

SECTION A (10 questions each carrying 1 mark)

- Find the value of i) $\tan 15^\circ$ ii) $\sin\left(\frac{-19\pi}{3}\right)$ iii) $\cot\left(-\frac{15\pi}{4}\right)$ iv) $\cos 600^\circ$ v) $\sin \frac{31\pi}{3}$
vi) $\cos(-1710^\circ)$ vii) $\operatorname{cosec}(-1410^\circ)$ viii) $\sin 75^\circ \cos 15^\circ - \cos 75^\circ \sin 15^\circ$ ix) $\tan \frac{\pi}{8}$
- Find the general solution of i) $\sin 3x = 0$ ii) $\cos 3x = 0$ iii) $\sin \theta = \frac{\sqrt{3}}{2}$
- i) A wheel makes 240 revolutions in one minute. Through how many radians does it turn in one second?
ii) In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
- Express $\frac{3-i}{5+6i}$ in standard form.
- Solve $x^2 - 2x + \frac{3}{2} = 0$.
- Solve i) $4x + 3 < 6x + 7$ ii) $-5 \leq \frac{2-3x}{4} \leq 9$ for real x iii) $(-2 - \frac{x}{4}) \geq 2(x+3)$
- Find the multiplicative inverse of $2 - 3i$
- Find the value of 'x' and 'y' if $(1-i)x + (1+i)y = 1 - 3i$
- Simplify i) $i^{10} + i^{11} + i^{12} + i^{13}$ ii) $i^{-39} + i^{-38} + i^{-37} + i^{-36}$.
- What is meant by inductive thinking? (induction means the generalisation from particular cases or facts)

SECTION B (12 questions each carrying 4 marks)

- i) Prove that $\frac{\sin 3x + \sin 5x + \sin 7x + \sin 9x}{\cos 3x + \cos 5x + \cos 7x + \cos 9x} = \tan 6x$
ii) Prove that $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$
iii) Prove that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \frac{x-y}{2}$
- i) Prove that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$
ii) If $\tan A = k \tan B$, show that $\sin(A+B) = \frac{k+1}{k-1} \sin(A-B)$
- i) Prove that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$
ii) Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$
- i) Find the general solution of $\cos 3x + \cos x - \cos 2x = 0$
ii) Solve $2 \cos^2 x + 3 \sin x = 0$
- In any triangle ABC prove the following: i) $\frac{b^2 - c^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$ ii) $a(b \cos C - c \cos B) = b^2 - c^2$
- i) Prove using PMI that $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$ For $n \in N$
ii) $\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$
iii) $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$
iv) $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{n+1}$
- i) Prove by P.M.I that $1.2 + 2.2^2 + 3.2^3 + \dots + n.2^n = (n-1)2^{n+1} + 2$
ii) Prove using PMI that $x^{2n} - y^{2n}$ is divisible by $x + y$
iii) Show that $n(n+1)(n+5)$ is divisible by 3.

18. i) If $(x + iy)^{\frac{1}{3}} = u + iv$, then Prove that $\frac{x}{u} + \frac{y}{v} = 4(u^2 - v^2)$

ii) If $x - iy = \sqrt{\frac{a - ib}{c - id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

iii) If $a + ib = \frac{c+i}{c-i}$, show that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$

19. Find the square root of i) $-7 + 24i$ ii)

20. A) Convert into polar form i) $\frac{1+3i}{1-2i}$ ii) $\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ iii) $\frac{-16}{1+i\sqrt{3}}$

B) Express in the modulus amplitude form i) $3 - \sqrt{3}i$ ii) $(1-i)^4$

21. Find the conjugate and additive inverse of $\frac{Z_1 Z_2 - Z_3}{Z_1 + Z_3}$ when $z_1 = 1 - 2i$, $z_2 = -1 + i$ and $z_3 = 1 + i$

22. Solve the following system of inequalities and represent the solution on the number line
 $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x + 47$

SECTION C (6 questions each carrying 6 marks)

23. i) Derive the formula for $\cos(x + y)$ and $\cos(x - y)$ using the unit circle.

ii) If $\sin x = \frac{-3}{5}$ and x in quadrant III find the values of $\sin x/2$, $\cos x/2$ and $\tan x/2$

iii) Prove that: $\frac{\sin x \sin 2x + \sin 3x \sin 6x}{\sin x \cos 2x + \sin 3x \cos 6x} = \tan 5x$

24. i) If $\tan((\alpha + \theta) = n \tan(\alpha - \theta)$, Show that $(n + 1)\sin 2\theta = (n - 1)\sin 2\alpha$

ii) Prove that $\cos^2 x + \cos^2(x + 120^\circ) + \cos^2(x - 120^\circ) = 3/2$

iii) In any triangle ABC prove the following:

a) $\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$

b) $(b - c)\cot \frac{A}{2} + (c - a)\cot \frac{B}{2} + (a - b)\cot \frac{C}{2} = 0$

25. Prove using PMI for $n \in N$, $3^{2n+2} - 8n - 9$ is divisible by 8

26. i) Prove using PMI for $n \in N$, $1.3 + 3.5 + 5.7 + \dots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$.

ii) $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

27. Solve graphically $x + 2y \leq 10$, $x + y \geq 1$, $x - y < 0$, $x \geq 0$, $y \geq 0$

28. Find real A if $\frac{3 + 2i \sin A}{1 - 2i \sin A}$ is a) purely real, b) purely imaginary

29. If a and b are two complex numbers such that $|b| = 1$ find the value of $\left| \frac{b - a}{1 - \bar{a}b} \right|$