PRACTICE PAPER SECOND TERM EXAM 2011

PREVIOUS YEAR QUESTIONS

SECTION A (1 Mark)

1. a) Find the value of cos 15° . b) Find the value of tan 15^0. C) Prove: $\frac{\cos 31^0 + \sin 31^0}{\cos 31^0 - \sin 31^0} = \tan 76^0$

d) Find the value of cot 75^0. e) Find the value of sin($\frac{-19\pi}{3}$). f) Find the general solution of sin3x = 0

g) Find the value of tan 75^0. h) Find the general solution of sinθ = $\frac{\sqrt{3}}{2}$

2. a) Find the conjugate of $\frac{1}{1+i}$ b) Find the multiplicative inverse of z = 2+3i.

c) Find the multiplicative inverse of 2- 2i.

d) Find the modulus of $\frac{1}{1+i}$. e) Express $\frac{3-i}{5+6i}$ in standard form. f) Find the multiplicative inverse of $\frac{2+3i}{3-2i}$

g) Find the modulus of the complex number $\frac{1}{2+2i}$. h)Express the following expression in the form a + ib: $\frac{2-\sqrt{-25}}{1+\sqrt{-36}}$.

3. **Solve : i) $x^2 - 6x + 14 = 0$.** ii) $x^2 - x + 2 = 0$. iii) $x^2 - 2x + \frac{3}{2} = 0$ iv) Solve $x^2 + x + \frac{1}{\sqrt{2}} = 0$

4. Solve : a) $-5 \le \frac{2-3x}{4} \le 9$ b) $4x + 5 < 6x + 9$ c) 4x+3<6x+7 for real x. d) 8-3x < 2 When x is a natural number.

5. a) How many six digit telephone numbers can be formed if each number starts with 176 and no digit appears more than once.
b) How many three digit even numbers can be formed with the digits 1,2,3,4,5,6,,7.

c) State the fundamental theorem of counting. d) Find the middle term(s) in the expansion of $\left(x + \frac{1}{x}\right)^{10}$

e) Find the number of sides of a polygon of 35 diagonals. f) If ***m*** parallel lines in a plane are intersected by ***n*** parallel lines, find the no. of parallelograms formed. g)If ${}^nC_7 = {}^nC_8$ Find ${}^nC_7$. h) Find x if $\frac{1}{9!} + \frac{1}{10!} = \frac{1}{11!}$.

6. a) Find the number of terms in the expansion of $(1 + 2x + x^2)^{20}$. b)Find the coefficient of a^5b^7 in the expansion of $(a-2b)^{12}$.

c) Find the 4th term from the end in the expansion of $(\frac{3}{x^2} - \frac{x^3}{6})^7$. **d)** Find the middle term in the expansion of $\left(3 - \frac{x^3}{6}\right)^8$ e) Expand $\left(x^2 + \frac{2}{x}\right)^5$ $x \neq 0$. f) Find the number of terms in the expansion of $(9 - 12x + 4x^2)^{40}$.

7. a) Which term of the sequence $\sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots\ldots is\ 729$ b)For what values of x, the numbers $-\frac{2}{7}$, x, $-\frac{7}{2}$ are in G.P.? **c) Find the 2nd term of an A.P. whose 6th and 8th terms are 12 and 22 respectively. d)** Find k so that 3k-1, k+1, k+3 are in A.P. e) Which term of 18, -12, 8 …… is $\frac{512}{729}$. f) The third term of a G.P. is 4. Find the product of its first five terms.

8. **a) Find the value of p for which the lines px + 3y = 4 and 3x – 4y = 7 are perpendicular. b)** Reduce the line $\sqrt{3}x - y + 8 = 0$ in the normal form. c) Find the value of x for which the points (x, -1), (2, 1) and (4, 5) are collinear. d) Reduce the equation 3x - 4y + 10 = 0 into slope - intercept form and find the slope and y - intercept. e) Find the distance between the parallel lines 3x + 4y + 7 = 0 and 6x + 8y + 18 = 0. f) Express $\sqrt{3}x + y = 1$ in normal form

SECTION B

1. **Solve i) $2\cos^2 x + 3\sin x = 0$. ii) $4\cos x - 3\sec x = \tan x$ iii) $\sin 3x + \sin x - \sin 2x = 0$ iv) $\cos 3x + \cos x - \cos 2x = 0$**
2. **Prove that i) $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ ii) $\cos^2 x + \cos^2(x + 120^0) + \cos^2(x - 120^0) = 3/2$**

iii) $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$ iv) $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$

v) $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x+y}{2}$ vi) $\frac{\cos 8x \cos 5x - \cos 12x \cos 9x}{\sin 8x \cos 5x + \cos 12x \sin 9x} = \tan 4x$

3. If tanA = k tanB, show that sin(A + B) = $\frac{k+1}{k-1}$ sin(A – B). ii) Prove that cos20˚cos40˚cos60˚cos80˚ = $\frac{1}{16}$

4. If tan ($(\alpha+\theta)=n\tan(\alpha-\theta), Show that$ (n +1)sin 2θ =(n-1) sin 2α

5. Prove that $\frac{\sin 3x+\sin 5x+\sin 7x+\sin 9x}{\cos 3x+\cos 5x+\cos 7x+\cos 9x}$ = tan6x ii) $\frac{\sin A.\sin 2A+\sin 3A.\sin 6A}{\sin A.\cos 2A+\sin 3A.\cos 6A}$ = tan 5A

6. Prove that i) tan3xtan2xtanx = tan3x – tan 2x – tanx ii) cotxcot2x—cot2xcot3x –cot3xcotx =1

7. Find the equation of the straight line passing through the intersection of the lines 4x + 7y -3 = 0 and 2x – 3y +1 = 0 and has the equal intercepts on the axes.

a. Find the value of k if the points (2k-1, -3), (7, -1) and (0, 3) are the vertices of the triangle of area 3 square units
b. Find the equation of set of points equidistant from (-1,-1) and (4, 2).
c. Find the equation of the line through the point of intersection of 2x + y = 1 and x + 3y = - 2 and with x intercept 3
d. If three points (h, 0), (a, b) and (0, k) lie on a line, Show that $\frac{a}{h}+\frac{b}{k}=1$.
e. Find the equation of the perpendicular bisector of the line segment joining the points A (2, 3) and B(6, -5)
f. Find the points on the y axis which are at a distance of $5\sqrt{2}$ from the points (3,-2,5)
g. Two lines passing through the points (2,3) intersect each other at an angle of 60^0. If slope of one line is 2, find the equation of the other line.
h. Find the equation of a line passing through (2,2) and cutting off intercepts on the axes whose sum is 9.
i. Find the equation of a straight line which passes through (3, 4) and the sum of whose intercepts on the coordinate axes is 14.
j. Find the equation of the line passing through the intersection of the lines 2x + 3y - 2 = 0 and x – 2y + 1 = 0 and perpendicular to the line 5x – 4y + 1 = 0.
k. Find the equation of a line perpendicular to 5x - 2y = 7 and passes through the midpoint of the line joining (4,-1) and (2,5)
l. Determine the equation of a line passing through (4,5) and make equal angles with the lines 5x - 12y + 6= 0 and 3x = 4y + 7
m. If the angle between two lines is $\frac{\pi}{4}$ and slope of one line is $\frac{1}{2}$,find the slope of the other line
n. A line perpendicular to the line joining the points (2,-3) and (1,2) divides it in the ratio 1:2.Find the equation of the line
o. In the triangle ABC whose vertices are A(1, 4), B(-3, 2) and C(-5, -3), find the equation of the altitude from the vertex B. Also find the area of triangle ABC.
p. If p is the length of the perpendicular from the origin to the line whose intercepts on the axes a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$
q. If the lines 2x + y - 3 = 0 ,5x + ky - 3 = 0 and 3x – y – 2 = 0 are concurrent, find k
r. If the lines y = 3x + 1 and 2y = x + 3 are equally inclined to the line y = mx + 4, find the value of m.

8. Find the sum of the sequence 8, 88, 888, 8888, ------- to n terms.

a. Find the sum to n terms of the series 3 + 7 + 13 + 21 + 31 + …….
b. Find the sum to n terms of $1^2 + (1^2+2^2) + (1^2+2^2+3^2) +$……..
c. Find sum to n terms of 3 x 8 + 6 x 11 + 9 x 14 ……..
d. Find the sum of the series $3.2^2+4.5^2+5.8^2+$..... up to **n** terms.
e. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M between a and b, then find the value of n.
f. Find n if $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the G.M. between a and b.
g. If the p^{th}, q^{th} and r^{th} terms of the A.P are a, b, c respectively, prove that a(q - r) + b(r - p) + c(p - q) = 0
h. If the p^{th}, q^{th}, r^{th} terms of a G.P are respectively a, b, c respectively, Prove that $a^{q-r}.b^{r-p}.c^{p-q}=1$
i. The sum of the first p terms of an AP is equal to sum of the first q terms, Find the sum of the first (p + q) terms

9. Prove using PMI that $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$ For n$\in N$

1. $\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+.....+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$

2. $1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+.....+\frac{1}{(1+2+3+.....+n)}=\frac{2n}{n+1}$

3. $1.3+2.3^2+3.3^3+...+n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$

10. Express In the polar form i) $\frac{1+3i}{1-2i}$ ii) $\frac{2+i6\sqrt{3}}{5+i\sqrt{3}}$ iii) $\frac{-16}{1+i\sqrt{3}}$ iv) $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$

1. **If a+ ib =** $\frac{c+i}{c-i}, showthat\ a^2+b^2=1 and \frac{b}{a}=\frac{2c}{c^2-1}$

2. If α and β are two complex numbers such that $|\alpha|=1$ find the value of $\left|\frac{\alpha-\beta}{1-\overline{\alpha}\beta}\right|$

3. **If** $(x+iy)^{\frac{1}{3}}$ **= a + ib, show that 4(a² – b²) =** $\frac{x}{a}+\frac{y}{b}$

4. If x – iy = $\sqrt{\frac{(x+i)^2}{2x^2+1}}$ Prove that $a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$

5. Find real value of θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real

11. Find the number of sides of a polygon having 44 diagonals.

1. **Find r if 5** $^4P_r = 6\ ^5P_{r-1}$
2. How many different words can be formed using the letters of the word "DAUGHTER" in each of the following:
   a) beginning with D b) beginning with D and ending with R
   c) vowels being always together d) vowels occupying even places
3. **Find the number of permutations of the letters of the world MATHEMATICS. In how many of these arrangements i) Do all the vowels occur together ii) Do the vowels never occur together iii) Do the words begin with M and end in S**
4. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements
   i) do the words start with P? ii) do all vowels occur together?
5. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in dictionary, what will be the 50 th word?

12. Find the term independent of x in the expansion of $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}\quad x>1$

a. Find $(a+b)^4$ -- $(a-b)^4$ and hence find $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$

b. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is

$\sqrt{6}:1.$

c. Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5.7.....(2n-1)2^n x^n}{n!}$

SECTION C

13. Prove that $\frac{\sin x\sin 2x+\sin 2x\sin 5x+\sin 3x\sin 10x}{\sin x\cos 2x+\sin 3x\cos 6x+\sin 2x\cos 11x}=\tan 7x$

If sinx = $\frac{-3}{5}$ **and x in quadrant III find the values of sinx/2, cosx/2 and tan x/2.**

14. Solve the following in equations graphically i) x+2y ≤ 10; x + y ≥ 1, x – y ≤ 0; x ≥ 0; y ≥ 0

15. i) The sum of two number is 6 times their geometric means, show that numbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$

ii) The sum of three numbers in G.P is 56. If we subtract 1,7,21 from these numbers in that order we obtain an AP .Find the no.s

16. a) The coefficients of the $(r-1)^{th}$, r^{th}, $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1:7:42 Find n and r

a. The coefficients of the $(r-1)^{th}$, r^{th}, $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio 1:3:5.Find n and r

b. In the expansion of $(1+x)^n$ the three consecutive coefficients are 462,330 and 165. Find n

c. The Coefficients of a^{r-1}, a^r, a^{r+1} in the expansion of $(1+a)^n$ are in arithmetic progression. Prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$

17. a) Find the image of the point (-8, 12) with respect to the line 4x + 7y + 13 = 0.

a. **Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.**

b. Find the image of point (1, 2) in the line x + y - 1= 0

c. If a and b are the lengths of the perpendiculars from the origin to the lines
$x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\operatorname{cosec}\theta = k$,respectively, prove that $a^2 + 4b^2 = k^2$.

d. **Prove that the product of the lengths of the perpendicular from the point**
$\left(\sqrt{a^2-b^2},0\right)$ and $(\sqrt{a^2-b^2},0)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2

e. Find the distance of the point A(2,3)from the line 3y =2x+9 measured along a line making angle 45^0 with the x axis

f. Find the equation of a line passing through the intersection of the lines x-3y+1 =0 and 2x+5y-9=0 and whose distance from the origin is 2 units

g. A straight line passes through the point (2,3) and its segment intercepted between the axes is bisected at that point. Find its equation

18. a) Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements
i) do the words start with P? ii) do all vowels occur together?

a. In how many distinct permutations of the letters in MISSISSIPPI do the four I's not come together.

b. A candidate is required to attempt six out of ten questions which are divided into sections each containing five questions and he is not permitted to attempt more than four questions from each section. In how many ways can he make up this choice?

c. An examination paper consists of 12 questions divided into parts A and B. Part A contains 7 questions and part B contains 5 questions. A candidate is required to attempt 8 questions selecting at least 3 from each part. In how many ways can the candidate select the questions?

d. A group consists of 4 girls and 6 boys. In how many ways can a team of 4 members be selected if the team has
a) At most 2 girls b) at least one boy and one girl c) at least 2 girls

e. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, find the values of n and r.