1. d.r.'s : 1, −1, −2

d.c.'s : $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

2. show that d.r.'s of $\vec{AB}$ & $\vec{BC}$ are proportional. Since B is a common point A, B, C are collinear

3. $\vec{a}+\vec{b} = 4\hat{i}+0\hat{j}+2\hat{k}$

$\hat{e} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{4\hat{i}+2\hat{k}}{\sqrt{20}}$

$= \frac{2\hat{i}+\hat{k}}{\sqrt{5}}$

4. $p\hat{i}+p\hat{j}+p\hat{k} = \vec{a}$ (let)

then $\vec{a}$ is a unit vector if $|\vec{a}|=1$

$\Rightarrow \sqrt{p^2+p^2+p^2} = 1$

$\Rightarrow 3p^2 = 1 \Rightarrow p = \pm\frac{1}{\sqrt{3}}$

5. $\frac{7}{\sqrt{6}}(\hat{i}-\hat{j}+2\hat{k})$

6. $\frac{2}{-6} = \frac{-3}{m} \Rightarrow m = 9$

7. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

8. $\sqrt{5^2+n^2} = 13 \Rightarrow n = \pm 12$

9. $\pm\frac{1}{\sqrt{3}}$ (refer Q. 4)

10. $\vec{p} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{q} = 4\hat{i}+\hat{j}-2\hat{k}$

midpt's position vector $= \frac{\vec{p}+\vec{q}}{2}$

$= 3\hat{i}+2\hat{j}+\hat{k}$

11. let $\vec{p} = 2a-3b$

12. $\frac{2}{\lambda} = \frac{-3}{-6}$ ($\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ for parallel)

$\Rightarrow \lambda = 4$

13. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -2 \\ 1 & -2 & 4 \end{vmatrix} = 8\hat{i}-14\hat{j}-9\hat{k}$

$\hat{e} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{8\hat{i}-14\hat{j}-9\hat{k}}{\sqrt{341}}$

14. $\vec{a}\times\vec{b} = -7\hat{i}-13\hat{j}+5\hat{k}$

$\hat{e} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-7\hat{i}-13\hat{j}+5\hat{k}}{9\sqrt{3}}$

reqd. vector $= 6\left[\frac{-7\hat{i}-13\hat{j}+5\hat{k}}{9\sqrt{3}}\right]$

$= \frac{2}{3\sqrt{3}}(-7\hat{i}-13\hat{j}+5\hat{k})$

15. $\cos\theta = \frac{5}{9}$ $\therefore \theta = \cos^{-1}(5/9)$

16. $\vec{a}\cdot\vec{b} = 0 \Rightarrow \lambda = -2$

17. $\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$

$\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$ $\therefore \perp$

18. $\vec{b}+\vec{c} = 2\hat{i}+3\hat{j}+\hat{k}$

$(\vec{b}+\vec{c})\cdot\frac{\vec{a}}{|\vec{a}|} = -1$

19. $\frac{60}{\sqrt{114}}$

20. $\hat{i}\cdot(\hat{i})+\hat{j}\cdot(-\hat{j})+\hat{k}\cdot\hat{k}$
$= 1 - 1 + 1 = 1$

Solving ①, ② & ③ ~~multiply eqn ①~~

① + ② gives $(3x - 2z = 4) \times 2 \Rightarrow 6x - 4z = 8$

② − ③ gives $x - 4z = -2 \Rightarrow x - 4z = -2$

$$5x = 10$$

$$\therefore x = 2$$

$x = 2,\ y = -1,\ z = 1$

Hence $\vec{d} = 2\hat{i} - \hat{j} + \hat{k}$

(32) let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$; $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$\bar{b} + \bar{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$|\bar{b} + \bar{c}| = \sqrt{(2+\lambda)^2 + 36 + 4} = \sqrt{(2+\lambda)^2 + 40}$

Unit vector along $\bar{b} + \bar{c} = \dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = \hat{e}$ (let)

Given $\vec{a} \cdot \hat{e} = 1 \Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \left[\dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}\right] = 1$

$\Rightarrow \dfrac{(2+\lambda)(1) + 6(1) + (-2)(1)}{\sqrt{(2+\lambda)^2 + 40}} = 1$

$\Rightarrow \cancel{2} + \lambda + 6 - \cancel{2} = \sqrt{(2+\lambda)^2 + 40} \Rightarrow (\lambda + 6)^2 = (2+\lambda)^2 + 40$

Solving $\lambda = 1$

(33) $\bar{a} + \lambda\bar{b} = (1-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$

$\vec{c} = 3\hat{i} + \hat{j} + 0\hat{k}$

Given $(\bar{a} + \lambda\bar{b}) \perp \bar{c} \Rightarrow (\bar{a} + \lambda\bar{b}) \cdot \bar{c} = 0$

$\Rightarrow 3(1-\lambda) + 1(2+2\lambda) + 0(3+\lambda) = 0 \Rightarrow \lambda = 5$

(34) Find $\vec{AB} \times \vec{AC}$

reqd. unit vector $= \dfrac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|}$

(21) $|\hat{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$8 = 2\times5\times\sin\theta \Rightarrow \sin\theta = \frac{4}{5}$

$\Rightarrow \cos\theta = \frac{3}{5}$

$\therefore \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 2\times\cancel{5}\times\frac{3}{\cancel{5}}$
$= 6$

(22) i) $|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$

$\Rightarrow |\bar{a}+\bar{b}|^2 = |\bar{a}-\bar{b}|^2$

$\rightarrow (\bar{a}+\bar{b})\cdot(\bar{a}+\bar{b}) = (\bar{a}-\bar{b})\cdot(\bar{a}-\bar{b})$

$\Rightarrow |\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\vec{b} = |\bar{a}|^2+|\bar{b}|^2-2\bar{a}\cdot\bar{b}$

$\Rightarrow 4\bar{a}\cdot\bar{b} = 0 \Rightarrow \bar{a}\cdot\bar{b} = 0$

$\therefore \bar{a}\perp\vec{b} \qquad \theta = 90^\circ$

ii) $\bar{a}\cdot\bar{b} = \cancel{2}\sqrt{3}\times\frac{1}{\cancel{2}} = \sqrt{3}$

iii) $|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$25 = 5\times13\sin\theta \Rightarrow \sin\theta = \frac{5}{13}$

$\therefore \cos\theta = \frac{12}{13}$

$\Rightarrow \bar{a}\cdot\bar{b} = 5\times\cancel{13}\times\frac{12}{\cancel{13}} = 60$

iv) $|\bar{a}-\bar{b}|^2 = (\bar{a}-\bar{b})\cdot(\bar{a}-\bar{b})$
$= |\bar{a}|^2+|\bar{b}|^2-2\bar{a}\cdot\bar{b}$
$= 4+25-2\times10$
$= 9$

$\therefore |\bar{a}-\bar{b}| = 3$

v) $\cos\theta = \frac{1}{9} \quad \therefore \theta = \cos^{-1}(1/9)$

vi) $\frac{\bar{b}\cdot\bar{a}}{|\bar{a}|} = \frac{3}{2}$

vii) $|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$\sqrt{3} = 1\times2\sin\theta \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \theta = 60^\circ$

viii) $\cancel{|\bar{a}|}\cancel{|\bar{b}|}\cos\theta = \cancel{|\bar{a}|}\cancel{|\bar{b}|}\sin\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\div \cos\theta \qquad \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$

$\therefore \theta = 45^\circ$

ix) $|\hat{a}+\bar{b}|^2 = (\bar{a}+\bar{b})\cdot(\hat{a}+\bar{b})$
$= |\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\bar{b}$
$= 9+4+2\times6 = 25$

$\therefore |\hat{a}+\bar{b}| = 5$

x) $\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

$\Rightarrow \cos\theta = \frac{\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}} = \frac{1}{2} \quad \therefore \theta = 60^\circ$

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta = \cancel{2}\sqrt{3}\times\frac{\sqrt{3}}{\cancel{2}}$
$= 3$

xi) $\cos\theta = \frac{\cancel{12}\,\cancel{6}^{3}}{\cancel{2}\times10\,5} = 3/5$

$\therefore \sin\theta = \frac{4}{5}$

$|\hat{a}\times\bar{b}| = 1\cancel{0}^{2}\,2\times\frac{4}{\cancel{5}} = 16$

xii) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix} = 0$

$(6p-81)\hat{i}+\hat{j}(27-2p)+\hat{k}(0) = 0$

$\Rightarrow 6p-81 = 0 \quad \& \quad 27-2p = 0$

$\Rightarrow p = 27/2$

(23) $(\bar{a}+\bar{b})\times(\bar{a}-\bar{b}) = 2\hat{i}+4\hat{j}+10\hat{k}$

$\hat{e} = \frac{2\hat{i}+4\hat{j}+10\hat{k}}{\sqrt{120}} = \frac{\hat{i}+2\hat{j}+5\hat{k}}{\sqrt{30}}$

(24). $|\bar{b}| = 1 \; ; \; |\bar{a}| = 2$

(25). i) $\lambda = 2/3$ ii) $\lambda = -15$

(26) i) $\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$, ii) $\frac{10}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})$ iii) $8\sqrt{3}$ iv) $4\sqrt{3}$

(27) i) $\theta = 0°$, ii) $49/2$

(28) let $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$ then $\bar{a}\times\bar{c} = \bar{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = 0\hat{i}+\hat{j}-\hat{k}$

$\Rightarrow (z-y)\hat{i}+\hat{j}(x-z)+\hat{k}(y-x) = 0\hat{i}+\hat{j}-\hat{k}$

comparing like terms, $z-y=0$, $x-z=1$ & $y-x=-1$

$\Rightarrow y = z$ — ①, $x = 1+z$ — ② "

Also $\bar{a}\cdot\bar{c} = 3 \Rightarrow x+y+z = 3$ — ③

sub. ① & ② in ③ $(1+z)+z+z = 3 \Rightarrow z = 2/3$

$\therefore y = 2/3$ & $x = 5/3$

Hence $\vec{c} = \frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$.

(29) let $\bar{d} = x\hat{i}+y\hat{j}+z\hat{k}$ since $\bar{d}\perp\bar{a}$, $\bar{d}\cdot\bar{a} = 0 \Rightarrow x-y = 0 \Rightarrow \boxed{x=y}$ — ①

Also $\bar{d}\perp\vec{b} \Rightarrow \bar{d}\cdot\bar{b} = 0 \Rightarrow 3y-z = 0 \Rightarrow \boxed{z = 3y}$ — ②

Since $\bar{c}\cdot\bar{d} = 1 \Rightarrow \boxed{7x-z=1}$ — ③

Sub ① & ② in ③ $7y-3y = 1 \Rightarrow y = 1/4, x = 1/4, z = 3/4$

Hence $\vec{d} = \frac{1}{4}\hat{i}+\frac{1}{4}\hat{j}+\frac{3}{4}\hat{k}$.

(30) Similar to above question. $x+4y+2z = 0$ — ①
$3x-2y+7z = 0$ — ②
$2x-y+4z = 18$ — ③

solving ① & ② by cross multiplication method

$\frac{x}{28+4} = \frac{-y}{1} = \frac{z}{-2-12} \Rightarrow x = 32K, y = -K, z = -14K$

Sub in ③, we get $K = 2$

$\therefore d = 64\hat{i}-2\hat{j}-28\hat{k}$

(31) let $\bar{d} = x\hat{i}+y\hat{j}+z\hat{k}$ then $x-y+z = 4$ — ①
$2x+y-3z = 0$ — ②
$x+y+z = 2$ — ③