# MATHEMATICAL TOOLS

Physical constants:-

1. Mass of an electron ($M_e$) = 9.1x$10^{-31}$kg.
2. Mass of a proton ( $M_p$) = 1.6725 x $10^{-27}$kg.
3. Mass of a neutron ($M_n$) = 1.6746 x $10^{-27}$kg.
4. Charge of an electron (e) = -1.6 x $10^{-19}$c
5. Speed of light in vacuum (c) = 3 x$10^8$ m/sec.
6. Planck Constant (h) = 6.6256 x $10^{-34}$ J x sec .
7. Universal Gravitation constant (G) = 6.67 x $10^{-11}$ $Nm^2/ kg^2$.
8. Avogadro Number ($N_A$) = 6.023 x$10^{23}$ $mol^{-1}$.
9. Boltzmann constant (K) = 1.38 x $10^{-23}$ J/K
10. Stefan Constant ($\sigma$) = 5.67 x $10^{-8}$ W $m^{-2}$ $K^{-4}$.
11. Wien Displacement Constant (b) = 2.898 x$10^{-3}$ m K
12. Solar Constant (S) = 1.388 x $10^3$ W $m^{-2}$
13. Mass of the sun ($M_S$) = 2x $10^{30}$kg.
14. Mass of the earth ($M_E$) = 5.98x $10^{24}$kg
15. Radius of the earth ($R_e$) = 6400 Km. = 6.4x$10^6$m.
16. Density of earth 5.522x$10^3$kg/$m^3$.
17. Average angular velocity of the earth = 7.29x $10^{-5}$rad./sec
18. Average distance between the sun and earth = 1.5x$10^{11}$m.
19. Average distance between moon and the earth =3.84x $10^8$m.
20. Magnetic Moment of the earth = 6.4x $10^{21}$ Amp. X $m^2$.

## Conversion Coefficients

1. 1Light year = 9.46x$10^{15}$m.
2. 1 A.U. = 1.496 x $10^{11}$m.
3. 1Å = $10^{-10}$m.
4. 1 Pound = 0.4536kg = 453.6 gm
5. 1 Fermi = $10^{-15}$m.
6. 1 C.S.L.= 1.4 x Mass of the sun.
7. 1 Shake = $10^{-8}$sec
8. 1ev = 1.6 x$10^{-19}$ Joule.
9. 1 Horse Power = 746 Watt.

## Quadratic Equation

An equation of second degree is called a quadratic equation. It is of the form :-

$$ax^2 + bx + c = 0$$

The roots of a quadratic equation are

$$X = \frac{-b \pm (b^2 + 4ac)^{1/2}}{2a}$$

## Binomial Theorem

If n is any integer, positive or negative or a fraction and x is any real number, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{\lfloor 2} + \ldots$$

If $|x| << 1$ , then $(1+x)^n = 1 + nx$.

## Mensuration :-

1. Area of a circle = $\pi r^2 = \pi D^2/4$
2. Surface area of a sphere = $4\pi r^2 = \pi D^2$
3. Volume of a sphere = $4/3\ \pi r^3$
4. Surface area of a cylinder = $2\pi r(r+l)$
5. Volume of a cylinder $= \pi r^2 l$
6. Curved surface area of a cone = $\pi rl$
7. Volume of a cone = $1/3\pi r^2 h$
8. Surface area of a cube = $6x(\text{side})^2$
9. Volume of a cube = $(\text{side})^3$

# Fundamental Trigonometric relations

$$\text{Cosec}\theta = \frac{1}{\text{Sin}\theta}$$

$$\text{Sec}\theta = \frac{1}{\text{Cos}\theta}$$

$$\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{1}{\text{Tan}\theta}$$

$$\text{Tan}\theta = \frac{\text{Sin}\theta}{\text{Cos}\theta}$$

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$1 + \tan^2\theta = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Cosec}^2\theta$$

Sin(A+B) = SinACosB + CosASinB

Cos (A + B) = CosACosB – SinASinB

Sin (A – B) = SinACosB – CosASinB

Cos (A – B) = CosACosB + SinASinB

$$\text{Tan}(A+B) = \frac{\text{TanA}+\text{TanB}}{1-\text{TanATanB}}$$

Sin 2A = 2SinACosA

Cos 2A = $2Cos^2A - 1 = 1 - 2Sin^2A = Cos^2A - Sin^2A$

Sin (A+B) + Sin(A-B) = 2 SinACosB

Cos(A+B) + Cos (A-B) = 2CosACosB

$$\mathrm{CosC} + \mathrm{CosD} = 2\mathrm{Cos}\frac{(\mathrm{C+D})}{2}\mathrm{Cos}\frac{(\mathrm{C-D})}{2})$$

# Logarithms

$Log_a mn = Log_a m + Log_a n$

$Log_a (\frac{m}{n}) = Log_a m - Log_a n$

$Log_a m = Log_b m \times Log_a b$

$Log_{10}10^3 = Log_{10}1000 = 3$

$Log_a 1 = 0$

$Log_a a = 1$

Average Values

$< Sin\theta > = 0$ , $< Cos\theta > = 0$

$< Sin^2\theta > = ½$

$< Cos^2 \theta > = ½$

Approximate Values

If angle ($\theta$) small then $\theta \longrightarrow 0$

$Sin\theta \cong \theta$

$Cos\theta \cong 1$

$Tan\theta \cong \theta$

## Differential Formulae

1. Differentiation of a constant C is zero

$$\frac{dc}{dx} = 0$$

2. $\frac{d(cy)}{dx} = c\frac{dy}{dx}$

3. $\frac{d(x^n)}{dx} = nx^{n-1}$

4. $\frac{d[f(x) \pm g(x)]}{dx} = \frac{df(x)}{dx} \pm \frac{dg(x)}{dx}$

5. $\frac{d\{f(x)g(x)\}}{dx} = \frac{f(x)dg(x)}{dx} + \frac{g(x)df(x)}{dx}$

6. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{\{g(x)\}^2}$

7. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

8. $\frac{de^x}{dx} = e^x$

9. $\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx}$

10. $\frac{dlog_e x}{dx} = \frac{1}{x}$

11. $\frac{d(a^x)}{dx} = a^x log_{e^a}$

12. $\frac{dlog_a x}{dx} = \frac{1}{x} log_{e^a}$

13. $\frac{d(\sin x)}{dx} = \cos x$

14. $\frac{d(\cos x)}{dx} = -\sin x$

15. $\frac{d(\tan x)}{dx} = sec^2 x$

16. $\frac{d(\cot x)}{dx} = -cosec^2 x$

17. $\frac{d(cosecx)}{dx} = -\text{cosec}\, x\ cot\ x$

18. $\frac{d(secx)}{dx} = \sec x\ tan\ x$

## Integral Formulae

1. ∫dx =x +c  Where c = constant
2. $\int x^{n+1}dx = \frac{x^{n+1}}{n+1} + C$
3. $\int dx/x = \log_e x + c$
4. ∫Sinx dx = - Cosx +c
5. ∫Sin ax dx = -<u>Cos ax</u>
   a
6. ∫ Cos x dx = Sin x +c
7. $\int Sec^2 x\, dx = \tan x + c$
8. $\int Cosec^2 x\, dx = -Cot\ x + c$

9. $\int \text{Sec } x \tan x \, dx = \text{Sec } x + c$
10. $\int \text{Cosec } x \text{ Cot } x \, dx = - \text{Cosec } x + c$
11. $\int e^x \, dx = e^x + c$

# Physical World And Measurement

There are four fundamental forces which govern both macroscopic and microscopic phenomena. There are

(i) Gravitational force
(ii) Nuclear force
(iii) Electromagnetic force
(iv) Weak force

The relative strengths of these forces are

$$F_g : F_w : F_e : F_s = 1 : 10^{25} : 10^{36} : 10^{38} .$$

All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.

(a) Fundamental quantities (b) Derived quantities.

The units of the fundamental quantities called fundamental units , and the units of derived quantities called derived units.

System of units:-

(a) MKS (b) CGS (c) FPS (d) SI

- *The dimensions of a physical quantity are the powers to which the fundamental quantities are raised to represent that physical quantity.*
- The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time, is called dimensional equation.
- According to this principle of homogeneity a physical equation will be dimensionally correct if the dimensions of all the terms in the all the terms occurring on both sides of the equation are the same.
- If any equation is dimensionally correct it is not necessary that must be mathematically correct too.
- There are three main uses of the dimensional analysis-

(a) To convert a unit of given physical quantities from one system of units to another system for which we use

$$n_2 = n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$$

(b) To check the correctness of a given physical relation.

(c) To derive a relationship between different physical quantities.

- Significant figures: - The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

For counting of the significant figure rule are as:

(i) All non- zero digits are significant figure.
(ii) All zero between two non-zero digits are significant figure.
(iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant. But such zeros are significant if they come from a measurement.
(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant.
(v) All zeros to the right of a decimal point are significant.
(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. Single zero conventionally placed to the left of the decimal point is not significant.
(vii) The number of significant figures does not depend on the system of units.

- In addition or subtraction, the result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.
- In multiplication or division, the result should be reported to the same number of significant figures as that of the number with minimum of significant figures.
- Accuracy refers to the closeness of a measurement to the true value of the physical quantity and precision refers to the resolution or the limit to which the quantity is measured.
- Difference between measured value and true value of a quantity represents error of measurement.
  It gives an indication of the limits within which the true value may lie.

  Mean of n measurements

$$a_{mean} = \frac{a_1+a_2+a_3+\cdots\ldots+a_n}{n}$$

Absolute error ( $\Delta a$ ) = $a_{mean}$ - $a_i$ Where $a_i$ = measured value

It may be - positive, negative or zero.

(i) Mean absolute error

(ii) Relative error - it is the ratio of the mean absolute error to the true value.

$$\delta a = I\ \Delta a\ I / a_{mean}$$

(iii) The relative error expressed in percent is called percentage error.

The error is communicated in different mathematical operations as detailed below:

(i) For $x = (a \pm b)$, $\Delta x = \pm ( \Delta a + \Delta b)$

(ii) For $x = a \times b$ , $\Delta x/x = \pm ( \Delta a/a + \Delta b/b)$

(iii) For $x = a/b$ , $\Delta x/x = \pm ( \Delta a/a + \Delta b/b)$

(iv) For $x= a^n b^m /c^p$ $\Delta x/x = \pm ( n\Delta a/a +m\ \Delta b/b + p\Delta c/c$

## Very short answer type questions, (1 mark question)

Q1. State one law that holds good in all natural processes.

Ans. One such laws is the Newton's gravitation law, According to this law everybody in this nature are attracts with other body with a force of attraction which is directly proportional to the product of their masses and inversely proportionally To the square of the distance between them.

Q2: Among which type of elementary particles does the electromagnetic force act?

Ans : Electromagnetic force acts between on all electrically charged particles.

Q3. Name the forces having the longest and shortest range of operation.

Ans : longest range force is gravitational force and nuclear force is shortest range force.

Q4. If 'slap' times speed equals power, what will be the dimensional equation for 'slap'?

Ans . Slap x speed = power

Or slap = power/speed = $[MLT^{-2}]$

Q5. If the units of force and length each are doubled, then how many times the unit of energy would be affected?

Ans : Energy = Work done = Force x length

So when the units are doubled, then the unit of energy will increase four times.

Q6. Can a quantity has dimensions but still has no units?

Ans : No, a quantity having dimension must have some units of its measurement.

Q7. Justify L +L = L and L – L =L.

Ans: When we add or subtract a length from length we get length, So L +L =L AND L – L =L, justify.

Q8. Can there be a physical quantity that has no unit and no dimensions?

Ans : yes, like strain.

Q9. Given relative error in the measurement of length is 0.02, what is the percentage error?

Ans: percentage error = 2 %

Q10. If g is the acceleration due to gravity and λ is wavelength, then which physical quantity does represented by $\sqrt{g\lambda}$.

Ans. Speed or velocity.

## Short answer type questions (2 marks)

Q1.If heat dissipated in a resistance can be determined from the relation:

$H = I^2Rt$ joule , If the maximum error in the measurement of current, resistance and time are 2% ,1% , and 1% respectively, What would be the maximum error in the dissipated heat?

Ans: % error in heat dissipated is ±6 %.

Q2. Name any three physical quantities having the same dimensions and also give their dimensions.

Ans : Any group of physical quantities, like work , energy and torque and their dimensions [ $ML^2 T^{-2}$].

Q3. In Van der Wall's equation $( P + a/V^2)( V – b ) = RT$, Determine the dimensions of a and b.

Ans : [a] = [$ML^5 T^{-2}$] and [b] = [ $M^0L^3T^0$].

Q4. Give the limitations of dimensional analysis.

Ans ……………………………

Q5. If $X= a+ bt^2$ , where X is in meter and t is in second . find the unit of a and b?

Ans : unit of a is meter and unit of b is m/sec$^{2.}$

Q6. What is meant by significant figures ? State the rules for counting the number of significant figures in a measured quantity?

Ans. …………………………

Q7. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual relative errors.

Ans : For $x = a/b$ , $\Delta x/x = \pm ( \Delta a/a + \Delta b/b)$

Q8. Deduce the dimensional formulae for the following physical quantities.

A) Gravitational constant.

B) Power

C) coefficient of viscosity

D) Surface tension.

Ans: (A) gravitational constant = $[M^{-1} L^3 T^{-2}]$,

B) Power = $[ML^2T^{-3}]$

C) Coefficient of viscosity = $[ ML^{-1} T^{-1}]$

D) Surface tension = $[ ML^0T^{-2}]$

Q9. Name the four basic forces in nature. Arrange them in the order of their increasing strengths.

Ans : (i) Gravitational force (ii) Electromagnetic force

(iii) nuclear force (iv) Weak force

The relative strengths of these forces are

$$F_g : F_w : F_e : F_s = 1 : 10^{25} : 10^{36} : 10^{38} .$$

Q10. Convert 1 Newton force in to Dyne.

Ans : $1N = 10^5$ Dyne.

## Short answer type questions (3marks)

Q1. If E,M,J and G respectively denote energy, mass, angular momentum and gravitational constant, Calculate the dimensions of $EJ^2/M^5G^2$

Q2. The frequency $\nu$ of vibration of stretched string depends on its length L its mass per unit length m and the tension T in the string obtain dimensionally an expression for frequency $\nu$.

Q3. What is meant by significant figures .State the rules for counting the number of significant figures in a measured quantity?

Q4. A physical quantity X is given by $X = A^2B^3/C\sqrt{D}$ , If the percentage errors of measurement in A,B,C and D are 4%,2%,3% and 1% respectively, then calculate the % error in X.

Q5. If two resistors of resistance $R_1=(4 \pm 0.5)\Omega$ and $R_2=(16 \pm 0.5)\Omega$ are connected (1) In series and (2) Parallel . Find the equivalent resistance in each case with limits of % error.

Q6. The length of a rod measured in an experiment was found to be 2.48m, 2.46, 2.50m and 2.48m and 2.49m, Find the average length , the absolute error in each observation and % error.

Q7. A famous relation in physics relates moving mass m to the rest mass $m_0$ of a particle in terms of its speed v and the speed of the light c. A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = m_o / (1 - v^2)^{1/2}$$

Guess where to put the missing c.

Q8. A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 4.2 $kgm^2s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the units of time is ϒ sec. show that a calorie has a magnitude 4.2 $\alpha^{-1} \beta^{-2} \Upsilon^{2}$ in terms of the new units.

Q9. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively, what are the dimensions of Y in MKS system?

Q10. In an experiment, on the measurement of g using a simple pendulum the time period was measured with an accuracy of 0.2 % while the length was measured with accuracy of 0.5%. Calculate the percentage error in the value of g.

## Long answer question ( 5 marks )

Q1. Explain:

(i) Absolute error (iii) Mean absolute error
(ii) Relative error (iv) percentage error
(v) Random error

Q2. Convert:

( i) Gravitational constant (G) = 6.67 x $10^{-11}$ $Nm^2\ kg^{-2}$ to $cm^3\ g^{-1}\ s^{-2}$ (ii) The escape velocity v of a body depends on, the acceleration due to gravity 'g' of the planet and the radius R of the planet, Establish dimensionally for relation for the escape velocity.

Q3. Name the four basic forces in nature. Write a brief note of each, hence compare their strengths and ranges.

## HOTs

Q1. What are the dimensions of $^1/u_0\epsilon_0$, where symbols have their usual meaning.

Ans : $[\ M^0\ L^2\ T^{-2}]$

Q2.What is the dimensions of $(^1/_2)\epsilon_0E^2$ , Where E electric field and $\epsilon_0$ permittivity of free space.

Ans : [ $M^1 L^{-1} T^{-2}$ ]

Q3. The pairs of physical quantities that have the same dimensions are:

(a) Reynolds's number and coefficient of friction,
(b) Curie and frequency of a light wave
(c) Latent heat and gravitational potential
(d) Planck's constant and torque.

Ans : (a), (b).

Q4. If L,C,R represent inductance , capacitance and resistance respectively, the combinations having dimensions of frequency are

(a) $^1/\sqrt{CL}$ (b) L/C (c) R/L (d ) R/C

Ans : (a) and (c).

Q5. If the error in radius is 3%, what is error in volume of sphere?

(a) 3 % (b) 27 % (c) 9 % (d) 6 %

Ans : ( c ) 9%.

# KINEMATICS

*rest and Motion are relative terms, nobody can exist in a state of absolute rest or of absolute motion.

*One dimensional motion:- The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.

*Two dimensional motion:- The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.

*Three dimensional motion:- The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

*The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

Displacement ≤ Actual distance

*Speed:- It is rate of change of distance covered by the body with respect to time.

Speed = Distance travelled /time taken

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \to 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

**Velocity =Displacement /Time**

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

**Acceleration = Change in velocity /time taken**

It is a vector quantity, Its SI unit is meter/ sec.$^2$ and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

| For accelerated motion, | For deceleration motion |
|---|---|
| $V = u + at$ | $v = u - at$ |
| $S = ut + {}^1/_2\, at^2$ | $S = ut - {}^1/_2\, at^2$ |
| $V^2 = u^2 + 2as$ | $V^2 = u^2 - 2as$ |
| $Sn = u + {}^a/_2(2n - 1)$ | $Sn = u - {}^a/_2(2n - 1)$ |

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec$^2$.

*Relative Motion:- The rate of change of distance of one object with respect to the other is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A.

*Relative velocity of object A with respect to object B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

When both objects are move in same direction, then the relative velocity of object B with respect to the object A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

When the object B moves in opposite direction of object A .

$$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$$

When $V_A$ and $V_B$ are incident to each other at angle Θ

$$V_{AB} = (V^2_A + V^2_B - 2V_AV_B\cos\Theta)^{1/2}$$

*Scalars :- The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

*Vectors :- The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc.

For examples : Displacement, velocity, acceleration, force , momentum etc.

- **Addition of Vectors :-**
  (i) Only vectors of same nature can be added.
  (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = ( A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan \beta = B \sin\Theta / ( A + B \cos\Theta)$ ,

Where Θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

(iii) Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(iv) Vector addition is associative,

$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + ( \vec{B} + \vec{C})$

(v) R is maximum if Θ = 0 and minimum if Θ = 180 $^0$.

**Subtraction of two vectors :-**

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of B from A = vector addition of A and (-B),

$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (\overrightarrow{-B})$

Where $R = [ A^2 + B^2 + 2AB \cos(180 - \Theta) ]^{1/2}$ and

$\tan\beta = B \sin(180 - \Theta) / [ A + B \cos(180 - \Theta) ]$, Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector subtraction is not commutative $\overrightarrow{(A} - \vec{B}) \neq ( \vec{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - ( \vec{B} - \vec{C})$

Rectangular components of a vector in a plane :- If A makes an angle Θ with x-axis and $A_x$ and $B_y$ be the rectangular components of A along X-axis and Y- axis respectively, then

$$\vec{A} = \vec{A}_x + \vec{B}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Here $A_x = A\cos\Theta$ and $A_y = A\sin\Theta$

And $A = (A_x^2 + A_y^2)^{1/2}$

And $\tan\Theta = A_y/A_x$

Dot product or scalar product : - The dot product of two vectors A and B, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A}\,.\,\vec{B} = AB\cos\Theta$$

(i) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

**Cross or Vector product :-**

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.

If Θ is the smaller angle between A and B, then

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad \hat{\jmath} \times \hat{k} = \hat{\imath}$ $\quad \hat{k} \times \hat{\imath} = \hat{\jmath}$
$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad \hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

(iii) If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t ,

  $V = [(u^2 - 2ugt\sin\theta + g^2t^2)]^{1/2}$
- Horizontal range

  $R = u^2 \sin 2\Theta/g$

  For maximum range $\Theta = 45^0$,

  $R_{max} = u^2/g$
- Flight time

  $T = 2u \sin\Theta/g$
- Height

  $H = u^2 \sin^2\Theta/2g$

  For maximum height $\Theta = 90^0$

  $H_{max.} = u^2/2g$

**Very Short answer type questions ( 1 marks )**

Q1. What does the slope of v-t graph indicate ?

Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?

Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.

Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are $h_1$ and $h_2$ respectively what is $h_1/h_2$.

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : $90^{0}$.

Q8. What is the angular velocity of the hour hand of a clock ?

Ans : $W = 2\pi/12 = \pi/6$ rad $h^{-1}$,

Q9. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .

Ans : Null vector .

**Short Answer type question ( 2 marks )**

Q1. Derive an equation for the distance travelled by an uniform acceleration body in $n^{th}$ second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n-1)$

Q2. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s

Ans. Differentiation of the given equation eq. w.r.t. time

We get $a = 18 + 18t$

At $t = 2$ sec.

$a = 54$ m/sec$^2$.

Q3.what is relative velocity in one dimension, if $V_A$ and $V_B$ are the velocities of the body A and B respectively then prove that $V_{AB}=V_A-V_B$?

Ans. Relative Motion:- The rate of change of separation between the two object is called relative velocity. The relative velocity of an object B with respect to the object A when both are in motion is the rate of change of position of object B with respect to the object A .

*Relative velocity of object A with respect to object B

$V_{AB} = V_A - V_B$

When both objects are moving in same direction , then the relative velocity of object B with respect to the object A

$V_{BA} = V_B - V_A$

Q4. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$R = u^2 \sin 2\Theta/g$$

For maximum range $\Theta = 45^0$,

$$R_{max} = u^2/g$$

and Height

$$H = u^2 \sin^2\Theta/2g$$

For $\Theta = 45^0$

$$H = u^2/4g = 1/4 \text{ of the } R_{max.}$$

Q6. State the parallelogram law of vector addition. Derive an expression for magnitude and direction of resultant of the two vectors.

Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB\cos\Theta)^{1/2}$

And $\tan\beta = B\sin\Theta / (A + B\cos\Theta)$ ,

Where $\Theta$ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And $\beta$ is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q7. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,

Ans. Because bullet follow parabolic trajectory under constant downward acceleration.

Q8. Derive the relation between linear velocity and angular velocity.

Ans : Derive the expression

$$V = r\,\omega$$

Q9. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .

Q10. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?

Ans: we know that

$$H_{max.} = R_{max}/2$$

So $h = R/2$

Or $R = 2h$

## Short answer questions ( 3 marks )

Q1. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q2. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles $\Theta_1$ and $\Theta_2$ projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0.}$

Q3. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.

Q4: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

(i) Zero

Q5. Two vectors **A** and **B** are inclined to each other at an angle Θ. Using triangle law of vector addition, find the magnitude and direction of their resultant.

Q6. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Q7. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Q8. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. ( g = 9.8 m/sec$^2$)

Q9. Find the value of λ so that the vector $\vec{A} = 2\hat{i} + \lambda\,\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each.

Q10. Show that a given gun will shoot three times as high when elevated at angle of $60^0$ as when fired at angle of $30^0$ but will carry the same distance on a horizontal plane.

Long answer question ( 5 marks)

Q1. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.

Q2. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q3. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \le |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q4. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q5. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

## HOTS

Q1. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : $60^0$

Q2. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q3. A cricket ball is hit at $45^0$ to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $uCos45^0$ is present on highest point.)

Q4. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q5. A projectile can have the same range R for two angles of projection. If $t_1$ and $t_2$ be the time of flight in the two cases, then prove that $t_1t_2 = 2R/g$

**ans** : for equal range the particle should either be projected at an angle Θ and ( 90 - Θ) ,

then $t_1 = 2u\ Sin\Theta/g$

$t_2 = 2u\ Sin(90 - \Theta)/g = 2u\ Cos\Theta/g$

$t_1t_2 = 2R/g$ .