Class: XII | Mathematics | WORKSHEET

MATRICES & DETERMINANTS

1. If $0 < x < \pi$, and the matrix $\begin{bmatrix} 2\cos x & 1 \\ 3 & 2\cos x \end{bmatrix}$ is singular , find x.
2. Determine the values of x for which the matrix $\begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$ is singular.
3. If A(x_1, y_1) , B(x_2, y_2) and C(x_3, y_3) are the vertices of an equilateral triangle with each side equal to a units, then prove that $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$
4. Using determinants, determine whether the points (a,b+c) , (b,c+a) and (c,a+b) form a triangle or not.
5. If the points (a,0) , (0,b) and (1,1) are collinear , prove that a + b = ab
6. If A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$
7. Show that the matrix A = $\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$ & hence find A^{-1}.
8. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}
9. Find a 2×2 matrix A such that $A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
10. Find the matrix X satisfying the equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} .X. \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
11. Show that $\begin{bmatrix} 1 & -\tan(\theta/2) \\ \tan(\theta/2) & 1 \end{bmatrix} \times \begin{bmatrix} 1 & \tan(\theta/2) \\ -\tan(\theta/2) & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
12. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ such that $A^2 = \lambda A - 2I$. Hence find A^{-1}
13. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the numbers a and b such that $A^2 + aA + bI = 0$. Hence find A^{-1}

14. Use product $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ to solve the equations

$x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$.

15. Using matrices, solve: $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$, $\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$, $\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

16. If $\begin{bmatrix} -1 & 3 & 4 \\ 5 & -1 & 2 \end{bmatrix}$ is additive inverse of $\begin{bmatrix} 2x & -3 & y \\ x+t & -z & 2z \end{bmatrix}$. Find x, y, z and t

17. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. Show that A^2 = $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

18. If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - xA + yI = 0$. Find the value of x and y

19. Express $\begin{bmatrix} 6 & 1 & 5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{bmatrix}$ as the sum of skew and skew symmetric matrices.

20. $\begin{bmatrix} 0 & x+2 & 2-x \\ 1-2x & 0 & 2x-1 \\ 3x-8 & x-8 & 0 \end{bmatrix}$ is a skew symmetric, find value of x.

21. Prove that value of the determent $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

22. Solve for x. $\begin{vmatrix} -x & -6 & -1 \\ -2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$.

23. Find value of x, If matrix A is not invertible. A= $\begin{vmatrix} 4 & -3 & 1 \\ -6 & 7 & -4 \\ 1 & -2 & x \end{vmatrix}$

24. Classify the following system of equations as consistent or inconsistent. If consistent solve it.

$x - y + 3z = 6$, $x + 3y - 3z = -4$ and $5x + 3y + 3z = 10$

Without actual expansion prove the following:

25. $\begin{vmatrix} 1 & \mathrm{a} & \mathrm{b}+\mathrm{c} \\ 1 & \mathrm{b} & \mathrm{c}+\mathrm{a} \\ 1 & \mathrm{c} & \mathrm{a}+\mathrm{b} \end{vmatrix} = 0$

26. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

27. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$

28. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

29. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$

30. $\begin{vmatrix} b^2c^2 & bc & (b+c) \\ c^2a^2 & ca & (c+a) \\ a^2b^2 & ab & (a+b) \end{vmatrix} = 0$

31. $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^z+a^{-z})^2 & (a^z-a^{-z})^2 & 1 \end{vmatrix} = 0$

32. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

Using properties of determinants prove the following:

33. $$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

34. $$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

35. $$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

36. $$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

37. $$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$$

38. $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

39. $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

40. $$\begin{vmatrix} 1+x & 2 & 3 \\ 1 & 2+x & c \\ 1 & 2 & 3+x \end{vmatrix} = x^2(x+6)$$

41. $$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

42. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

43. $\begin{vmatrix} (b^2+c^2) & ba & ca \\ ab & (c^2+a^2) & cb \\ ac & bc & (a^2+b^2) \end{vmatrix} = 4a^2b^2c^2$

44. $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

45. $\begin{vmatrix} (a+b+c) & -c & -b \\ -c & (a+b+c) & -a \\ -b & -a & (a+b+c) \end{vmatrix} = 2(a+b)(b+c)(c+a)$

46. $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ *then prove thatxyz* $= -1$

47. *Solve for* x $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

48. $\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & 1 & 1+c^2 \end{vmatrix} = (1+a^2+b^2+c^2)^3$

49. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

50. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

51. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a+1)^3$

52. $\begin{vmatrix} 1+x & 2 & 3 \\ 1 & 2+x & c \\ 1 & 2 & 3+x \end{vmatrix} = x^2(x+6)$

53. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

54. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

55. $\begin{vmatrix} a & b & c \\ a^a & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

56. $\begin{vmatrix} 1 & 1 & 1 \\ a^a & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

57. Find x and y if $\begin{pmatrix} 2x+3y & 3 \\ -2 & 3x-y \end{pmatrix} = \begin{pmatrix} x+1 & 3 \\ -2 & x+y+3 \end{pmatrix}$

58. Find Matrix if X $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -2 & 5 \end{pmatrix}$

59. Express $\begin{pmatrix} 2 & -2 & 3 \\ -5 & 4 & 0 \\ -2 & 1 & 3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

60. If A = $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ Prove that $A^3 - 6A^2 + 11A - I_3 = 0$. Hence find A^{-1}

61. If A = $\begin{pmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ 0 & 1 & -3 \end{pmatrix}$ and $f(x) = x^3 + 2x^2 - 3x + 2$ find f(A).

62. If A= $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ Prove that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$

63. If A= $\begin{pmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{pmatrix}$ Prove that $A^n = \begin{pmatrix} \cos nA & \sin nA \\ -\sin nA & \cos nA \end{pmatrix}$

64. **Solve the following equations by matrix method**

a. x + y = 3 ; 3x - 2z = 3 ; 2y + z = 7

b. x + y + z = 0 ; 3x - 2z + z = - 4; 2y + z =3

c. $\frac{3}{x} + \frac{2}{y} - \frac{3}{z} = 3$; $\frac{1}{x} + \frac{4}{y} - \frac{3}{z} = 2$; $\frac{6}{x} + \frac{4}{y} - \frac{9}{z} = 5$

d. 5x + 7y + 2 = 0 and 4x + 6y + 3 = 0

65. **Solve the following equations by matrix method**

a. 2x + y + 3z = 3 ; 4x – y = 3 ; –7x + 2y + z = 2
[ans: x = –6, y = –27, z = 14]

b. 2x – y + 2z = 3 ; 2x + y + z = – 1; x – 3y + 2z = 6
[ans: x = – 1, y = –1, z = 2]

c. x + 2y + z = 7; x + 3z = 11 and 2x – 3y = 1
[ans: x = 2, y = 1, z = 3]

d. 2x – y – z = 3; x + y + 2z = 5; 2x – y + z = 7
[ans: x = 2, y = –1, z = 2]

e. x – y + z = 2; 2x – y = 0 and 2y – z = 1
[ans: x = 1, y = 2, z = 3]

f. x + y + z = 3 ; 2x – y + z = 2 and x – 2y + 3z = 2
[ans: x = 1, y = 1, z = 1]

g. $x + y - z = 1$; $3x + y - 2z = 3$ and $x - y - z = -1$ [ans: x = 2, y = 1, z = 3]

h. $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} - + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ [ans: x = 2, y = 3, z = 5]

i. $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$; $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$; $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$ [ans: x = 1/2, y = 1/3, z = 1/5]

66. If A = $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ find A^{-1} and hence solve:

$2y + 5z = 2 + x$; $2x + z = 15 + 3y$; $y + z = x - 3$

67. **Find the inverse of the following matrices, if they exist, by elementary transformations:**

a. $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ b. $\begin{pmatrix} 5 & 7 & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ c. $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 1 & 1 & 3 \end{pmatrix}$