and C are events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$ $P(A \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$ then prove that $P(B \cap C)$ lies in the interval [0.23, 0.48]

24. For a post three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is twice that of C. The post is filled. What are the probabilities of A, B and C being selected?

25. A and B are two candidates seeking admission is college. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Show that the probability of B being selected is atmost 0.8.

26. S = {1, 2, 3, -----, 30}, A = {x : x is multiple of 7} B = { x : x is multiple of 5}, C = {x : x is a multiple of 3}. If x is a member of S chosen at random find the probability that

(i) $x \in A \cup B$

(ii) $x \in B \cap C$

(iii) $x \in A \cap C'$

27. A number of 4 different digits is formed by using 1, 2, 3, 4, 5, 6, 7. Find the probability that it is divisible by 5.

28. A bag contains 5 red, 4 blue and an unknown number of m green balls. Two balls are drawn. If probability of both being green is $\frac{1}{7}$ find m.

## ANSWERS

1. {0,1,2,3}

2. {Red, Black}

3. {T, HT, HHT, HHHT.........}

4. $\{HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT\}$

5. Getting a number 8 when a die is rolled

6. Getting a number less then 7 when a die is rolled

7. A = {HHH, HHT, HTH, THH}

B = {HTT, THT, HTT}

C = {TTT}

8. $2^n$

9. $(E \cap F \cap G) \cup (E' \cap F \cap G) \cup (E \cap F' \cap G) \cap (E \cap F \cap G')$

10. $(E \cap F' \cap G) \cup (E' \cap F \cap G') \cup (E' \cap F' \cap G)$

11. $(E \cap F' \cap G')$

12. $(E \cap F \cap G') \cup (E \cap F' \cap G) \cup (E' \cap F \cap G)$

13. Impossible event

14. $\frac{8}{21}$

15. $\frac{1}{15}$

16. Mutually exclusive events.

17. 0

18. 1

19. (i) $\frac{1}{14}$ (ii) $\frac{15}{56}$

20. 32

21. $\frac{23}{28}$

22. $\frac{7}{8}$

23. $0.23 \leq P(B) \leq 0.48$

24. $\frac{3}{5}, \frac{3}{10}, \frac{1}{10}$

26. (i) $\frac{1}{3}$, (ii) $\frac{1}{15}$, (iii) $\frac{1}{10}$

27. $\frac{1}{7}$

28. 6

# MODEL TEST PAPER – I
# CLASS - XI

## MATHEMATICS

**Time : 3 hours** **Maximum Marks : 100**

***General Instructions :***

(i) All questions are compulsory.

(ii) Q. 1 to Q. 10 of Section A are of 1 mark each.

(iii) Q. 11 to Q. 22 of Section B are of 4 marks each.

(iv) Q. 23 to Q. 29 of Section C are of 6 marks each.

(v) There is no overall choice. However an internal choice has been provided in some questions.

### SECTION A

1. A = {1, 2, 3, 4, 5, 6}, B = {2, 3, 5, 7, 9}

   U = {1, 2, 3, 4, .....10}, Write $(A - B)'$

2. Express $(1 - 2i)^{-2}$ in the standard form a + ib.

3. Find $20^{th}$ term from end of the A.P. 3, 7, 11, .... 407.

4. Evaluate $5^2 + 6^2 + 7^2 + .... + 20^2$

5. Evaluate $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x}}{x}$

6. Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1 + x + x^2} - 1}{x}$

7. A bag contains 9 red, 7 white and 4 black balls. If two balls are drawn at random, find the probability that both balls are red.

8. What is the probability that an ordinary year has 53 Sundays?

9. Write the contrapositive of the following statement :

"it two lines are parallel, then they do not intersect in the same plane."

10. Check the validity of the compound statement "80 is a multiple of 5 and 4."

## SECTION B

11. Find the derivative of $\frac{\sin x}{x}$ with respect to x from first principle.

**OR**

Find the derivative of $\frac{\sin x - x \cos x}{x \sin x + \cos x}$ with respect to *x*.

12. Two students Ajay and Aman appeared in an interview. The probability that Ajay will qualify the interview is 0.16 and that Aman will quality the interview is 0.12. The probability that both will qualify is 0.04. Find the probability that—

    (a) Both Ajay and Aman will not qualify.

    (b) Only Aman qualifies.

13. Find domain and range of the real function $f(x) = \frac{3}{1 - x^2}$

14. Let R be a relation in set A = {1, 2, 3, 4, 5, 6, 7} defined as R = {(a, b): a divides b, a ≠ b}. Write R in Roster form and hence write its domain and range.

**OR**

Draw graph of f(x) = 2 + |x – 1|.

15. Solve : $\sin^2 x - \cos x = \frac{1}{4}$.

16. Prove that $\cos 2\theta . \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 5\theta \sin \frac{5\theta}{2}$.

17. If $x$ and $y$ are any two distinct integers, then prove by mathematical induction that $x^n - y^n$ is divisible by $(x - y)$ $\forall n \in N$.

18. If $x + iy = (a + ib)^{1/3}$, then show that $\frac{a}{x} + \frac{b}{y} = 4\left(x^2 - y^2\right)$

**OR**

Find the square roots of the complex number 7 – 24i

19. Find the equation of the circle passing through points (1, –2) and (4, –3) and has its centre on the line $3x + 4y = 7$.

**OR**

The foci of a hyperbola coincide with of the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Find the equation of the hyperbola, if its eccentricity is 2.

20. Find the coordinates of the point, at which yz plane divides the line segment joining points (4, 8, 10) and (6, 10, –8).

21. How many words can be made from the letters of the word 'Mathematics', in which all vowels are never together.

22. From a class of 20 students, 8 are to be chosen for an excusion party. There are two students who decide that either both of them will join or none of the two will join. In how many ways can they be choosen?

## SECTION C

23. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had taken

    (i) atleast one of the three subjects,

    (ii) only one of the three subjects.

24. Prove that $\cos^3 A + \cos^3\left(\frac{2\pi}{3} + A\right) + \cos^3\left(\frac{4\pi}{3} + A\right) = \frac{3}{4}\cos 3A.$

25. Solve the following system of inequations graphically

$$x + 2y \leq 40,\ 3x + y \geq 30,\ 4x + 3y \geq 60,\ x \geq 0,\ y \geq 0$$

**OR**

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

26. Find n, it the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left[\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right]^n$ is $\sqrt{6}$ : 1.

27. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio $\left(3 + 2\sqrt{2}\right) : \left(3 - 2\sqrt{2}\right)$.

28. Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

29. Calculate mean and standard deviation for the following data

| ***Age*** | ***Number of persons*** |
|---|---|
| 20 – 30 | 3 |
| 30 – 40 | 51 |
| 40 – 50 | 122 |
| 50 – 60 | 141 |
| 60 – 70 | 130 |
| 70 – 80 | 51 |
| 80 – 90 | 2 |

**OR**

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that an observation 12 was misread as 8. Calculate correct mean and correct standard deviation.

# ANSWERS

## SECTION A

1. {2, 3, 5, 7, 8, 9, 10}
2. $\frac{-3}{25} + \frac{4}{25} i$
3. 331
4. 2840
5. 2
6. $\frac{1}{2}$
7. $\frac{18}{95}$
8. $\frac{1}{7}$
9. "If two lines intersect in same plane then they are not parallel."
10. True

## SECTION B

11. $\frac{\cos x}{x} - \frac{\sin x}{x^2}$ or $\frac{x^2}{(x \sin x + \cos x)^2}$

12. 0.76, 0.08

13. Domain = R – {–1, 1}, Range = $(-\infty, 0) \cup [3. \infty)$

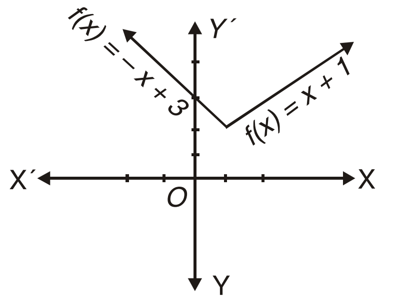
14. R = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 4) (2, 6), (3, 6)}

    Domain = {1, 2, 3}

    Range = {2, 3, 4, 5, 6, 7}

**OR**

$$f(x) = \begin{cases} x + 1 & \text{If } x \geq 1 \\ -x + 3 & \text{If } x < 1 \end{cases}$$




15. $x = 2n\pi \pm \frac{\pi}{3}, \quad n \in z$

18. $\pm (4 - 3i)$

19. $15x^2 + 15y^2 - 94x + 18y + 55 = 0$

**OR**

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

20. (0, 4, 46)

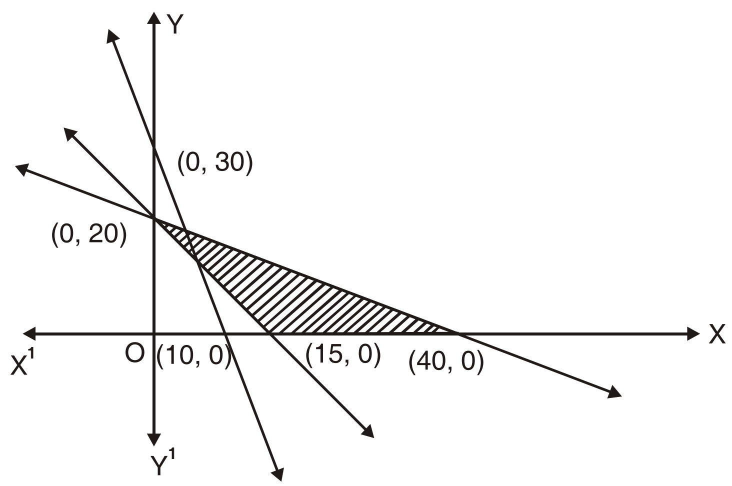
21. 4868640

22. 62322

## SECTION C

23. 2, 11

25.



**OR**

The number of litres of the 30% solution of acid must be more than 120 litres but less than 300 litres.

26. $n = 10$

27. $(-1, -4)$

29. 55.1, 11.87 OR 10.2, 1.99

# MODEL TEST PAPER – II
# CLASS - XI

## MATHEMATICS

**Time : 3 hours** **Maximum Marks : 100**

***General Instructions :***

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, B and C.

(iii) Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iv) There is no overall choice. However, an internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

### SECTION A

1. Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

2. Find the value of sin 1845°.

3. Write the negation of the following statement : 'Sum of 2 and 3 is 6'.

4. Write the converse of the statement : 'If the sum of digits of a number is divisible by 9 then the number is divisible by 9'.

5. Write the solution of $3x^2 - 4x + \frac{20}{3} = 0$.

6. Find the sum of the series

   $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) + ....$ to n terms.

7. A die is thrown. Find the probability of getting a number less than or equal to 6.

8. Five marbles are drawn from a bag which contains 7 blue marbles and 4 black marbles. What is the probability that all will be blue?

9. Find the general solution of $\cos 3\theta = -\frac{1}{2}$.

10. What is y-intercept of the line passing through the point (2, 2) and perpendicular to the line 3x + y = 3?

## SECTION B

11. Evaluate : $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$

**OR**

$$\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2}$$

12. Differentiate cot $x$ with respect to $x$ by the first principle.

13. Find the square root of –5 + 12 i

14. How many diagonals are there in a polygon with $n$ sides?

15. Prove the following by the principle of mathematical induction

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}, \ n \in N$$

**OR**

Using principle of mathematical induction prove that

$4^n + 15n - 1$ is divisible by 9 for all n $\in$ N.

16. Find the domain and range of $f(x) = \frac{1}{\sqrt{x - 5}}$

17. Find the value of $n$ so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

**OR**

Find the sum of the following series upto $n$ terms :

.6 + .66 + .666 + ......

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

19. Find the length of the axes, eccentricity and length of the latus-rectum of the hyperbola $25x^2 - 36y^2 = 225$.

**OR**

Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$.

20. Using section formula, prove that the three points (–4, 6, 10), (2, 4, 6) and (14, 0, –2) are collinear.

21. On her vacations Veena visits four cities (A, B, C, D) in a random order. What is the probability that she visits.

    (i) A before B?

    (ii) A before B and B before C?

22. Prove that

    $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$.

## SECTION C

23. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find :

    (i) How many read none of the three magazines?

    (ii) How many read magazine C only?

24. The $3^{rd}$, $4^{th}$ and $5^{th}$ terms in the expansion of $(x + a)^n$ are respectively 84, 280 and 560, find the values of $x$, $a$ and $n$.

**OR**

The coefficients of $(r - 1)^{th}$, $r^{th}$ and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find $n$ and $r$.

25. Find the sum of the following series upto $n$ terms :

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + ......$$

26. Prove that

$$\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$$

27. Solve the following system of inequalities graphically :

$$x + 2y \leq 10,\ x + y \geq 1,\ x - y \leq 0,\ x \geq 0,\ y \geq 0.$$

28. Find the general solution of

$$\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$$

**OR**

Prove that

$$\sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right) = -\frac{3}{4}\sin 3x$$

29. Find the mean deviation about the median for the following data :

| Marks | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 |
|---|---|---|---|---|---|---|
| Number of girls | 8 | 10 | 10 | 16 | 4 | 2 |

## ANSWERS

1. 16

2. $\frac{1}{\sqrt{2}}$

3. It is false that sum of 2 and 3 is 6.

4. If a number is divisible by 9 then the sum of the digits of the number is divisible by 9.

5. $\frac{2+4i}{3}$

6. $\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

7. 1

8. $\frac{1}{22}$

9. $\frac{2n\pi}{3} \pm \frac{2\pi}{9}$, $n \in z$.

10. $\frac{4}{3}$

11. $\frac{1}{2}$ or $\frac{b^2 - a^2}{2}$

12. $-\text{cosec}^2 x$

13. $\pm (2 + 3i)$

14. $\frac{n(n-3)}{2}$

16. $(5, \infty)$; $(0, \infty)$

17. $n = -\frac{1}{2}$ or $\frac{2n}{3} - \frac{2}{27}\left(1 - 10^{-n}\right)$

19. Length of transverse axis = 6, lengths of conjugate axis = 5, $e = \frac{\sqrt{61}}{6}$,

Length of latus rectum $= \frac{25}{6}$

**OR** $x^2 + y^2 + 4x - 2y = 0.$

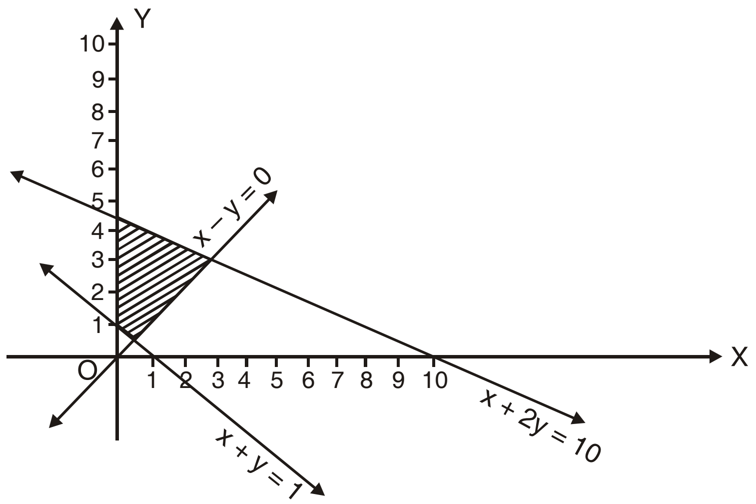
21. (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$

23. (i) 20 (ii) 30

24. $n = 7,\ a = 2,\ x = 1$ **OR** $n = 7$ and $r = 3.$

25. $\frac{n}{24}\left(2n^2 + 9n + 13\right)$

27.




28. $\theta = (2n + 1)\frac{\pi}{8},\ n\pi \pm \frac{\pi}{3},\ n \in z.$

29. 11.44